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Author(s)	Pinkam, Nantawat; Elibol, Armagan; Chong, Nak Young
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Description	

Informative Mobile Robot Exploration for Radiation Source Localization with a Particle Filter

Nantawat Pinkam, Armagan Elibol, Nak Young Chong
School of Information Science,
Japan Advanced Institute of Science and Technology,
1-1 Asahidai, Nomi, Ishikawa 923-1211
Email: {pinkam.nantawat, aelibol, nakyoung}@jaist.ac.jp

***Abstract*—In this work, we consider the localization problem of an unknown radiation source with measurement uncertainty by using robotic systems in a geometric environment. We proposed the scheme for localization of a radioactive source using the particle filter with information gain-based exploration. The traditional method to localize the radiation is to use the gradient descent algorithm. However, such the algorithm may fail to work in the case of uncertain measurements, which lead to an inaccurate outcome. On the other hand, a standard particle filter can be used to deal with the measurement uncertainty, but the estimated intensity result may be unstable since it only uses the current measurement update as a likelihood function. To solve the problem of measurement uncertainty and unstable intensity result, we proposed an exploration method using the information gain with particle filter. The algorithm takes the information of the particles in the filter to estimate the possible actions for the robot. The expected information gain from those actions can be used to select the best possible action for the robot. The proposed method has been verified by the simulations. The proposed strategy can decrease the time it takes to finish the task comparing to the conventional methods such as the lawn mowing algorithm and source estimation seeking algorithm.**

***Keywords*—Source localization, information theory, mobile robot**

I. Introduction

The environmental hazard is an important threat where chemical substance damages the environment, which has an unfavorable effect on living organisms. One of the causes that contaminate surroundings is radioactive material leakage, which is an increasing concern in national security [1]. This emerging threat can be either induced by a malicious attack or accidental release of radioactive material. Thus, the radiation source estimation can be a valuable tool in order to plan a counter-measure to the problem, including saving human life and clean up the leakage material [2].

The radioactive measurement gives a non-linear output [3], [4], yet introduces uncertainty in measurement [5]. In order to handle these issues, there are several studies addressing the problem of source localization and estimation. In [6], three estimation techniques were applied: the Maximum Likelihood Estimator (MLE), the Extended Kalman Filter (EKF), and the Unscented Kalman Filter (UKF) to estimate the location of the radiation source. The authors used the Cramér-Rao lower bound to find the lower bounds for those estimation algorithms. The best result is obtained using the MLE, but it requires a lot of computational power to calculate the maxima. In [7], it was used a helicopter to carry a detector and flew over the area to map the radiation source. The particle filter was used as the estimation technique with prior knowledge of the intensity of the source, type of $1/R^2$. In [8], it was used a mobile ground robot equipped with a detector to map a source in the environment with obstacles. The artificial force field algorithm with the control vector was used to map the radiation source while avoiding the obstacles. In [9], it was shown the application of multi-robot systems to deal with radiation mapping by developing the control algorithm to follow the increasing information gradients. Each robot has to visit each cell and determine which cell to go the following based on the information of the past.

The main problem of exploration in the radiation field is the measurement uncertainty [10], [11]. Each sensor measurement of the robot at the same position does not guarantee to have the same value. In a particular case, it is based on Poisson distribution. The particle filter is one of the tools that give flexibility over the non-linear system. One way to navigate through the exploration field to locate the radiation source is to follow the estimated source location of the particle filter [12]. However, that does not guarantee to be the optimum path, or the result may be a failure in the worst case due to the distance between the robot and the source. If the location of the robot is far away from the source or in low radiation area, the particle filter does not guarantee to give an accurate result due to lack of information. Therefore, in this work, the method for exploration in the radiation field by using the information gain is proposed. The robot will use the information gain-based exploration, which can calculate the best possible action for the robot by using the particles information (weight, intensity, and location).

The rest of the paper is organized as follows: Section 2 describes the background of the radiation measurement and shows the model for the radiation. Section 3 shows the detail process of the particle filter with our proposed intensity estimation algorithm. Section 4 describes the proposed exploration method using the information gain. Section 5 is the experimentation with the analysis of the result. The last part, Section 6, presents our conclusion.

