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Description	

Power Flow Management: Solvability Condition for a System with Controllable and Fluctuating Devices

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***Abstract*—Renewable energy sources such as wind power or photo-voltaic power are environmental-friendly, but the fluctuation of the output power can affect the stability and quality of power network due to their intermittent nature. From such point of view, a power flow control is introduced which assigns power levels for controllable power devices and connections between power devices to absorb the power fluctuations caused by fluctuating devices. While, the system controller needs to handle various aspects such as transient behavior, latency of system control, cost efficiency etc., the issue whether the system has a feasible solution or not is one of the most important issues. In the paper, we discuss solvability conditions for a system which includes both controllable and fluctuating power devices (i) with given power generation and consumption levels of fluctuating devices, and (ii) with any power level of fluctuating sources and loads. Hence, the ultimate objective is to design a system which has robustness against power fluctuations caused by fluctuating devices.**

***Index Terms*—Power flow management, power fluctuations, distributed power sources, renewable energy resources.**

I. INTRODUCTION

To face the increase of power demand, constraints on scalability of the existing power system, and reduction in environmental costs of nonrenewable energy sources have promoted the development of distributed energy generation. This means, energy conversion units are situated close to the energy consumer, and large units are substituted by smaller ones. A distributed energy system is an efficient, reliable, and environment friendly alternative to the conventional energy systems. As one of the main forms of distributed energy generation, renewable energy outputs fluctuate due to the intermittent characteristics and irregular nature [1].

Renewable energy sources such as wind power and photovoltaic power are environment-friendly but the generated power varies greatly, resulting in a risk of fluctuations and is uncontrollable. Increasing the amount of renewable energy would degrade the power quality and stability of the power grid. From the view point of the effects on power system, power fluctuations caused by power loads has also the same effect [2]. Therefore, the crucial task of electrical power management systems is to keep balance between dynamic changing power supply and consumption patterns.

In order to manage power fluctuations effectively, cooperation with controllable power devices seems to be promising technology [3]–[5]. Information technology is now being used throughout power grids, where embedded sensors, actuators, and controllers are used for continuous power monitoring, control, and management [7]–[9]. Based on the high controlability of smart power sensors, actuators, and controllers, the amount and direction of power flow at each power source and load can be accurately controlled by the user, which provide the technical basis for the realization of our proposed *power flow control problem*.

For the implementation of power flow control for practical situations in which fluctuating power devices change power levels dynamically, a power control mechanism is required for each time instance. Hence, the goal of this control problem is to find the power levels for controllable power devices, and power flows between power devices. Since the real system changes at each time instance, the issue whether the system (i.e., power flow control problem) has a feasible solution or not in particular time instance is an important issue to solve.

In this paper, we discuss power flow control in each time instance and the solvability conditions for a system which includes controllable and fluctuating power devices (i) with given generated power and demand of fluctuating power devices, (ii) with any situation/value of fluctuating devices. Hence, the ultimate objective of this paper is to design a system which has robustness against power fluctuations caused by fluctuating power devices. The power flow management has been discussed in past with respect to different objectives and optimization techniques [5], [6] but there is no discussion about solvability issue even though it is an important issue which is the main focus of our studies. There exists some numerical methods which provide only yes/no answer to solvability of power flow control problem. Compared with such methods, our proposed solvability conditions for a system not only answer the feasibility of the system but also provide the detailed understanding of the system behavior to answer the reasons of existence of feasible solution under several constraints.

The rest of the paper is organized as follows; Section II shows categorization of power devices and power generation/consumption limitations for each source/load to show the range of operation of that particular power device. The solvability condition of the system including controllable devices with given power levels for fluctuating devices is discussed in Section III. The sufficiency of theorem by introducing definitions and examples is shown in Section IV. The solvability condition of a system for any power levels of fluctuating

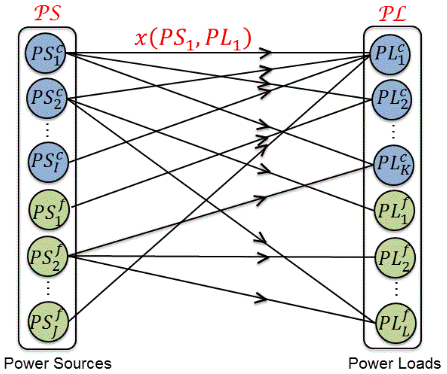

Fig. 1. Representation of power devices and connections.

devices is discussed to show the robustness against power fluctuations, in Section V. Finally, Section VI gives concluding remarks.

