A study on relationships between some subrecursive function classes and complexity classes

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In computer science, various classes of computational complexity have been studied, as represented by the class $\mathcal{F}$PTIME of polynomial time computable functions, and there are many unsolved problems such as the P vs NP problem. On the other hand, in computability theory, classes of subrecursive functions has been studied for a long time. Although many complexity classes are contained in the class of elementary functions $\mathcal{E}$, little research has been done on relationship between classes of subrecursive functions smaller than the class $\mathcal{E}$ and complexity classes. For example, the relationship between the class $\mathcal{E}^2$ or the class $\mathcal{M}^2$ and the class $\mathcal{F}$PTIME of polynomial time computable functions is not known at all, where the class $\mathcal{E}^2$ is the second class in the Grzegorczyk hierarchy using bounded recursion, and the class $\mathcal{M}^2$ is the second class in the hierarchy obtained by replacing bounded recursion with bounded minimisation.

In this thesis, we try to elucidate the relationship between the second class $\mathcal{E}^2$ in the Grzegorczyk hierarchy and the class $\mathcal{F}$PTIME of polynomial time computable functions. We define a class of functions $\mathcal{E}^{2+}$. The class $\mathcal{E}^{2+}$ is an extension of the second class $\mathcal{E}^2$ in the Grzegorczyk hierarchy to which course-of-values recursion whose values are bounded by 1 is added. And we also define simultaneiously and recursively two classes of functions $\mathcal{C}_{\mathbb{N}}$ and $\mathcal{C}_{\mathbb{W}}$. Functions in the class $\mathcal{C}_{\mathbb{N}}$ take two data types as their arguments, natural numbers and binary strings, and their values are natural numbers, and functions in the class $\mathcal{C}_{\mathbb{W}}$ take the two data types as their arguments, and their values are binary strings. Then, we associate functions in $\mathcal{E}^{2+}$ with functions in $\mathcal{C}_{\mathbb{N}}$, and also associate functions in $\mathcal{F}$PTIME with functions in $\mathcal{C}_{\mathbb{W}}$. Using these relations, with respect to their set classes $\mathcal{E}^{2+}_*$ and PTIME, we show that PTIME is contained in $\mathcal{E}^{2+}_*$.

Furthermore, we try to elucidate the relationship between the second class $\mathcal{M}^2$ in the hierarchy of bounded minimisation and the function class $\mathcal{F}$LH of the logtime hierarchy. We define simultaneiously and recursively two classes of functions $\mathcal{D}_{\mathbb{N}}$ and $\mathcal{D}_{\mathbb{W}}$. Functions in the class $\mathcal{D}_{\mathbb{N}}$ take two data types as their arguments, natural numbers and binary strings, and their values are natural numbers, and functions in the class $\mathcal{D}_{\mathbb{W}}$ take the two data types as their arguments, and their values are binary strings. Then, we associate functions in $\mathcal{M}^2$ with functions in $\mathcal{D}_{\mathbb{N}}$, and also associate functions in $\mathcal{F}$LH with functions in $\mathcal{D}_{\mathbb{W}}$. Using these relations, with respect to their set classes $\mathcal{M}^2_*$ and LH, we show that LH is contained in $\mathcal{M}^2_*$.

Keywords: Subrecursion; Grzegorczyk hierarchy; Implicit computational complexity; Complexity classes; Polynomial-time functions; Logtime hierarchy; Function algebra.