

Title	Adaptive Rescheduling of Energy Consumption Based on User Preferences for the Future Smart Grid
Author(s)	Charoen, Prasertsak; Javaid, Saher; Lim, Yuto; Tan, Yasuo; Charoenlarnopparut, Chalie
Citation	2018 IEEE Innovative Smart Grid Technologies - Asia (ISGT Asia): 36-41
Issue Date	2018
Type	Conference Paper
Text version	author
URL	<a href="http://hdl.handle.net/10119/16127">http://hdl.handle.net/10119/16127</a>
Rights	This is the author's version of the work. Copyright (C) 2018 IEEE. 2018 IEEE Innovative Smart Grid Technologies - Asia (ISGT Asia), 2018, pp.36-41. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.
Description	

# Adaptive Rescheduling of Energy Consumption Based on User Preferences for the Future Smart Grid

Prasertsak Charoen, Saher Javaid,  
Yuto Lim and Yasuo Tan

School of Information Science  
Japan Advanced Institute of Science and Technology City  
Ishikawa, Japan

Chalie Charoenlarnnopparut  
School of ICT

Sirindhorn International Institute of Technology  
Pathum Thani, Thailand

**Abstract**— In the future smart grid, Demand Side Management (DSM) will be implemented to facilitate utility companies and consumers in order to achieve mutual benefits such as minimizing total energy cost and reducing consumption peaks. The optimal energy consumption scheduling is calculated based on user preferences in advance, e.g., day-ahead schedules. Most of the prior works in the literature have reported good results under the assumption that users are committed to the optimal schedules and do not deviate from their preferences, which may not be true in practice. In this paper, we consider the implications of allowing users to change their preferences and request for new schedules at any time. Based on user constraints, we propose single and multi-user adaptive rescheduling algorithms. The algorithms reschedule deviating energy consumption optimally thus reducing the total energy cost. Simulation results show that the total energy cost of the community can be reduced by as much as 11.4% in specific scenarios.

**Index Terms**-- Demand side management; energy consumption scheduling; game theory; smart grid; user preference

## I. INTRODUCTION

The concept of DSM in the Smart Grid is related to all activities pertaining to the alteration of the consumers' demand profiles in order to closely match the supply. DSM can be employed to facilitate the integration of distributed generation (DG) which can provide a significant reduction in both energy generation and transmission. Also, DSM aims to support a transition of high penetration rate in renewable energy sources (RES) and reduction of carbon emission [1]-[4]. The main idea is that the users deploy Home Energy Management System (HEMS) solutions in their homes and participate in DSM programs of the utility company by using smart meters. The HEMS can be used to automatically schedule and manage the load consumption based on real-time pricing provided by the utility.

Various designs and mechanisms have been proposed in the smart grid literature based on HEMS to manage the flexibility offered by DSM. Among them, a game theoretic framework is a promising mathematical tool to analyze the interaction between utility companies and consumers [5]-[10]. Those interactions can be viewed from a technical point of view as

well as a social viewpoint, such as interactions between smart meters and utility company control centers or service agreements between the utility companies and their customers. For instance, game theory can be applied in designing DSM models for developing scheduling of flexible appliances, energy pricing, and billing mechanisms.

The work by Mohsenian-Rad *et al.* [6] originally formulated the energy consumption of houses in a community as a game. In this game, each user has specific preferences regarding the use of flexible appliances and seeks to find an energy consumption schedule in advance that optimizes a payoff function which is mainly a function of the energy cost. The utility company distributes the total energy cost to all users in the form of electricity bills. The cost function depends on the total users' load. This implies that a change in the load of one user would impact the total cost, which in turn impacts the individual bills of the users. The author also proposed a daily billing mechanism which calculates the electricity bill for each user proportionally to the energy consumption of the entire day. Later, in [8], [9] proposed hourly billing mechanisms, where hourly costs are shared among users respectively to their consumption. This improves the system fairness level in terms of each user's contribution in the system.

One of the major drawbacks in [6]-[9] lies in the rigid assumptions regarding user behaviors. Those works assumed that every user is committed to the optimal schedule assigned ahead of time. The assumption is backed up by the fact that the users do not benefit financially if their power consumption deviates from the optimal schedules. However, this assumption rarely holds true in practice, and consumption deviation can occur at any time after the schedules have been assigned. For example, changes in user preferences is one of the causes that lead to changes in using appliances. Recently, only the work in [10] considered the assumption of schedule violations and further extended the DSM model in [8]. The proposed mechanism fairly distributes the cost of schedule violations across users with deviating consumption while maintaining the same cost for the users that obey the schedules.

