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# A Study on Accepting Power of Space-bounded One-pebble Two-dimensional Turing Machines

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The theory of computational complexity is one of the main fields in computer science. One of the central subjects of the theory of computational complexity is clarifying a difference of the accepting powers of determinism, nondeterminism and alternation of various computing models. For the class of the language accepted by the space-bounded deterministic, nondeterministic and one-dimensional Turing machines, the following facts are known:

- (i) In  $o(\log \log n)$  space-bounded deterministic, nondeterministic and alternating Turing machines have the same accepting power as finite automata.
- (ii) Alternating Turing machines are stronger in accepting power than deterministic and nondeterministic Turing machines for spaces between  $\log \log n$  and  $o(\log n)$ .

It is unknown whether accepting power of nondeterministic Turing machines is stronger than deterministic Turing machines for spaces more than  $\log \log n$  and whether accepting power of alternating Turing machines is stronger than nondeterministic Turing machines for spaces more than  $\log n$ .

Also in one-pebble one-dimensional Turing machines which is allowed to use of one-pebble on the input tape of one-dimensional Turing machine, it is known that  $\log \log n$  space-bounded deterministic and nondeterministic machines have the same accepting power as finite automata. However, it is unknown whether accepting power of one-pebble nondeterministic Turing machines is stronger than one-pebble deterministic Turing machines for spaces more than  $\log \log n$ , and whether accepting power of alternating one-pebble Turing machines is stronger than nondeterministic one-pebble Turing machines for spaces more than  $\log \log n$ .

On the other hand, for two-dimensional Turing machines with two-dimensional input tapes, it is shown that even when it limits to a square tape, in  $o(\log n)$  space-bounded, alternating machines are stronger than nondeterministic machines, and nondeterministic machines are stronger than deterministic machines for spaces less than  $o(\log n)$ .

This paper introduces a one-pebble two-dimensional Turing machine (p2-tm) which is allowed to use of one pebble on the input tape of a two-dimensional Turing machine, and investigates a relationship among the accepting powers of space-bounded deterministic, nondeterministic and alternating p2-tms.

P2-tm can be used as a computing model which measures the complexity of two-dimensional patterns. Chapter 2 of this paper gives definitions and notations related to this paper. Chapter 3 of this paper investigates a difference between the accepting powers of space-bounded deterministic and nondeterministic p2-tms. Let  $L(n) : N \rightarrow N$  (where  $N$  denotes the set of natural numbers) be a function with one variable  $n$ . A p2-tm  $M$  whose input tapes are restricted to square tapes is called  $L(n)$  space-bounded if it uses at most  $L(n)$  cells of the storage tape for any input tape with  $n$  rows and  $n$  columns ( $n \geq 1$ ). Chapter 3 shows that, even when limited to a square tape, for any function  $L(n) = o(\log n)$ ,  $L(n)$  space-bounded nondeterministic p2-tms are stronger in accepting power than  $L(n)$  space-bounded deterministic p2-tms. In fact, Chapter 3 shows that there is a set of square tapes accepted by a nondeterministic one-pebble two-dimensional finite automaton, but not accepted by any  $o(\log n)$  space-bounded deterministic p2-tm. Chapter 4 investigates a difference between the accepting powers of nondeterministic and alternating p2-tms. Let  $L(m, n) : N^2 \rightarrow N$

be a function with two variables  $m$  and  $n$ , and let  $M$  be a p2-tm.  $M$  is called  $L(m, n)$  space-bounded if  $M$  uses at most  $L(m, n)$  cells of the storage tape for any input tape with  $m$  rows and  $n$  columns ( $m, n \geq 1$ ). Chapter 4 shows that for any function  $L(m, n) = f(m) + g(n)$  (where  $f(m) = o(\log m)$  and  $g(n) = o(\log n)$ ),  $L(m, n)$  space-bounded alternating p2-tms are stronger in accepting power than  $L(m, n)$  space-bounded nondeterministic p2-tms. In fact, Chapter 4 shows that there is a set of two-dimensional tapes accepted by an alternating one-pebble two-dimensional finite automaton, but not accepted by any  $L(m, n) = f(m) + g(n)$  space-bounded nondeterministic p2-tm (where  $f(m) = o(\log m)$  and  $g(n) = o(\log n)$ ). Finally, Chapter 5 gives conclusion and future problems.