II. SYSTEM MODEL

The proposed system model consists of distributed power sources, power loads, and connections between them. This section shows representation and categorization of power devices (i.e., power sources and loads) based on their characteristics and functionality. It also includes the power generation/consumption limitations for each source/load to show the range of operation and performance of that particular power device. The connections between power sources and loads shows the flow of power from a particular power source to a particular power load(s).

A. Representation and Categorization of Power Devices

A power source, PS, can be defined as an electric device, which supply electric power to power loads, e.g., photo-voltaic, wind turbine, utility grid, etc. A power load, PL, is an electric device, which consumes electric power supplied by power sources. All power devices (i.e., sources and loads) are divided into two categories based on their types, characteristics, and functionality such as, *Controllable* PS^c/PL^c, and *Fluctuating* PS^f/PL^f. A controllable power device PS^c/PL^c can control its power (supply/consume) against power fluctuations whereas, fluctuating power device PS^f/PL^f cannot control its power. All power sources can be represented with unique identifiers as, $\mathcal{PS} = \{PS_1^c, PS_2^c, \cdots, PS_I^c, PS_1^f, PS_2^f, \cdots, PS_J^f\} = \{PS_1, PS_2, PS_3, \ldots, PS_{I+J}\}$, where I and J show the total numbers of controllable and fluctuating power sources, respectively. Similarly, all power loads with both types can be indexed as, $\mathcal{PL} = \{PL_1^c, PL_2^c, \cdots, PL_K^c, PL_1^f, PL_2^f, \cdots, PL_L^f\} = \{PL_1, PL_2, PL_3, \ldots, PL_{K+L}\}$ where K and L show the total numbers of controllable and fluctuating power loads. The actual power levels (i.e., generation and consumption) of sources and loads with both types can be represented as, ps_i^c, ps_j^f, $p\ell_k^c$ and $p\ell_\ell^f$, respectively.

A connection is a pair of a power source and a power load, (PS_m, PL_n). In order to represent connections between power sources and loads, a bipartite graph is introduced as shown in Fig. 1, which consists of a set of power sources ($\mathcal{PS}$), a set of power loads ($\mathcal{PL}$), and a set $\mathcal{X}$ of connections between power sources and loads as, $\mathcal{X} \subseteq \mathcal{PS} \times \mathcal{PL}$. Each connection (PS_m, PL_n) is associated with some power level in Watt $x(PS_m, PL_n)$ to show the amount of power supplied from a power source PS_m to a power load PL_n via this connection, which is always non-negative real number.

B. Power Generation/Consumption Limitation

Each power device PS/PL has a minimum and maximum power generation/consumption limitation, which shows the range of operation and performance of that power device. One of the most obvious criteria is the generation capacity of a PS within minimum and maximum limits. The minimum power generation limit, ps_i^{c-min}, and maximum limit, ps_i^{c-max}, shows the capacity of a controllable power source. That is, the controllable power source can supply power to connected power loads between its capacity range, which can be defined as,

$$ps_i^{c-min} \leq ps_i^c \leq ps_i^{c-max} \tag{1}$$

Similarly, ps_j^f shows power supply of jth fluctuating power source. The minimum and maximum power generation limits can be defined as, ps_j^{f-min}, and ps_j^{f-max}, respectively. According to the definition of fluctuating device, the generated power of a fluctuating power source is based on the physical constraints and weather conditions. The generated capacity of a fluctuating power source can be written as,

$$ps_j^{f-min} \leq ps_j^f \leq ps_j^{f-max} \tag{2}$$

All power loads are also bounded between minimum and maximum consumption limits, which shows the range of operation for that particular type of PL. Since the power consumption can be controlled accurately based on the assigned power, the controllable load can operate on any power level between minimum and maximum limit based on the available power supply. The consumption range can be defined as,

$$p\ell_k^{c-min} \leq p\ell_k^c \leq p\ell_k^{c-max} \tag{3}$$

The power consumption of fluctuating load changes dynamically between minimum and maximum limits as,

$$p\ell_\ell^{f-min} \leq p\ell_\ell^f \leq p\ell_\ell^{f-max} \tag{4}$$

Note that, all minimum power levels by power sources and power loads with both types are non-negative real numbers.

