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Algebra and Relational Semantics for Lambek Calculus

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1 Introduction

Classical logic and intuitionistic logic are formulated as Gentzen systems having three structural rules, i.e. exchange rules, weakning rules and contraction rules. Logics lacking some or all structural rules are called substructural logics. Various kinds of substructural logics have been intoroduced and studied in recent years. For example,**FL** is a logic obtained from intuitionistic logic by removed contraction, exchange and weakening rules. The logic **FL** is slightly stronger than the original Lambek Calculus. Linear logic lacks both contraction and weakening rules, and BCK logic does not have contraction rule.

In the present thesis, we will discuss two kinds of semantics for Lambek Calculus and **FL**. The first one is algebraic semantics for extentions of **FL**. Algebras for **FL** are given by (noncommutative) residuated lattices that have been studied from purely algebraic point of view since 1930. In recent years, their close connection with substructural logics has been clarified. The second is relational semantics for Lambek Calculus. Relational semantics can be regarded as a particular algebraic semantics, in which all elements are binary relations on a given set and algebraic operations are interpreted by some (natural) operations on relations. In the latter half of the thesis, we will intoroduce the proof of Lambek Calculus with respect to relational semantics, based on the paper Andreka and Mikulas.

2 Research work

There are two main topics in this thesis. The first is on a relation between lattice ordered groups and residuated lattices, and the second is the completeness of Lambek Calculus with respect to relational semantics.

We discuss first some relations between residuated lattices and lattice ordered groups. Residuation is a fundamental concept of ordered structures and categories. Consider any lattice-ordered monoid with monoid operation $\bullet$. A residuated lattice is defined as a lattice-ordered monoid with residuals $\backslash$ and $/$ that satisfy the property $x \bullet y \leq z \iff x \leq z/y \iff y \leq x\backslash z$. It is easy to see that any lattice ordered group forms a residuated lattice by defining $z/y = z \bullet y^{-1}$ and $x\backslash z = x^{-1} \bullet z$. When we discuss residuated lattices as algebras for substructural logics, it is natural to assume that they are bounded. That is, each of them has both the least element ($\bot$) and the greatest ($\top$). In this thesis, we show that each lattice ordered group can be embedded into one with the least and the greatest.

Next, we discuss the completeness of Lambek Calculus with respect to relational semantics. Lambek Calculus was introduced by Lambek (1958). The problem of whether Lambek Calculus is complete with respect to relational semantics has been raised several times by e.g. van Benthem (1989) and van Benthem (1991). Lambek Calculus is formulated as a Gentzen sequent system which has no structural rules ,cf.contraction, exchange and weakening rule. Thus it has only logical rules, rules for $\backslash, /$ and $\bullet$. Relational semantics for Lambek Calculus was suggested by Johan van Benthen around 1988. In this paper we will explain the proof of the completeness of Lambek Calculus with respect to relational semantics, following the paper by Andreka and Miklas. It is necessary to intoroduce relational models and K-models for some relational structures. Relational structures are basic structures that have $\bullet, \backslash$ and $/$ as binary operations and $\leq$ a binary relation on the algebra. K-model is a model for each relational structure.

Relational model is an ordered pair consisting of a transitive binary relation and a mapping which determines an interpretation. At first step of proof, the completeness of Lambek Calculus with respect to ordered residuated semigroups is shown. In the next step, each ordered residuated semigroup is shown to be isomorphic to a representable relational structure. This step is hard step where we need to use transfinite induction. In last step, it is shown that semantics of relational model agrees with semantics determined by representable relational structures. This completes the proof of the completeness of Lambek Calculus with respect to relational semantics.