Hardware-Assisted Fast Drawing of Various Voronoi Diagrams and Application

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In this study, we focus on efficiency of computing Voronoi diagrams using a hardware-assisted technique, and propose an efficient algorithm for a contour interpolation problem. Further, we generalize a Voronoi diagram in various ways.

Recently, a new paradigm for a *discrete Voronoi diagram* with assistance of a hardware has been proposed. The discrete Voronoi diagram is a Voronoi diagram defined in a discrete space, which is generated by digitizing a continuous space. That is, when n sites are given in a discrete space, the discrete Voronoi diagram is a tessellation of the space, where each region consists of all points that are closer to one site than to any other. This technique called *hardware-assisted technique*, is a method for improving the efficiency of an algorithm by executing some tasks of the algorithm in hardware. With improvement in the processing speed and the storing capacity of computers, even if an algorithm assumes the discrete Voronoi diagram, a sufficiently practical Voronoi diagram can

be computed. So, the hardware-assisted technique has attracted many researcher's attention.

For computing a discrete Voronoi diagram, we can improve the processing speed by using graphics hardware. And this approach has good features which are robust against degeneracy and numerical errors in geometrical problems, and simple implementation with graphics libraries. Further, since the Voronoi diagram obtained in this method is a digital image, we can acquire information from a characteristic digital image, in addition to the information acquired by a computational geometrical method. Taking these advantages, it is beginning to be applied in various fields of application, such as the map simplification and the sport teamwork analysis.

A key concept for the hardware-assisted technique is to render a polygonal mesh approximation to each site's distance function. Each site is assigned a unique color ID, and the corresponding distance mesh is rendered in that color using a parallel projection. We make use of the Z-buffer depth comparison operation of the graphics hardware. The Z-buffer depth test compares the new depth value to previously stored value. If the new value is less, the Z-buffer records the new distance, and the color buffer records the site's ID. In this way, each pixel in the frame buffer will have a color corresponding to the site to which it is closest. And generalized Voronoi diagrams can be constructed by handling the distance function. In this paper, we formulate *Minkowski distance Voronoi diagram*, *elliptic distance Voronoi diagram*, *weighted distance Voronoi diagram*, and *Hausdorff distance Voronoi diagram*.

In this method, the Voronoi diagram is a digital image, which differs from one in the theoretical approach of computational geometry. Therefore, it is necessary to carry out post-processing, when we want to know about geometric structures: a set of Voronoi vertices or Voronoi edges, Voronoi boundaries, neighbors and so on. The post-processing to under-

stand their information throws away the advantage of hardware-assisted technique, since it needs $O(NM)$ times for a $N \times M$ scene.

Meanwhile, there are many applications the Voronoi diagram of digital image can use as it is. In fact, we can compute to transform metric and to compute area of Voronoi region in $O(1)$ time, although it depends on executable operations or function of a graphics hardware. Furthermore, the expression of Voronoi diagram can be applied to directly manipulation of a digital picture. This idea is natural and sophisticated. Thus, it is distinguishable on the problem efficiently solved using the digital image representation of a Voronoi diagram, and the problem which cannot be solved unless it uses the geometric information which a Voronoi diagram has.

In this paper, we consider a *contour interpolation* as one of applications which can be intervene flexibly. The interpolation of contour map is to add interpolated contours to part of areas without data in original map. In general, interpolation approach are distinguished actual measur method and approximation method. We choose later which approximate smoothing data, viewpoint of the information science.

Given a set of contour lines, where each contour is approximated by polygonal chain without self-intersection, the interpolated contours consist of all Voronoi edges of original contours. Furthermore, we propose an efficient algorithm for constructing some interpolated contours of different height between adjacent contours. These contours are generated by computing a weighted Voronoi diagram.