C. Power Flow Control Problem

As the physical power by a fluctuating power device varies a lot due to its nature and operation mode, the power flow on each connection must be changed according to the fluctuating environment. Here, it is assumed that the power levels of fluctuating devices are measured with power sensors at each time instance. In order to accommodate power fluctuations

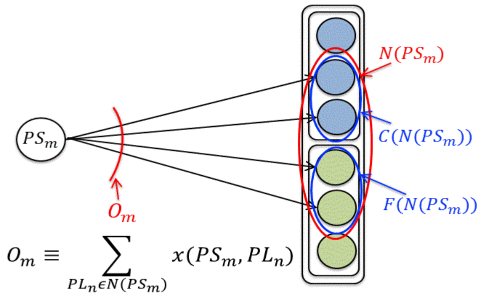

Fig. 2. A Power source with connections.

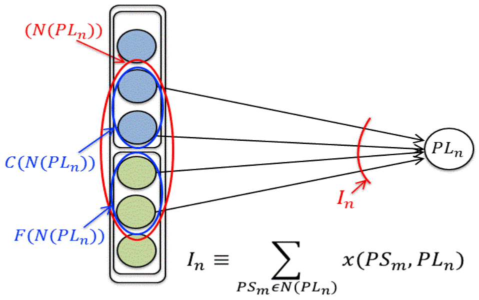

Fig. 3. A Power load with connections.

caused by fluctuating power devices, a power flow control algorithm is introduced which uses measured power levels of fluctuating power devices and computes power levels for controllable power devices and connections under the power balance constraint such that the total power generated by all power sources is fully consumed by power loads, and all power loads receive sufficient power from power sources.

Each connection connects a PS to its neighbor on the other side of the connection. The set of neighbors of PS_m is denoted as, $N(PS_m)$, which comprises of $C(N(PS_m))$, and $F(N(PS_m))$ to show the set of controllable and fluctuating power devices, respectively. As for the representation of neighboring devices, please refer to Figs. 2, and 3.

The sum of all outgoing power flows, O_m, of power source PS_m can be written as,

$$O_m = \sum_{PL_n \in N(PS_m)} x(PS_m, PL_n)$$

At the end of power flow control, the power generation ps_m of power source PS_m must be equal to the sum of all outgoing flows, O_m, defined as,

$$O_m = ps_m \tag{5}$$

Similarly, the sum of all incoming flows, I_n, of a power load, PL_n, can be computed as,

$$I_n = \sum_{PS_m \in N(PL_n)} x(PS_m, PL_n)$$

The power consumption $p\ell_n$ of power load PL_n must be equal to the sum of all incoming power flows to this PL as,

$$I_n = p\ell_n \tag{6}$$

For the implementation of power flow control for practical situations in which fluctuating power devices change power levels dynamically, a power control mechanism is required for each time instance. Hence, the goal of control problem is to find the power levels for controllable power devices, and power flow for each connection such that Eqs. (5), and (6) are satisfied along with the power generation and consumption limitations given in Eqs. (1)- (4).

III. SOLVABILITY OF THE SYSTEM WITH GIVEN POWER LEVELS FOR FLUCTUATING DEVICES

In this section, solvability condition is discussed for the system consisting of controllable power devices as well as fluctuating power devices. Since, the generated power and demand for fluctuating power sources and loads are given for each time instance, the problem is to find the power levels for controllable power sources, loads, and power flow assignment for each connection for the given system.

The following theorem shows the necessary and sufficient conditions for the solvability of power flow control problem.

Theorem- 1

The power flow control problem has a feasible solution if and only if the following two conditions are satisfied, where $C(\bullet)$ denotes the set of controllable devices in set $\bullet$, and $F(\bullet)$ shows the set of fluctuating devices in set $\bullet$.