In this paper, we relax the assumptions used in [6]-[9] by allowing users to change their preferences at any time,

deviating from their original promises. The revised user preferences may result in different power consumption. Under this new assumption, the conventional models fail to achieve optimality. Our assumptions differ from [10], as we allow users to either increase or decrease energy consumption based on their new preferences. Based on the energy consumption game framework presented in [9], we propose adaptive energy consumption rescheduling algorithms to cope with the deviating users. The objective is to minimize the total energy generation cost by providing options for users to request new schedules.

The rest of this paper is organized as follows: Sec. II introduces the models of the power grid and the energy consumption game. In Sec. III, the details of the adaptive energy consumption rescheduling algorithms are presented. Numerical simulation results and discussion are given in Sec. IV. The conclusion of the paper is drawn in Sec. V.

## II. SYSTEM MODEL

The system model in this paper is based on the energy consumption game in [9] where the interaction of each user is coordinated. We consider a community power grid composed of a set of  $\mathcal{N} = \{1, \dots, N\}$  users that share a single energy source provided by a utility company. Each user is representing a house owning a HEMS coordinated with a smart meter. Each HEMS is capable to schedule appliances and compute its own electricity bill,  $B_n$ , using real-time pricing announced by the utility company. The communication network provides HEMS with two-way communication among users and also to the utility. Fig. 1 shows the community power system and communication networks. Without loss of generality, we assume that each user has a single flexible appliance such as a PHEV or a washing machine. A set of scheduling time period  $\mathcal{H} = \{1, \dots, H\}$  is divided into hourly time slots e.g.,  $H = 24$  for a day.

### A. Energy generation cost functions

The utility company provides energy to the community and responsible for generating and distributing electricity. The cost of energy can be calculated at each hour  $h \in \mathcal{H}$ . With the assumption that marginal costs increase with demand, we can assume that the cost function  $C_h(\cdot)$  is *increasing* and *strictly convex* [6]. In general, this cost function can be the actual cost of electricity production or an artificial cost signal that sent to users' HEMS for computing energy consumption optimization. In this paper, we assume the cost function  $C_h(L_h)$  as a quadratic function of the total load  $L_h = \sum_{n=1}^N x_n^h$ , where  $x_n^h$  denotes loads at period  $h$  of user  $n \in \mathcal{N}$ :

$$C_h(L_h) = a_h L_h^2 + b_h L_h + c_h \quad (1)$$

where  $a_h > 0$ ,  $b_h \geq 0$  and  $c_h \geq 0$  are the coefficients of the cost function. The cost depends on the load in each period  $h$  as at peak times it is more expensive to produce energy.

### B. Energy generation cost functions

Each user  $n$  has a flexible appliance whose operation time can be scheduled. Users can set their time preferences for the

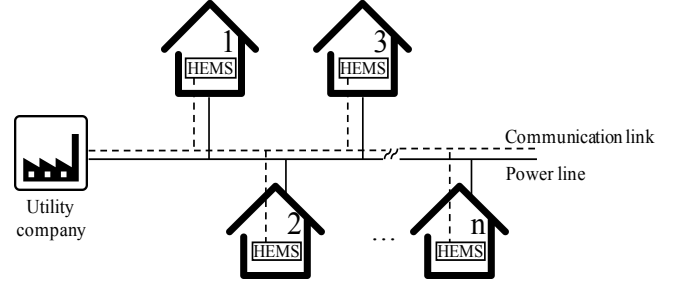


Fig. 1 Overview of the community system model

appliances to operate within a time frame  $[\alpha_n, \beta_n]$ , where  $1 \leq \alpha_n \leq \beta_n \leq H$ . The user's HEMS needs to fulfill the energy requirement for the appliance to finish its task, denoted as

$$E_n = \sum_{h=\alpha_n}^{\beta_n} x_n^h \quad (2)$$

and

$$x_n^{h,min} < x_n^h < x_n^{h,max} \quad (3)$$

where  $x_n^h = 0$  when  $h$  is not in the preferred user  $n$ 's period  $\mathcal{H}_n$ .  $x_n^{h,min}$  and  $x_n^{h,max}$  denote the minimum and maximum energy level in each hour  $h$  respectively. Thus, the energy consumption vector for user  $n$  can be expressed for the whole schedule periods as

