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Natural Deduction Systems for Substructural Logics and a Proof-Theoretic Study Based on Them

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1 Natural deduction

Natural deduction systems were introduced by G. Gentzen before his introduction of sequent calculi. The basic idea behind them is to formalize logical reasoning in the way as what we do in mathematics. Proof theoretic study of natural deduction system has been developed much by D. Prawitz [5]. Prawitz proved normal form theorem, which corresponds cut elimination theorem in sequent calculi. The theorem plays an important role in proof theoretic study of natural deducion. As its consequences, we can derive Craig's interpolation theorem both for classical logic and intuitionistic one, and also disjunction property of intuitionistic logic, for instance.

It is known that there exsists a close correspondence, called Currry-Howard isomorphism, between proof figures in natural deduction sytem for intuitionistic logic and terms of lambda calculus. Moreover, if a given proof figure is of a normal form, the corresponding lambda term is normal (as lambda terms).(See [1],[2],[3])

Most of proof theoretic studies of substructural logics up to now are based on sequent calculi, but only a little has been done on natural deduction sytems for substructural logics. Our main goal of the present thesis is to develop a proof theoretic study of substructural logics based on natural deduction systems.

2 Variable sharing property of the logic FLec

It is known that the variable sharing property holds for substructural logics without weakening rule, where the *variable sharing property* for a logic **L** means that if $A \supset B$ is provable in **L** there exists a propositional variable which is common to both formulas A and B. We assume here that our language has no constant symbols.

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We give first a proof of variable sharing property for **FLec**, the sequent system obtained from **LJ** by deleting weakening rule, by introducing the notion of *good sequents*. We say a sequent $\Pi \to \Lambda$ is good if and only if there exsits at least two formulas in the sequent $\Pi \to \Lambda$ (i.e. either both Π and Λ are noempty, or Λ is empty and Π contains at least two formulas.) Of course, each initial sequent is good.

The outline of our proof goes as follows. Suppose $\Gamma \to A$ is provable in **FLec** with nonempty Γ. By the cut elimination theorem for **FLec**, $\Gamma \to A$ has a cut-free proof figure **P**. Then, we can show that there exists a branch in **P** starting from $\Gamma \to A$ and going up to an initial sequent, which consists only of good sequents. By using induction along this branch and by introducing the notion of *partitions* , we can prove that the variable sharing property holds for every good sequent in this branch.

3 A proof of variable sharing property using natural deduction system

Next, we will show variable sharing property using natural deduction system for **FLec**. Since we have no simple natural deduction system for the full system of **FLec**, our result is restricted only to the implicational fragment. First we introduce a natural deduction system **Nec**, which is obtained from **NJ** by claiming that at least one assumption must be cancelled when we apply the introduction rule for implication. However, we have a certain difficulty in showing variable sharing property by using induction on the depth of the normal proof figure in **Nec**. So, we introduce a modified system, called **Nec*** , which corresponds to *strong β normal form* of lambda terms for **Nec**. Also, by using partitions of assumptions, we prove the following.

> Suppose that $\Gamma \vdash A$ is provable in **Nec***. For any partition $\langle \Gamma_1, \Gamma_2 \rangle$ of Γ such that Γ_1 is nonempty, $V(\Gamma_1) \cap V(\Gamma_2 \cup \{A\}) \neq \emptyset$.

4 Craig's interpolation theorem for FLec and a modification of Maehara's method

A standard proof theoretic method of proving the interpolation theorem is Maehara's method. However, as it is pointed out in [4], we have troubles in eliminating constant symbols for substructural logics without weakening rule. To overcome this difficulty, we use the idea of good sequents and apply Maehara's method only to a branch consisting only of good sequents. Since variable sharing property holds for good sequents, it is not necessary to introduce constant symbols. Thus, our proof is simpler than the proof in [6].So we have the following.

> If $\Gamma \to A$ is provable in **FLec** and Γ is nonempty, there exists a formula D such that (1) $\vdash \Gamma \to D$, (2) $\vdash D \to A$, and (3) $V(\{D\}) \subseteq V(\Gamma) \cap V(\{A\})$.

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