1-1 $$\forall S \subseteq PS, \sum_{PS_i^c \in C(S)} ps_i^{c-min} + \sum_{PS_j^f \in F(S)} ps_j^f \leq \sum_{PL_k^c \in C(N(S))} p\ell_k^{c-max} + \sum_{PL_\ell^f \in F(N(S))} p\ell_\ell^f$$

1-2 $$\forall T \subseteq PL, \sum_{PS_i^c \in C(N(T))} ps_i^{c-max} + \sum_{PS_j^f \in F(N(T))} ps_j^f \geq \sum_{PL_k^c \in C(T)} p\ell_k^{c-min} + \sum_{PL_\ell^f \in F(T)} p\ell_\ell^f$$

Here, we show the necessity of the conditions. Let $x : X \rightarrow R_+$ be a feasible solution and let S be an arbitrary subset of power sources, then Eqs. (5), (6) are satisfied for every PS and PL as,

$$\sum_{PS_j^f \in F(S)} ps_j^f = \sum_{PS_j^f \in F(S)} O_j^f$$

$$\sum_{PS_i^c \in C(S)} ps_i^c = \sum_{PS_i^c \in C(S)} O_i^c$$

$$\sum_{PL_\ell^f \in F(N(S))} I_\ell^f = \sum_{PL_\ell^f \in F(N(S))} p\ell_\ell^f$$

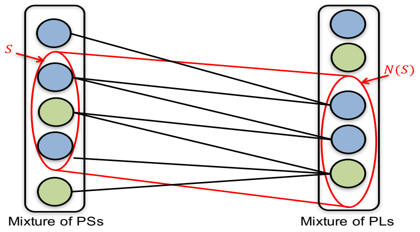

Fig. 4. Illustration of a subset S of power sources and its neighbor set $N(S)$.

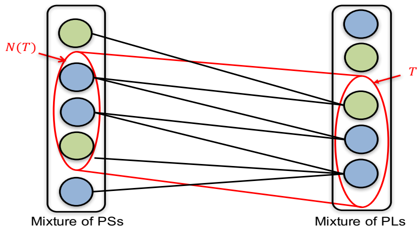

Fig. 5. Illustration of a subset T of power loads and its neighbor set $N(T)$.

$$\sum_{PL_k^c \in C(N(S))} I_k^c = \sum_{PL_k^c \in C(N(S))} p\ell_k^c$$

Since, each power source in S is supplying power to loads in $N(S)$, but the power loads in $N(S)$ can receive power from other power sources not in S (see Fig. 4), which can be expressed as,

$$\sum_{PS_m \in S} O_m \leq \sum_{PL_n \in N(S)} I_n \tag{7}$$

On the other hand, the total sum of all outgoing power flows on each connection in S can be written as,

$$\begin{aligned} \sum_{PS_m \in S} O_m &= \sum_{PS_i^c \in C(S)} ps_i^c + \sum_{PS_j^f \in F(S)} ps_j^f \\ &\geq \sum_{PS_i^c \in C(S)} ps_i^{c-min} + \sum_{PS_j^f \in F(S)} ps_j^f \end{aligned} \tag{8}$$

The subset of loads $N(S)$ contain controllable and fluctuating power devices and total incoming power flows can be represented as,

$$\begin{aligned} \sum_{PL_n \in N(S)} I_n &= \sum_{PL_k^c \in C(N(S))} p\ell_k^c + \sum_{PL_\ell^f \in F(N(S))} p\ell_\ell^f \\ &\leq \sum_{PL_k^c \in C(N(S))} p\ell_k^{c-max} + \sum_{PL_\ell^f \in F(N(S))} p\ell_\ell^f \end{aligned} \tag{9}$$

By combining Eqs. (7), (8), and (9), we can conclude the necessity of the condition *1-1*, which satisfies that system has a feasible solution. Now, we will show the necessity of condition *1-2*. From the view point of power loads, for any subset T of power loads (see Fig 5), each power load in T is receiving power supplied from power sources in $N(T)$, but the power sources in $N(T)$ can supply power to loads outside T. This can be expressed as,

$$\sum_{PS_m \in N(T)} O_m \geq \sum_{PL_n \in T} I_n \tag{10}$$

Since T includes both types of power loads, therefore, the total sum of all incoming flows on each connection in T can be written as,

$$\begin{aligned} \sum_{PL_n \in T} I_n &= \sum_{PL_k^c \in C(T)} p\ell_k^c + \sum_{PL_\ell^f \in F(T)} p\ell_\ell^f \\ &\geq \sum_{PL_k^c \in C(T)} p\ell_k^{c-min} + \sum_{PL_\ell^f \in F(T)} p\ell_\ell^f \end{aligned} \tag{11}$$

The subset of power sources $N(T)$ contains both types of power sources, and total outgoing power flows can be shown as,

$$\begin{aligned} \sum_{PS_m \in N(T)} O_m &= \sum_{PS_i^c \in C(N(T))} ps_i^c + \sum_{PS_j^f \in F(N(T))} ps_j^f \\ &\leq \sum_{PS_i^c \in C(N(T))} ps_i^{c-max} + \sum_{PS_j^f \in F(N(T))} ps_j^f \end{aligned}$$

From Eqs. (10), (11), and (12), we have shown the necessity of the condition *1-2*. In further discussions, we will show the sufficiency of the theorem by giving examples and definitions so that the conditions *1-1* and *1-2* are satisfied.