$$\mathbf{x}_n = [x_n^1, x_n^2, \dots, x_n^H]. \quad (4)$$

Note that the total energy consumption of appliance does not change during scheduling, only the operation time (i.e., ON/OFF) is shifted. The set of feasible energy consumption for user  $n$  respects the constraints given in (2) and (3) and can be defined as

$$\mathcal{X}_n = \left\{ \mathbf{x}_n \mid \sum_{h=\alpha_n}^{\beta_n} x_n^h = E_n; x_n^h = 0, \forall h \in \mathcal{H} \setminus \mathcal{H}_n; x_n^{h,min} < x_n^h < x_n^{h,max} \forall h \in \mathcal{H}_n \right\} \quad (5)$$

### C. Energy consumption game

The utility company is responsible for designing a billing mechanism and distributing the cost of energy generation to all users participating in the power system. For simplicity, we assume a budget-balance case where the total generation cost is equal to the sum of all users' bills. One of the billing mechanisms is hourly proportional billing where the total energy cost is divided between users at each time period, with respect to the energy they consumed. The bill for user  $n$  is given by

$$B_n = \sum_{h=1}^H \frac{x_n^h}{\sum_{m=1}^N x_m^h} C_h(L_h). \quad (6)$$

The HEMS of each user locally seeks to find the energy consumption scheduling vector  $x_n$  that minimizes the user's electricity bill  $B_n$  by solving the following optimization problem:

$$\min_{x_n \in \mathcal{X}_n} \sum_{h=1}^H b_n^h(x_n; x_{-n}) \quad (7)$$

where  $x_{-n}$  is the energy consumption of all other users except  $n$  and  $\sum_{h=1}^H b_n^h = B_n$ . Since the total cost is affected by the total load of all users, the choice of one user's consumption profile also affects all other users. That is, the electricity bill of user  $n$  does not only depend on the user's consumption but also depends on all other users' consumption in the same hour. Therefore, we can formalize the problem as an energy consumption game among users based on game theory framework:

- Players: Users  $n \in \mathcal{N} = \{1, \dots, N\}$
- Strategies: the energy consumption schedule vector  $x_n \in \mathcal{X}_n$  for each user
- Payoffs: negative billing for each user  $-B_n$

The goal for the utility company is to minimize the cost of electricity generation. To find the optimal energy consumption schedule vectors, at first, each user  $n \in \mathcal{N}$  sets the user preference  $[\alpha_n, \beta_n]$  in HEMS. Then, HEMS initially solves the optimization problem (7) locally as its best response strategy for  $x_n$  by randomly assuming a vector  $x_{-n}$ . The resulting schedule is shared with other users through a broadcast message. Once a user receives the update messages from other users, the HEMS updates its local knowledge of other users' aggregated loads  $x_{-n}$ . Next, HEMS iteratively solves (7) and shares the new schedule again until no user announces an update. In this way, the iterative player best response strategy will converge to the Nash Equilibrium and provide optimal total energy cost ([6], Thm. 3). Fig. 2 illustrates an example of day-ahead scheduling where each user iteratively computes its schedule based on given preferred time period and energy constraints. The resulting optimal scheduled time  $h_n^*$  is shown for each user.

The assumption that every user must commit to the assigned energy consumption schedule is necessary for prior works to achieve the optimal total energy cost. However, in practice, users may want to change their preferences after the schedule is assigned. Without a scheduling algorithm to deal with such users, the prior works fail to achieve an optimal total energy cost and the user's action may lead to increase consumption during peak hours which, in turn, increase the total cost and users' electricity bills. In the next section, adaptive energy consumption rescheduling algorithms are proposed to address the deviating users.

### III. PROPOSED ADAPTIVE ENERGY CONSUMPTION RESCHEDULING ALGORITHMS

Most of the works in the literature assumed that every user would fully commit to the schedules, that is, during the scheduling period (i.e.,  $\mathcal{H} = \{1, \dots, 24\}$ ) no user will change its

