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Efficient proof-search algorithms using sequent calculi for nonclassical logics

Satoru Maruyama (010113)

School of Information Science,
Japan Advanced Institute of Science and Technology

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1 Introduction

There are various approaches to automated theorem proving for nonclassical logics. In our research, we made first a theoretical study of efficient proof-search algorithms which use cut-free sequent systems and made comparisons among them. Then, we implimented a theorem prover for intuitionistic logic by using the cut-free system introduced by Dyckhoff and Hudelmaier. The program is written by using Standard ML.

2 Proof-search and sequent systems

2.1 Wang's system

It is known that if we have a cut-free system for a given logic then we can derive usually the decidability by using the subformula property. But, the usual procedure for showing the decidability is quite inefficient even in the case of the sequent calculus LK for classical logic. For, in the proof-seach we need to check whether a sequent which is newly produced has already been obtained before or not. This checking, called loop-checking, needs both memories and time, and hence it becomes a main source of inefficiency in the proof-search.

For classical propositional logic, the system introduced by Wang, obtained from LK by modifying their rules, is known to be effective. In fact, it is not necessary to make a loop-checking in Wang's system, since it has no explicit applications of contraction rule. Moreover, in each rule of the system the provability of the upper sequent(s) is equivalent to that of the lower sequent. Therefore, we can avoid trial and error, and our proof-search can be carried out in a completely mechanical way.

2.2 Dyckhoff and Hudelmaier's system

The method used in Wang's system doesn't work well for intuitionistic logic. Dragalin introduced a system for intuitionistic logic, which resembles Wang's system (see Figure 1). But, in the application of ($\supset\rightarrow^*$) rule, we cannot assure that the left-upper sequent is simpler than the lower sequent. Therefore, to make sure the termination of the proof-search we still need to use loop-checking.

In order to overcome the difficulty, Dyckhoff and Hudelmaier introduced independently a system shown below, at the beginning of 1990s. Their system is obtained from the system of Dragalin by replacing ($\supset\rightarrow^*$) by four rules shown in Figure 2.

$$\frac{}{A,\Gamma\rightarrow A,\Delta}\ (action^*) \qquad \frac{}{f,\Gamma\rightarrow\Delta}\ (f\rightarrow^*)$$

$$\frac{A,B,\Gamma\rightarrow\Delta}{A\wedge B,\Gamma\rightarrow\Delta}\ (\wedge\rightarrow^*) \qquad \frac{\Gamma\rightarrow A,\Delta\quad \Gamma\rightarrow B,\Delta}{\Gamma\rightarrow A\wedge B,\Delta}\ (\rightarrow\wedge^*)$$

$$\frac{A,\Gamma\rightarrow\Delta\quad B,\Gamma\rightarrow\Delta}{A\vee B,\Gamma\rightarrow\Delta}\ (\vee\rightarrow^*) \qquad \frac{\Gamma\rightarrow A,B,\Delta}{\Gamma\rightarrow A\vee B,\Delta}\ (\rightarrow\vee^*)$$

$$\frac{A\supset B,\Gamma\rightarrow A\quad B,\Gamma\rightarrow\Delta}{A\supset B,\Gamma\rightarrow\Delta}\ (\supset\rightarrow^*) \qquad \frac{A,\Gamma\rightarrow B}{\Gamma\rightarrow A\supset B,\Delta}\ (\rightarrow\supset^*)$$

Figure 1: Dragalin's system

In this system, we can avoid loop-checking. Though there still remain some trial and error, the algorithm using this sytem becomes much more efficient. To examine the efficiency of this system we have implimented the algorithm and made a theorem prover for intuitionistic logic.

$$\frac{B, A, \Gamma \to \Delta}{A \supset B, A, \Gamma \to \Delta} \ (\supset\to_1^*)$$

$$\frac{C \supset (D \supset B), \Gamma \to \Delta}{(C \wedge D) \supset B, \Gamma \to \Delta} \ (\supset\to_2^*)$$

$$\frac{C \supset B, D \supset B, \Gamma \to \Delta}{(C \vee D) \supset B, \Gamma \to \Delta} \ (\supset\to_3^*)$$

$$\frac{D \supset B, \Gamma \to C \supset D \quad B, \Gamma \to \Delta}{(C \supset D) \supset B, \Gamma \to \Delta} \ (\supset\to_4^*)$$

Figure 2: Dyckhoff and Hudelmaier's system

3 inplimentation

Our program is written by using Standard ML. Every input is given in the following way.

- For each propositional variable, a declaration 'Prop' is put at the beginning.
- A formula $A \wedge B$ is written as Land (Prop "A", Prop "B").
- A formula $A \vee B$ is written as Lor (Prop "A", Prop "B").
- A formula $A \supset B$ is written as Limp (Prop "A", Prop "B").
- When we write two or more formulas, we insert commas ',' between them.

Since it is sometimes necessary to supplement functions in the proof, an actual input becomes of the following form.

```
main (node ([Left side sequent], ([], [])),
 node ([Right side sequent], ([], [])));
```

A proof figure will be output when a input formula is provable. But, if it is not provable, then it is displayed only as 'NO_PRINCIPAL', since the exception of Standard ML happens in this case.

4 Conclusion

In this research, we have implimented a theorem prover for intuitionistic logic, by using the cut-free system by Dyckhoff and Hudelmaier. By examining the prover, we could have made sure its efficiency.