IV. SUFFICIENCY WITH DEMONSTRATION

In this section, our target is to show that the conditions *1-1* and *1-2* are satisfied, which corresponds to a feasible solution. We prove the sufficiency of the theorem by introducing some definitions and examples because mathematical proofs can exceed the page limit of this paper. The mathematical proof would be presented in another paper. Please note that, this paper discusses only the solvability issue for the existence of the feasible solution. The real time power control issue is not considered in this paper.

A. Alternating Path and Augmenting Path

Definition- 1: Each power device (PS/PL) could have three states; Power-High, Power-Balanced, and Power-Low.

- POWER-HIGH : A power source is called "power-high" node, when the sum of all outgoing power flows can be increased, this can be expressed as, $O_i^c < ps_i^{c-max}$, $O_j^f < ps_j^f$. A power load is called "power-high" node, when the sum of all incoming power flows can be increased as, $I_k^c > p\ell_k^{c-min}$, $I_\ell^f > p\ell_\ell^f$.
- POWER-BALANCED : When the total sum of all outgoing/incoming power flows to/from controllable PS/PL is ranging between its minimum and maximum power limitation is called "power-balanced" node specified as, $ps_i^{c-min} \leq O_i^c \leq ps_i^{c-max}$, and $p\ell_k^{c-min} \leq I_k^c \leq p\ell_k^{c-max}$. When generating/consuming power of fluctuating PS/PL is exactly equal to the sum of all outgoing/incoming power flows is called "power-balanced" node specified as, $O_j^f = ps_j^f$, and $I_\ell^f = p\ell_\ell^f$, respectively.
- POWER-LOW : A power source is called "power-low" node, when the sum of all outgoing power flows can be decreased as, $O_i^c > ps_i^{c-min}$, and $O_j^f > ps_j^f$. Similarly,

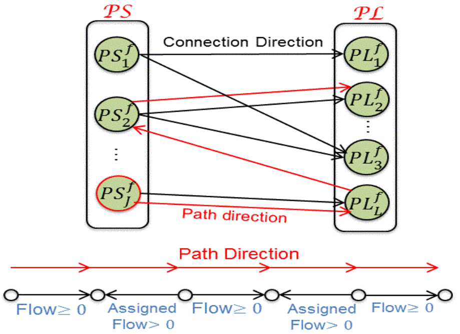

Fig. 6. Alternating Path.

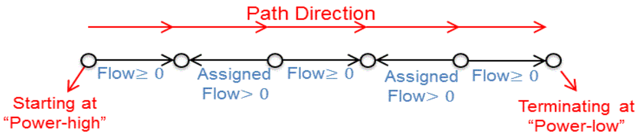

Fig. 7. Augmenting Path.

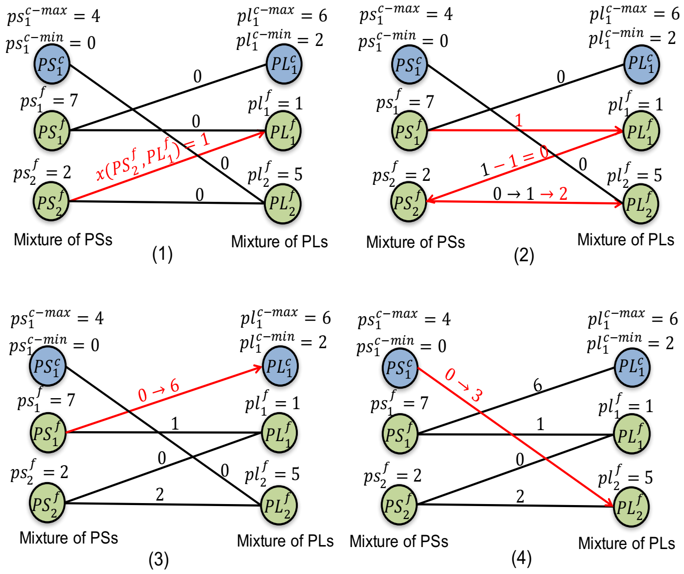

Fig. 8. Demonstration example.

a PL can be defined as "power-low" node when the sum of all incoming power flows can be decreased as, $I_k^c < p\ell_k^{c-max}$, and $I_\ell^f < p\ell_\ell^f$.