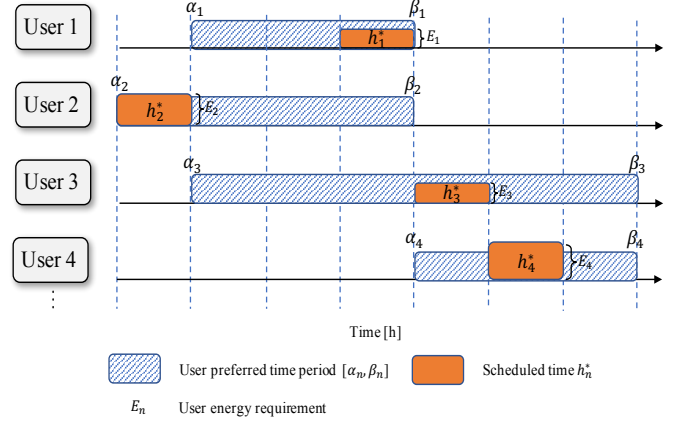


Fig. 2 Day-ahead energy consumption scheduling

preference and the appliance operation will be fulfilled according to the schedules. However, in practical scenarios, some users may not be able to commit to the day-ahead schedules. These deviating users may want to change their preferences. To accommodate such users, a rescheduling algorithm that can reduce total energy cost is required. Based on these assumptions, we proposed adaptive energy consumption rescheduling algorithms.

Let us consider a DSM program that schedules the users' flexible appliances for the next  $H = 24$  hours. At the beginning of the scheduling period, denoted as *Initialization period*, the utility company broadcast the energy cost and billing functions to all user in the community. Each user sets its appliance operation preference in HEMS. Then, each HEMS iteratively takes turn to calculate the optimal energy consumption schedule and shares the results with all other users until all schedules are finalized. Once the day-ahead schedules are determined, the scheduled appliances operation is fulfilled according to the day-ahead schedules. We denote this period as *Operation period*. A flow diagram of the initialization and operation periods with the proposed rescheduling algorithms are shown in Fig. 3. Next, we will explain our rescheduling algorithms for 2 cases depending on user constraints in the following sections.

#### A. Case I: Single user energy consumption rescheduling

In this case, we assume that only the deviating users (that is, users who changed their preference) can alter their schedules while the schedules of all other users remain the same. This is based on the constraints that for other users, once the day-ahead schedules are assigned, they do not want their schedules to be altered. During the operation period, for example at time  $h'_k$ , let us assume user  $k \in \mathcal{N}$  changes its preference from  $[\alpha_k, \beta_k]$  to  $[\alpha'_k, \beta'_k]$  for an appliance scheduled to operate at time  $h_k^*$ , where  $h_k^* > h'_k$ . To find new energy consumption schedule for user  $k$ , the HEMS associated with user  $k$  updates new preference and current aggregated loads information from all other users. Then, the HEMS locally calculates new energy consumption schedule  $x'_k$  constrained by the new preference  $[\alpha'_k, \beta'_k]$  and scheduling period  $\mathcal{H}' = \{h'_k, \dots, H\}$  as

$$\min_{x'_k \in \mathcal{X}'_k} \sum_{h=h'_k}^H b_k^h(x'_k; x_{-k}) \quad (8)$$

where the set of feasible energy consumption for user  $k$  follows the new constraints and is updated as

$$\mathcal{H}'_k = \left\{ x'_k \mid \sum_{h=\alpha'_k}^{\beta'_k} x_k^h = E_k; x_k^h = 0, \forall h \in \mathcal{H}' \setminus \mathcal{H}'_k \right\} \quad (9)$$

$$x_k^{h,min} < x_k^h < x_k^{h,max} \quad \forall h \in \mathcal{H}'_k$$

and  $\mathcal{H}'_k = \{\alpha'_k, \dots, \beta'_k\}$ . The resulting schedule consumption is shared with all other users and the utility for future use. The proposed algorithm provides a possibility for users to change their preferences while reducing the energy cost. Note that only user  $k$ 's schedule is changed while all other users' schedules remain the same. The example of the proposed single user rescheduling is illustrated in Fig. 4 where user 1 changed its preference at time  $h'_1$  from  $[\alpha_1, \beta_1]$  to  $[\alpha'_1, \beta'_1]$ . Then, the new scheduled time is recalculated and changed from  $h_1^*$  to  $h'_1$ , where other users' scheduled time remains the same.

