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Proof Theoretic Study of Embeddings between Substractural Logics

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1 Introduction

In the middle of 1930s, G. Gentzen introduced natural deduction systems and sequent calculi for classical logic and intuitionistic logic. Usually, sequent calculi have three structural rules, i.e. contraction, weakening and exchange rules, which have been regarded simply as auxiliary rules. But, from the middle of 1980s, logics which are obtained from classical logic LK and intuitionistic logic LJ by deleting some or all of these structural rules have attracted much attention. They include e.g. relevant logics, which have no weakening rule in general, the BCK logic which has no contraction rule, linear logic by Girard (1987) which has neither weakening rule nor contraction rule, and Lambek calculus by Lambek (1958), which has none of structural rules. Nowadays, these logics are said to be *substructural logics*.

In proof-theoretic study of substructural logics, the cut elimination theorem plays a key role. Many of important results like the decidability and the interpolation property come out as consequences of the cut elimination theorem. It is shown that in most of basic substructural logics, the cut elimination theorem holds (see e.g.[1]).

Main topic of the present thesis is to show embedding relations among substructural logics. We will consider two kinds of embeddings. The first one is Kolmogolv-style translation. A well-known result says that by Kolmogolv-style translation LK is embedded into LJ. We extend it and roughly speaking, we show that any basic substructural logic with the law of double negation can be embedded into a substructural logic without it as long as they have the same structural rules.

The second one is due to the method by Kiriyama and Ono. They proved in [3] that LJ can be embedded into FLec, i.e. LJ minus weakening rule. This result can be generalized and we show that any basic substructural logic with weakening rule can be embedded into a substructural logic without it.

2 Basic substructural logics

Substructural logic is formalized as a system obtained from Gentzen's sequent calculus LK and LJ by removing some or all of structural rules, i.e. exchange rule, weakening rule and contraction rule. Let FL be Full Lambek Calculus, which is obtained from LJ by deleting all of structural rules. The logic FL is the weakest basic substructural logic. By subscripts e, w, c, we mean exchange rule, weakening rule and contraction rule, respectively. By adding these subscripts to FL, we denote the logic obtained from FL by adding corresponding structural rules. Thus, FLe and FLew mean FL with exchange, and FL with both exchange and weakening rules. Moreover, by adding C at the top, we mean the logic obtained by adding the law of double negation. Thus, CFLew means the logic obtained from FLew by adding the law of double negation, for example.

3 Embedding by the Kolmogolv-style translation

The Kolmogorov-style translation is defined inductively as follows:

$$\begin{array}{ll} T(\bot) := \neg\neg\bot & T(\top) := \top \\ T(f) := f & T(t) := \neg\neg t \\ T(p) := \neg\neg p & T(A \wedge B) := \neg\neg(T(A) \wedge T(B)) \\ T(A \vee B) := \neg(\neg T(A) \wedge \neg T(B)) & T(A \supset B) := \neg\neg(T(A) \supset T(B)) \\ T(A + B) := \neg(\neg T(A) * \neg T(B)) & T(A * B) := \neg\neg(T(A) * T(B)) \end{array}$$

It is well known that $LJ \vdash T(\Gamma) \to T(D) \Leftrightarrow LK \vdash \Gamma \to D$. The result can be generalized as follows.

Theorem 3.1 *CFLe can be embedded into FLe. That is,*

$$FLe \vdash T(\Gamma) \to T(D) \Leftrightarrow CFLe \vdash \Gamma \to D.$$

This holds also between CFLew and FLew, and between CFLec and FLec.

4 Embedding by Kiriyama-Ono translation

Undecidability of the predicate logic FLec is shown in §3 of the paper [3] by using the fact that the predicate LJ can be embedded into the predicate FLec. Let us call the translation used there, Kiriyama-Ono translation. By using the similar idea, we show that the propositional FLew can be embedded into FLe. This time, we will use a simpler translation, in which constants are dispence with. The translation and the embedding relation are given below.

We will take an arbitrary sequent $\Gamma \to A$ and fix it through the following argument. Let $\mathcal{P}$ be the set of all propositional symbols appearing in $\Gamma \to A$. We difine a formula T by

$$T \stackrel{\text{def}}{=} \bigwedge_{p \in \mathcal{P}} (P \supset P)$$

Let $\Phi (= \Phi_{\mathcal{P}})$ be the set of all formulas which are construct by propositional symbols in $\mathcal{P}$. For each formula $B \in \Phi$, define formulas B^* as follows:

(1) B is $atomic(i.e\ B = P) \Rightarrow B^* = P \wedge T (= B \wedge T)$
(2) $B = C \circ D\ (\circ \in \wedge, \vee, \supset) \Rightarrow B^* = (C^* \circ D^*) \wedge T$
(3) $B = \neg C \Rightarrow B^* = (\neg C^*) \wedge T$

Theorem 4.1

$$FLew \vdash \Gamma \to A \Leftrightarrow FLe \vdash \Gamma^* \to A^*$$

We note that Theorem 4.1 holds also predicate logics.

5 Embedding LK into CFLec

By modifying Kiriyama-Ono translation and using the constant f, we can show the following translation of LK into CFLec. The translation and the embedding relation are given as follows.

We will take an arbitrary sequent $\Gamma \to \Delta$ and fix it through the following argument. Let $\mathcal{P}$ be the set of all propositional symbols appearing in $\Gamma \to \Delta$. We difine a formula T by

$$T \stackrel{\text{def}}{=} \bigwedge_{p \in \mathcal{P}} (P \supset P) \wedge (f \supset f)$$

Let $\Phi (= \Phi_{\mathcal{P}})$ be the set of all formulas which are construct by propositional symbols in $\mathcal{P}$. For each formula $B \in \Phi$, define formulas $|B|^+$, $|B|^-$ as follows:

(1) B is atomic (i.e. B is either P or f)$\Rightarrow |B|^- = B \wedge T, |B|^+ = B \vee f$
(2) $B = C \circ D\ (\circ \in \wedge, \vee, *) \Rightarrow |B|^- = (|C|^- \circ |D|^-) \wedge T, |B|^+ = (|C|^+ \circ |D|^+) \vee f$
(3) $B = C \supset D \Rightarrow |B|^- = (|C|^- \supset |D|^-) \wedge T, |B|^+ = (|C|^+ \supset |D|^+) \vee f$

Theorem 5.1

$$LK \vdash \Gamma \to \Theta \Leftrightarrow CFLec \vdash |\Gamma|^- \to |\Theta|^+$$

We note that the embedding relation hold also between CFLew and CFLe.

6 Conclusion

In the present thesis, we show many embedding results by using either Kolmogorov-style translation or modifications of Kiriyama-Ono translation.

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