Abstract

Keywords: infinite string rewriting, recursive path orders

Term rewriting is a well-known computational model which is Turing equivalent to lambda calculus, and has many applications e.g. automated theorem proving and functional programming, etc [8]. A significant property of term rewriting systems, named as convergence consisting of termination and confluence should be ensured in these fields. Termination ensures the finiteness of computation steps and confluence guarantees the uniqueness of result when different computation sequences arise. As a famous undecidable problem, the word problem will be totally unsolvable without convergence.

String Rewriting and Convergence. This thesis talks about a formalization of string rewrite systems with infinite rules and provides a decidable technique for termination proving. String rewrite systems are ordered equations of strings. Convergence means that different rewrite sequences from the same string can lead to same result finally. We hope that this property holds for all strings since convergence make it possible to solve word problems.

Word Problems and Solutions. A major purpose of string rewrite systems is to solve word problems for finitely presented monoids. The word problem for the equations E is described as follows:

Instance: Two strings u, v and axioms E

Question: Does $u = v$ hold under E?

Although the word problem is undecidable, the problem can still be solved by finding a convergent system R such that $\leftrightarrow_R^* = \leftrightarrow_E^*$ holds.

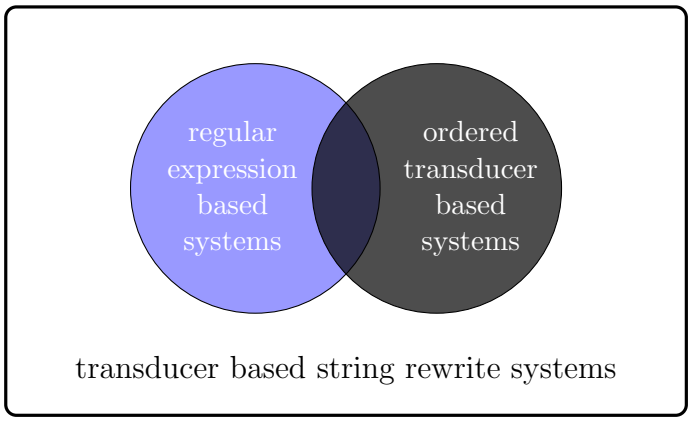
Automated Equational Reasoning. The formal methods to prove equation equivalence presented by word problems have been under research for decades. The technique [7] influenced by the standard completion procedure [6] and the extended version called unfailing completion [1] is used in most modern automated theorem provers.

However, some results [4, 3, 5, 9] in this field show that a finite convergent system of given equations does not always exist, which brings us difficulties when proving that equation equivalence does not hold.

Motivation and Approach. The above consequence gives us the question: If we are given an infinite string rewrite system claimed convergent, does it really have the convergence property?

This thesis uses finite state transducers to formalize these string rewrite systems which especially have infinite rules to check the confluence property. Transducers are a kind of abstract machine that recognizes pairs of strings. We only focus on termination, one of the necessary properties of convergence. Recursive path orders [2] provide a technique to prove termination. Instead of comparing the infinite many rules of given string rewrite systems, we develop a procedure to ensure that the language accepting by a transducer remains true under this order.

Unfortunately, the problem how to check an arbitrary finite transducer remains open. We develop two restricted classes of finite transducers indicated as follows.



Each of them can do string rewriting and check termination with the help of recursive path orders.

Contributions. The main contributions of the thesis are listed as:

- the regular expression based string rewriting,
- the recursive path order over regular expressions,
- the ordered transducer based string rewriting, and
- the recursive path order over ordered transducers.

References

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