### B. Case II: Multiple user energy consumption rescheduling

In case II, we further allow the rescheduling algorithm to alter schedules of other qualified users within their preferences. That is, after the new schedule for the deviating user is determined, the schedule of the user  $m \in \mathcal{N} \setminus \{k\}$  qualifies for rescheduling if the appliance operation time is scheduled later than the time that user  $k$  requests for rescheduling  $h'_k$ :

$$\{m \mid h_m^* \geq h'_k, \forall m \in \mathcal{N} \setminus \{k\}\} \quad (10)$$

where  $h_m^*$  is the scheduled time of user  $m$ 's appliance. Then, if the preference  $\alpha_m$  is earlier than  $h'_k$ , adjust the preference from  $[\alpha_m, \beta_m]$  to  $[h'_k, \beta_m]$ . Otherwise, use the same preference  $[\alpha_m, \beta_m]$ :

$$[\alpha_m, \beta_m] = \begin{cases} [h'_k, \beta_m], & \alpha_m < h'_k \\ [\alpha_m, \beta_m], & \text{otherwise} \end{cases} \quad (11)$$

Thus, the process continues from the single user rescheduling during operation period, after user  $k$  determined the new schedule and shared with all other users, all users' associated HEMS check the conditions in (10). The user that satisfied (10) updates his preference according to (11) and iteratively reschedules the energy consumption in (8) until all schedules are fixed. The user which does not qualify the condition in (10) only updates the current aggregated load information without rescheduling.

Fig. 5 shows an example of the proposed multiple user rescheduling algorithm. After user 1 changed his preference and determined a new schedule, user 2, 3 and 4 check their conditions in (10). User 2 scheduled time  $h_2^*$  is before the time of rescheduling  $h'_k$ . Thus, we cannot change the schedule of user 2. For user 3 and 4, the condition in (10) is satisfied. User 3 further checks the condition in (11) and adjusts his preference period to  $[h'_1, \beta_3]$ , where user 4 retains his preference period as  $[\alpha_4, \beta_4]$ . Then, user 1, 3 and 4 iteratively recalculate their schedule according to the procedure described above. The final scheduled times are shown for each user as  $h_n^{**}$ .

The advantage of the proposed rescheduling algorithm is that it can adaptively reschedule power consumption of

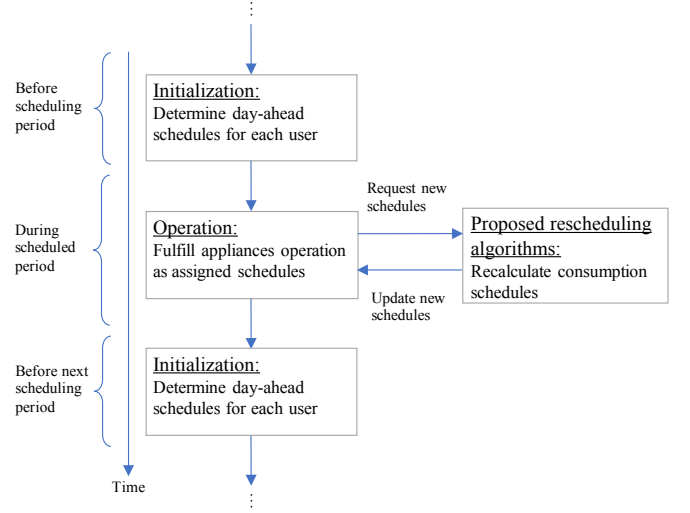


Fig. 3 A diagram of energy consumption scheduling with the proposed adaptive energy consumption rescheduling

