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A study on relationships between some subrecursive function classes and complexity classes

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In computer science, various classes of computational complexity have been studied, as represented by the class \mathcal{FPTIME} of polynomial time computable functions, and there are many unsolved problems such as the P vs NP problem. On the other hand, in computability theory, classes of subrecursive functions has been studied for a long time. Although many complexity classes are contained in the class of elementary functions \mathcal{E} , little research has been done on relationship between classes of subrecursive functions smaller than the class \mathcal{E} and complexity classes. For example, the relationship between the class \mathcal{E}^2 or the class \mathcal{M}^2 and the class \mathcal{FPTIME} of polynomial time computable functions is not known at all, where the class \mathcal{E}^2 is the second class in the Grzegorzcyk hierarchy using bounded recursion, and the class \mathcal{M}^2 is the second class in the hierarchy obtained by replacing bounded recursion with bounded minimisation.

In this thesis, we try to elucidate the relationship between the second class \mathcal{E}^2 in the Grzegorzcyk hierarchy and the class \mathcal{FPTIME} of polynomial time computable functions. We define a class of functions \mathcal{E}^{2+} . The class \mathcal{E}^{2+} is an extension of the second class \mathcal{E}^2 in the Grzegorzcyk hierarchy to which course-of-values recursion whose values are bounded by 1 is added. And we also define simultaneously and recursively two classes of functions $\mathcal{C}_{\mathbb{N}}$ and $\mathcal{C}_{\mathbb{W}}$. Functions in the class $\mathcal{C}_{\mathbb{N}}$ take two data types as their arguments, natural numbers and binary strings, and their values are natural numbers, and functions in the class $\mathcal{C}_{\mathbb{W}}$ take the two data types as their arguments, and their values are binary strings. Then, we associate functions in \mathcal{E}^{2+} with functions in $\mathcal{C}_{\mathbb{N}}$, and also associate functions in \mathcal{FPTIME} with functions in $\mathcal{C}_{\mathbb{W}}$. Using these relations, with respect to their set classes \mathcal{E}_*^{2+} and PTIME , we show that PTIME is contained in \mathcal{E}_*^{2+} .

Furthermore, we try to elucidate the relationship between the second class \mathcal{M}^2 in the hierarchy of bounded minimisation and the function class \mathcal{FLH} of the logtime hierarchy. We define simultaneously and recursively two classes of functions $\mathcal{D}_{\mathbb{N}}$ and $\mathcal{D}_{\mathbb{W}}$. Functions in the class $\mathcal{D}_{\mathbb{N}}$ take two data types as their arguments, natural numbers and binary strings, and their values are natural numbers, and functions in the class $\mathcal{D}_{\mathbb{W}}$ take the two data types as their arguments, and their values are binary strings. Then, we associate functions in \mathcal{M}^2 with functions in $\mathcal{D}_{\mathbb{N}}$, and also associate functions in \mathcal{FLH} with functions in $\mathcal{D}_{\mathbb{W}}$. Using these relations, with respect to their set classes \mathcal{M}_*^2 and LH , we show that LH is contained in \mathcal{M}_*^2 .

Keywords: Subrecursion; Grzegorzcyk hierarchy; Implicit computational complexity; Complexity classes; Polynomial-time functions; Logtime hierarchy; Function algebra.