II. Radiation Model

Radioactivity is a process of an unstable atom emits various types of radiation, including alpha particles (α), beta particles (β), X-rays, and gamma particles (γ). A Geiger counter and a scintillator are radiation measurement devices that are used to detect gamma rays [13]. The radiation reading from a measurement device follows the inverse square relationship. The intensity of a radiation source is inversely proportional to the distance relative to the instrument according to the following equation [3]:

$$\lambda_k = \frac{I}{(x_k - x_0)^2 + (y_k - y_0)^2 + (z_k - z_0)^2} + \lambda_b \quad (1)$$

where λ_k is the intensity at the measurement device, I is the intensity of the source, (x_k, y_k, z_k) is the position of the measurement device, (x_0, y_0, z_0) is the position of the source and λ_b is the background intensity at the measurement point. Since we use ground mobile robots, we set the altitude difference z to 0.

According to [5], the radiation measurement is uncertain because of the nature of radioactive decay. Each decay event is random, independent, and occurs at a fix mean rate λ that follows the Poisson statistics as the equation below:

$$f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (2)$$

where λ is the average number of count for the period of an exposure time τ. k is the exact measurement reading from the source.

We assume that the radiation detector attached to the robot location (x_k, y_k) is uniformly directional response and neglect the air attenuation. The measurement by a robot is independently distributed, and the exposure time τ of all measurement is constant [14].

III. Radiation Estimation Algorithm

Recursive Bayesian estimation (RBE) or Bayes filter is an excellent method to deal with sensor uncertainty [15]. It also provides real-time computational feasibility with localization accuracy [16]. Most recursive Bayesian estimators have the same concept, which predicts the system behavior by using the measurement to correct the prediction according to the following Bayes theorem:

$$P(\mathbf{x} \mid \mathbf{z}_{1:k}) = \frac{P(\mathbf{z}_k \mid \mathbf{x}, \mathbf{z}_{1:k-1}) P(\mathbf{x} \mid \mathbf{z}_{1:k-1})}{P(\mathbf{z}_k \mid \mathbf{z}_{1:k-1})} \quad (3)$$

where $\mathbf{x}$ is the estimated state. $\mathbf{z}$ is the observation. The posterior belief, $P(\mathbf{x} \mid \mathbf{z}_{1:k})$, is the product of the measurement model $P(\mathbf{z}_k \mid \mathbf{x}, \mathbf{z}_{1:k-1})$ and the prior belief $P(\mathbf{x} \mid \mathbf{z}_{1:k-1})$ divided by the normalization constant $P(\mathbf{z}_k \mid \mathbf{z}_{1:k-1})$.

In this work, we choose the particle filter as the mapping algorithm since it compromises efficiency and performance for the non-linear and non-Gaussian systems [17]. In addition, the particle filter is non-parametric. Thus, it provides heuristic, and it is more flexible than other algorithms. The particle filter or Sequential Monte Carlo (SMC) is the method representing the

Algorithm 1 Particle Filter

Initialize particles: $x_0 = rand(N, 1)$
Initialize particles' weights: $w_0 = 1/N$
while Not converge **do**
  for $i = 1$ to N **do**
    Update weight:
    $w_k^i = w_{k-1}^i \cdot \mathcal{N}(z_k^i, \lambda_k^i, \lambda_k^i)$
  end for
  Normalize weights: $w_k = \frac{w_k}{\sum_{i=1}^N w_k^i}$
  Calculate state estimate:
  $\hat{x}_k = \sum_{i=1}^N w_k^i \cdot x_k^i$
  Resampling process
end while

posterior belief using a set of random state samples as particles. Each particle is basically a hypothesis of the real state at time t with an associate weight to represent the accuracy of the hypothesis based on the state measurement. The detail of the particle filter algorithm is in Algorithm 1.