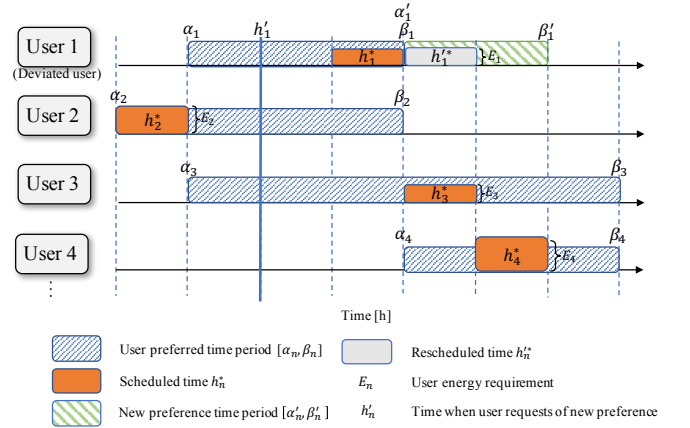


Fig. 4 Single user energy consumption rescheduling

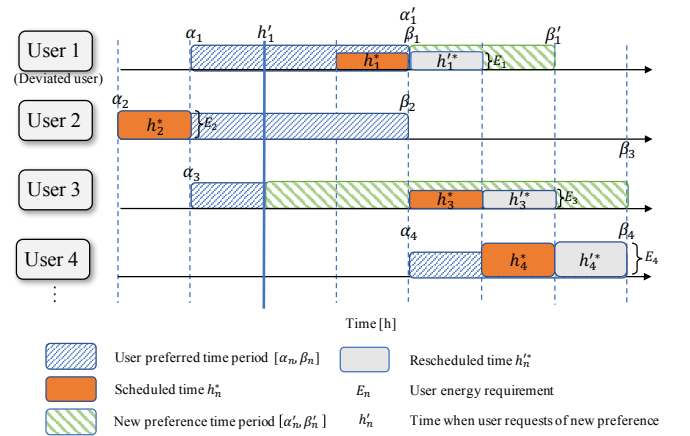


Fig. 5 Multiple user energy consumption rescheduling

deviating users and reflects practical scenarios where users' commitments cannot be guaranteed.

#### IV. RESULTS AND DISCUSSION

We numerically present and assess the performance of the proposed algorithms. In our considered system, we have 20 users scheduling for the next  $H = 24$  hours. The user preference values  $\alpha_n$  and  $\beta_n$  are randomly generated for each user. The energy cost functions are  $C_h(L_h) = 0.01L_h^2 + 2L_h$  for  $h < 12$  and  $C_h(L_h) = 0.03L_h^2 + L_h$  for  $h \geq 12$  as in [9]. Each user has a single flexible appliance to be scheduled, such as a washing machine, dishwasher, etc. The total energy  $E_n$  required for each appliance is randomly selected between 5 and 40kW. We randomly selected users that changed their preferences from the day-ahead schedules during 8 a.m. to 5 p.m. The deviated preferences values  $\alpha'_n$  and  $\beta'_n$  are randomly assigned at a later point in time. To establish a *base case*, we consider the case where no rescheduling happens. In this case, the deviating users randomly consume energy within their deviated preferences periods. Fig. 6 demonstrates a comparison of aggregated consumption profiles assigned by the conventional day-ahead scheduling and consumption profile of deviating users without rescheduling. The corresponding aggregated electricity cost in both cases are shown in Fig. 7. The simulation results show that the total energy cost when some users deviated their consumption is increased by 5.9% from 1034.7 cents to 1095.8 cents. This is because the deviating users consumed energy without taking into consideration the loads of other users in the community, causing higher peaks and energy generation cost.

Using these settings, we compared the performance of the proposed rescheduling algorithms against the base case. Fig. 8 shows comparison of 3 aggregated consumption profiles of 20 users in the cases of no rescheduling, single user rescheduling, and multiple user rescheduling. The corresponding aggregated energy cost of the algorithms are shown in Fig. 9. The total energy cost of the proposed single user rescheduling algorithm is 1009.7 cents, which is 7.8% reduction compared to the case of no rescheduling (1095.8 cents). The proposed multiple user rescheduling further reduced the total energy cost to 1000.5 cents, which is 0.9% reduction compared to the single user rescheduling case (8.7% reduction compared to the base case of no rescheduling). The single user rescheduling algorithm uses new preferences set by the deviating users and recalculates energy consumption schedules considering the loads of other users in the community. Thus, the resulting schedules are assigned in such a way that high demand is avoided during the same hour and lower the total energy cost. The multiple user rescheduling algorithm also recalculates the loads of other qualified users and further flattens the aggregated load profile, resulting in greater energy cost reduction.

In addition, we compared the total energy cost of our proposed reschedules algorithms to the case of no rescheduling by varying the number of deviating users from 1 to 10 (out of 20). Fig. 10 shows the normalized total energy cost for 3 cases; no rescheduling, single user rescheduling, and multiple user rescheduling. The results show that when the number of deviating users increase, the greater the cost reduction that the

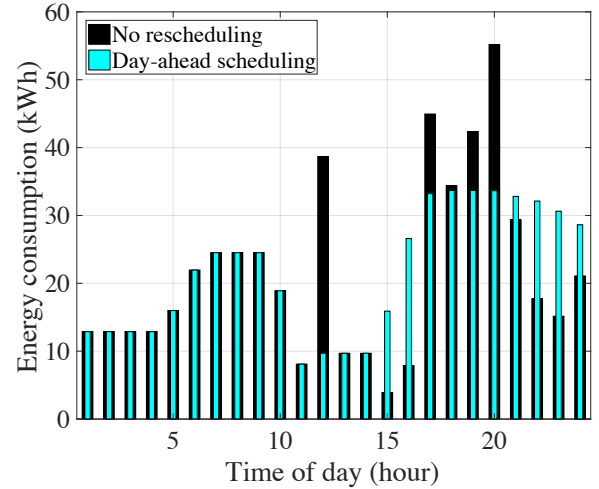


Fig. 6 Comparison of aggregated energy consumption of 20 users for day-ahead scheduling and no scheduling when user deviated consumption.