Definition- 2: A path is simply a collection of connections reachable from one device to another device. A path may contain edges "forward edges" having the same direction with the path as well as edges "backward edges" having opposite direction with the path. Every backward edge has positive power flow then the path is called alternating path (Fig. 6).

Definition- 3: An alternating path, which starts from "power-high" node and terminates on"power-low" node, is called an augmenting path (Fig. 7).

B. Demonstration

This subsection represents the sufficiency of the theorem to show the existence of feasible solution. We consider the system with three power sources and three loads with connections. One of the sources is selected as controllable (PS_1^c), and the other two sources are selected as fluctuating $(PS_1^f$ and $PS_2^f)$. Similarly, one of the load is selected as controllable (PL_1^c), and other two loads are fluctuating as PL_1^f and PL_2^f. In Fig. 8, given generated power levels by fluctuating sources are denoted as, $ps_1^f = 7$, and $ps_2^f = 2$. The power demand levels by fluctuating loads are specified as, $p\ell_1^f = 1$, and $p\ell_2^f = 5$, respectively. The controllable devices are bounded between maximum and minimum power limits as, $ps_1^{c-max} = 4$, and , $ps_1^{c-min} = 0$. The power limits for controllable load are given as, $p\ell_1^{c-max} = 6$, and $p\ell_1^{c-min} = 2$. At first, we show that the condition *1-1* is satisfied for all subsets of power sources.

Table I shows the list of all subsets of sources with neighbor subsets along with power generation and consumption computation according to condition *1-1*. For each subset, the sum of minimum power levels for controllable and given power levels of fluctuating is less or equal to the sum of maximum power levels for controllable load and given power levels for fluctuating loads. Similarly, we can show that the condition *1-2* is also satisfied and finally we found that the system satisfied conditions *1-1* and *1-2*, and we have feasible solution. As for the power flow assignment for each connection, initially it is considered as "zero". Since all sources are "power-high", the system will try to find an augmenting path to increase power by selecting a source arbitrarily. For example, the augmenting path started with PS_2^f and terminated at "power-low" node PL_1^f is selected and the power is increased on this connection by "1" to satisfy the power demand (Fig 8(1)). This makes this power load a "power-balanced" node. Similarly, we choose a path from PS_2^f to PL_2^f and increased power flow by "1". The next augmenting path is selected from PS_1^f to PL_2^f through PL_1^f and PS_2^f and power flow along this path is increased by "1" so that the power flow on each connection does not become negative as shown in the Fig. 8(2). Since PS_2^f and PL_1^f became "power-balanced" nodes, the next augmenting path is chosen from PS_1^f to PL_1^c for power increase by "6" (Fig 8(3)). Now all power devices are "power-balanced" except PS_1^c and PL_2^f, so system selected augmenting path starting from PS_1^c and terminating at PL_2^f to increase power by "3" (Fig. 8(4)). Here, we choose generated power by fluctuating

TABLE I

LIST OF SUBSET S OF $\mathcal{PS}$ AND $N(S)$

S	$N(S)$	ps_i^{c-min} $+ps_j^f$	$p\ell_k^{c-max}$ $+p\ell_\ell^f$
$\{PS_1^c\}$	$\{PL_2^f\}$	0	5
$\{PS_1^f\}$	$\{PL_1^c, PL_1^f\}$	7	7
$\{PS_2^f\}$	$\{PL_1^f, PL_2^f\}$	2	6
$\{PS_1^c, PS_1^f\}$	$\{PL_1^c, PL_1^f, PL_2^f\}$	7	12
$\{PS_1^c, PS_2^f\}$	$\{PL_1^f, PL_2^f\}$	2	6
$\{PS_1^f, PS_2^f\}$	$\{PL_1^c, PL_1^f, PL_2^f\}$	9	12
$\{PS_1^c, PS_1^f, PS_2^f\}$	$\{PL_1^c, PL_1^f, PL_2^f\}$	9	12

power devices as much as possible to save power supply of controllable power sources.