In this work, we use a mobile robot to explore and map the radiation. The estimation state $\mathbf{x}$ for each particle requires the location (x, y) of the source and its intensity I as:

$$\mathbf{x} = \begin{bmatrix} x & y & I \end{bmatrix} \quad (4)$$

A. Intensity Estimation of Particles

We can use Eq. 1 to calculate the particle expected intensity I at the location (x, y) as we have a k^{th} measurement z_k at the robot location (x_k, y_k). However, it may introduce an estimation accuracy problem since we only use the current measurement that only provides the information at a specific location. If we combine the previous measurements, which include measurement locations $(x_{1:k}, y_{1:k})$ and intensity measurement $\lambda_{1:k}$, the particles' intensity estimation will be more accurate.

From Eq. 1, we can estimate the particle's intensity I^i at (x^i, y^i) using the current measurement z_k by:

$$\frac{I^i}{d_k^2} + \lambda_b = z_k \quad (5)$$

where $d_k = \sqrt{(x_k - x^i)^2 + (y_k - y^i)^2}$. Consider the previous measurements from the first measurement to measurement n, they also can estimate the intensity I^i by:

$$\frac{I^i}{d_{k-1}^2} + \lambda_b = z_{k-1} \quad (6)$$

$$\frac{I^i}{d_{k-2}^2} + \lambda_b = z_{k-2} \quad (7)$$

$$\vdots \quad (8)$$

$$\frac{I^i}{d_{k-n}^2} + \lambda_b = z_{k-n} \quad (9)$$

combining those terms, we have:

$$I^i \cdot \sum_{j=0}^{n} d_{k-j}^{-2} = \sum_{j=0}^{n} (z_{k-j} - \lambda_b) \tag{10}$$

$$I^i = \frac{\sum_{j=0}^{n} (z_{k-j} - \lambda_b)}{\sum_{j=0}^{n} d_{k-j}^{-2}} \tag{11}$$

Thus, Eq. 11 can be used to estimate the particle's intensity I^i using previous and current measurements.

1) Likelihood Function: By using Eq. 2, we can calculate the likelihood $\mathbf{x}_i$ of particle i by using the k^{th} measurement z_k and the predicted intensity λ_k^i from Eq. 1. Here, we approximate the Poisson distribution to the Gaussian distribution to reduce the computational complexity [6]:

$$p(z_k; \mathbf{x}^i) = \mathcal{P}(z_k; \lambda_k^i) \tag{12}$$

$$\approx \mathcal{N}(z_k; \lambda_k^i, \lambda_k^i) \tag{13}$$

However, since we use the previous and current measurements $z_{1:k}$ at $(x_{1:k}, y_{1:k})$, the new likelihood function is the product of all likelihoods from all measurements as:

$$p(z_{1:k}; \mathbf{x}^i) \approx \prod_{j=1}^{k} \mathcal{N}(z_j; \lambda_j^i, \lambda_j^i) \tag{14}$$

The state estimation of the particle filter is usually the average state of the particles with the highest weights.

2) Resampling Process: The resampling process is on how to remove the lowest weight particles and resample them elsewhere in order to avoid the problem of degeneracy. The degeneracy problem is the usual problem in the particle filter, which is a few highest weight particles dominate the distribution while most particles will have weights close to zero [18]. The resampling process will remove a portion of the lowest particle weights and resample them at the high weight particles using the roulette wheel selection [19]. In addition, the particles that have zero weight will be resampled as well. It is to ensure that some of the high weight particles will be selected and also add diversity to the population from lower weight particles. The newly born particles $x_{k+1}^{index_L^j}$ will have the a uniform distribution and spawn near the highest weight particles x_k^{target} added by Gaussian noise.

$$x_{k+1}^{index_L^j} = x_k^{target} + \mathcal{N}(\mu, \sigma^2) \tag{15}$$

3) Termination Criteria: The algorithm will converge when the termination criteria are met. In this case, we set the termination criteria as: if the root mean square error of the estimated intensity is less than 5% for 5 iterations and if the robot has visited the area within 1.5m around the estimated location, the algorithm has converged. The reason why the robot has to visit the estimated location is to prevent the algorithm from ending too soon, which may lead to the false location of the radiation source.