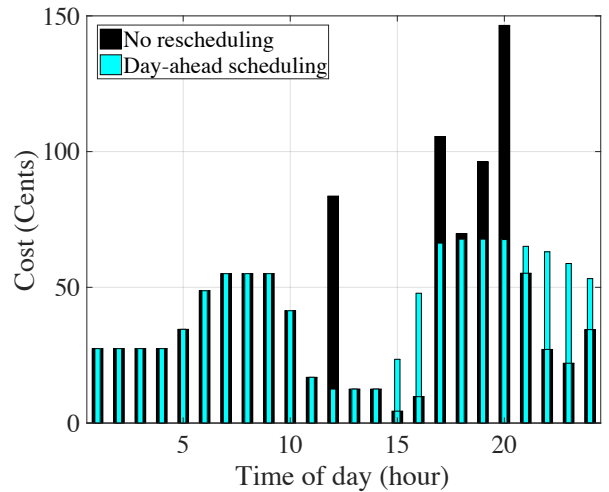


Fig. 7 Comparison of the corresponding aggregated energy cost of 20 users for day-ahead scheduling and no scheduling when user deviated consumption.

proposed rescheduling algorithms achieved. When 50% of users deviate their preferences, up to 11.4% reduction in cost is achieved by the proposed multiple user rescheduling. The marginal energy cost reduction between the single and multiple user rescheduling indicates that rescheduling the loads of other qualified users can further help reduce the total energy cost.

These results demonstrate the effectiveness of the proposed algorithms when it comes to avoid consumption peaks and thus flattening the total consumption profiles, resulting in lower total energy costs.

#### V. CONCLUSION

In this paper, we proposed adaptive energy consumption rescheduling algorithms for DSM programs. We considered the



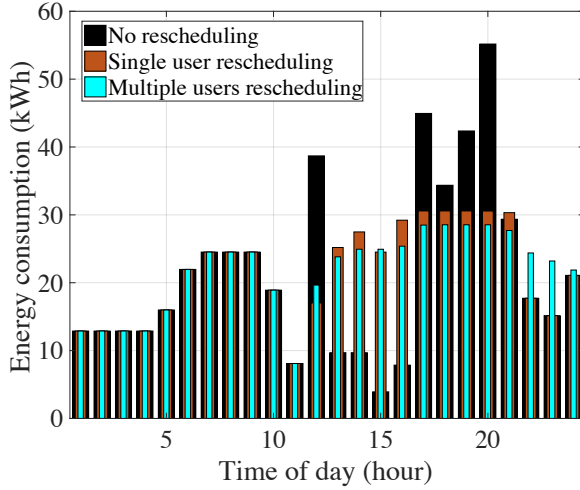


Fig. 8 Comparison of aggregated energy consumption of 20 users for no rescheduling, single user rescheduling and multiple user rescheduling

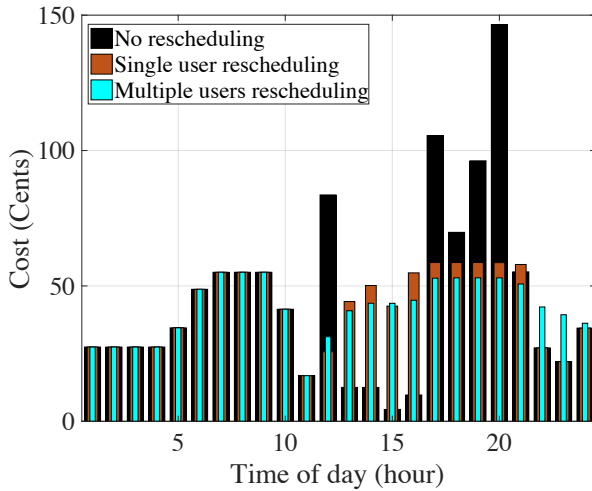


Fig. 9 Comparison of the corresponding aggregated energy cost of 20 users for no rescheduling, single user rescheduling and multiple user rescheduling