V. SOLVABILITY OF THE SYSTEM FOR ANY POWER LEVEL FOR FLUCTUATING DEVICES

In this section, solvability condition is discussed for the system consisting of controllable devices and any power assignment of fluctuating devices within its capacity range.

Theorem- 2

The power flow control problem always has a feasible solution if and only if the following two conditions are satisfied.

2-1 $\forall S \subseteq PS,$

$$\sum_{PS_i^c \in C(S)} ps_i^{c-min} + \sum_{PS_j^f \in F(S)} ps_j^{f-max} \leq \sum_{PL_k^c \in C(N(S))} p\ell_k^{c-max} + \sum_{PL_\ell^f \in F(N(S))} p\ell_\ell^{f-min}$$

2-2 $\forall T \subseteq PL,$

$$\sum_{PS_i^c \in C(N(T))} ps_i^{c-max} + \sum_{PS_j^f \in F(N(T))} ps_j^{f-min} \geq \sum_{PL_k^c \in C(T)} p\ell_k^{c-min} + \sum_{PL_\ell^f \in F(T)} p\ell_\ell^{f-max}$$

Here, we prove the sufficiency of above conditions. Let S be any subset of power sources and $N(S)$ be a subset of power loads, then condition *1-1* can be expressed as,

$$\begin{aligned}
&\sum_{PS_i^c \in C(S)} ps_i^{c-min} + \sum_{PS_i^f \in F(S)} ps_j^{f} \\
\leq &\sum_{PS_i^c \in C(S)} ps_i^{c-min} + \sum_{PS_i^f \in F(S)} ps_j^{f-max} \\
\leq &\sum_{PL_k^c \in C(N(S))} p\ell_k^{c-max} + \sum_{PL_\ell^f \in F(N(S))} p\ell_\ell^{f-min} \\
\leq &\sum_{PL_k^c \in C(N(S))} p\ell_k^{c-max} + \sum_{PL_\ell^f \in F(N(S))} p\ell_\ell^{f},
\end{aligned}$$

This concludes that if the condition *2-1* is satisfied, then condition *1-1* is always satisfied for any situation of fluctuating power devices. Similarly, for any subset of power loads T and its neighbor subset $N(T)$ of power sources, condition *1-2* can be defined as,

$$\begin{aligned}
&\sum_{PS_i^c \in C(N(T))} ps_i^{c-max} + \sum_{PS_j^f \in F(N(T))} ps_j^{f} \\
\geq &\sum_{PS_i^c \in C(N(T))} ps_i^{c-max} + \sum_{PS_j^f \in F(N(T))} ps_j^{f-min} \\
\geq &\sum_{PL_k^c \in C(T)} p\ell_k^{c-min} + \sum_{PL_\ell^f \in F(T)} p\ell_\ell^{f-max} \\
\geq &\sum_{PL_k^c \in C(T)} p\ell_k^{c-min} + \sum_{PL_\ell^f \in F(T)} p\ell_\ell^{f},
\end{aligned}$$

This shows that *2-2* is satisfied, then *1-2* is always satisfied for any value of fluctuating devices.

In order to guarantee the existence of feasible solution and the robustness of the given system for any generating demand/supply for fluctuating device within the power range, we show the necessity of the conditions. If we substitute $ps_j^f = ps_j^{f-max}$ and $p\ell_\ell^f = p\ell_\ell^{f-min}$ in condition *1-1*, then we get condition *2-1*, for any value of fluctuating device. Similarly, the result of substituting $ps_j^f = ps_j^{f-min}$ and $p\ell_\ell^f = p\ell_\ell^{f-max}$ in condition *1-2* is equivalent to condition *2-2* to show that the system always has a feasible solution for any situation of fluctuating power devices.

VI. CONCLUDING REMARKS

Energy saving and reduction in gas emissions jointly promoted the development of renewable energy sources, but the fluctuation of the output power together with dynamic power demand of power loads can affect the quality of power network due to their intermittent nature. In this paper, the increase of the power fluctuation in the future power systems due to uncontrollable power generation sources has been pointed out. From such point of view, a power flow control is introduced in each time instance and the solvability condition for a system to have a feasible solution is discussed with example demonstration. The robustness of the given system which consists of power sources, loads, and connections with any situation of fluctuating power devices is also discussed.

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