implications of allowing users to deviate from their original preferences and request new energy consumption schedules that are different from the assigned day-ahead optimal schedules. Both a single user and a multiple user energy consumption rescheduling algorithm were proposed. The single user rescheduling algorithm recalculates only the deviating user's schedule while the multiple user rescheduling algorithm further recalculates the qualified users' schedules. Simulation results confirmed that the proposed rescheduling algorithms reduce the total energy cost of the community from the conventional day-ahead scheduling by adaptively rescheduling user loads in response to their changing preferences. Furthermore, as the number of deviating users increases, there are more opportunities to reduce the energy cost. By being able to address changes in user preferences, the proposed algorithms will further help DSM programs to achieve a practical deployment in the future smart grid.

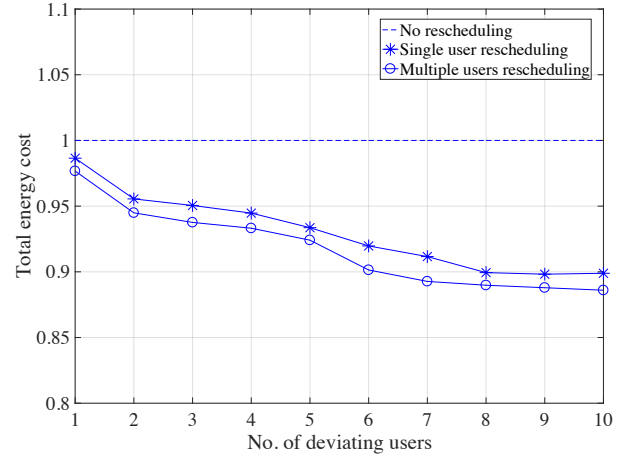


Fig. 10 Comparison of the aggregated energy cost of 20 users versus the number of deviating users for no rescheduling, single user rescheduling and multiple users rescheduling

## REFERENCES

- [1] A. Ipakchi and F. Albuyeh, "Grid of the future," *IEEE power and energy magazine*, vol. 7, no. 2, pp. 52-62, 2009.
- [2] J. S. Vardakas, N. Zorba, and C. V. Verikoukis, "A Survey on Demand Response Programs in Smart Grids: Pricing Methods and Optimization Algorithms," *IEEE Comm. Survey & Tutorials*, vol. 17, no. 1, 2015.
- [3] R. Deng, Z. Yang, M. Chow, and J. Chen, "A Survey on Demand Response in Smart Grids: Mathematical Models and Approaches," *IEEE Transactions on Industrial Informatics*, vol. 11, no. 3, 2015.
- [4] B. P. Esther and K. S. Kumar, "A Survey on Residential Demand Side Management Architecture, Approaches, Optimization Models and Methods," *Renewable and Sustainable Energy Reviews*, vol. 59, pp. 342-351, 2016.
- [5] W. Saad, Z. Han, H. V. Poor, and T. Basar, "Game-Theoretic Methods for the Smart Grid" *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 86-105, 2012.
- [6] A. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous Demand-Side Management Based on Game-Theoretic Energy Consumption Scheduling for the Future Smart Grid," *IEEE Transaction on Smart Grid*, vol. 1, no.3, pp. 320-331, Dec. 2010.
- [7] Z. Zhu, J. Tang, S. Lambetharan, W. H. Chin, and Z. Fan, "An Integer Linear Programming and Game Theory Based Optimization for Demand-side Management in Smart Grid," *IEEE International Workshop on Smart Grid Communications and Networks*, Houston, TX, Dec. 2011.
- [8] Z. Baharlouei, H. Narimani, and H. Mohsenian-Rad, "Tackling co-existence and fairness challenges in autonomous demand side management," in *Proc. IEEE GLOBECOM*, Anaheim, CA Dec. 2012, pp. 3159-3164.
- [9] Z. Baharlouei, M. Hashemi, H. Narimani, and H. Mohsenian-Rad, "Achieving Optimality and Fairness in Autonomous Demand Response: Benchmarks and Billing Mechanisms," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 968-975, 2013.
- [10] T. Assaf, A. H. Osman, and M. Hassan, "Fair Autonomous Energy Consumption Scheduling Based on Game-Theoretic Approach for the Future Smart Grid," *18th International Conference on Computer Modelling and Simulation*, Cambridge, UK, 2016.