IV. Information Gain-based Exploration

The entropy is the tool to measure the uncertainty of a random variable [20], [21]. In this case, we want to evaluate the uncertainty of the map using the entropy to model this in the probabilistic manner.

$$H[P(\mathbf{x})] = -\int_{\mathbf{x}} p_i \log p_i \tag{16}$$

$$\approx -\sum_{i=1}^{n} p_i \log p_i \tag{17}$$

where, $p_i = P(\mathbf{x} = \mathbf{x}_i)$. In our case, the particle filter entropy is as the following equation:

$$H[P(\mathbf{x}|z_t)] = -\sum_{i=1}^{\#particle} w_i p(x_i|z_t) \log p(x_i|z_t) \tag{18}$$

where, $\mathbf{x}$ is the distribution of the particles. w_i is the weight of each particle i. z_t is the observation that we obtain at the time step t. Then, the action of the robot a_t can be evaluated using the expected information gain. It is the change of the entropy for the particle filter when we apply the action as:

$$I(\hat{z}, a_t) = H[P(\mathbf{x}|z_t)] - H[P(\mathbf{x}, \hat{\mathbf{x}}|a_t, \hat{z})] \tag{19}$$

where $\hat{z}$ is the observation to be obtained when action a is taken. This value can be calculated by Eq. 1, using the estimated intensity value from the current particle filter time step t. $\hat{x}$ is the new distribution of particles introduces by the action a_t. The expected information gain can be obtained by integrating all the possible measurements $\hat{z}$ when the robot takes action a_t.

$$E[I(a_t)] = \int_{\hat{z}} p(\hat{z}|a_t, z_t) I(\hat{z}, a_t) dz \tag{20}$$

However, to compute the expected information gain is usually a complicated task. Instead, the utility function using the greedy aspect of the maximum information gain I when the robot takes action a as:

$$\mathbf{a}^* = \underset{a}{\operatorname{argmax}}\, H[P(\mathbf{x}|z_t)] - H[P(\mathbf{x}, \hat{\mathbf{x}}|a, \hat{z})] \tag{21}$$

The utility function becomes:

$$U(a) = I(a) - \alpha \text{cost}(a) \tag{22}$$

where α is the weight of the cost of the action a that the robot has to take. We choose the best action, *i.e.*, the action that gives the highest utility as the robot action.

Fig. 1 shows the overall radiation map that the robot has to explore. The radiation map is the map with different radiation levels corresponding to the radiation source. According to Eq. 1, radiation intensity will reduce inversely proportional to the distance from the radiation source. Fig. 2 shows the particles of the robot that are the possible location of the source.

However, as the robot progress through the area, the entropy of the map becomes less and less significant, according to Fig. 3. On the opposite, when the robot is getting closer to the

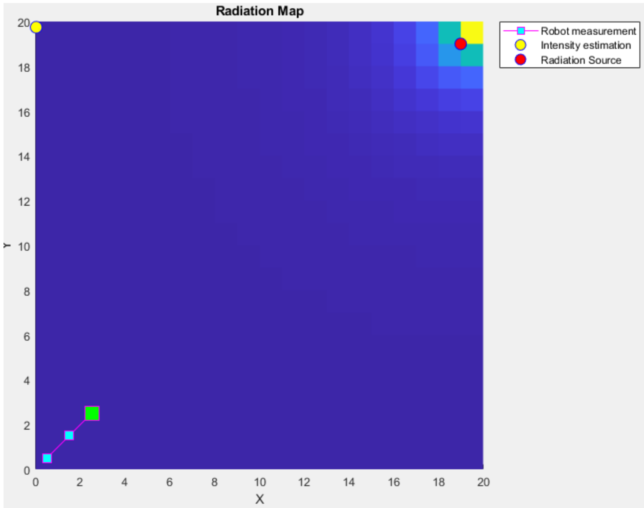

Fig. 1: The simulated environment with a robot represented by a green square. The small squares that follow the robot are the previous reading in the last two time steps $(k-1, k-2)$. The real location of the source represented by the red circle and the background of the map represents its corresponding radiation level. The estimated location of the radiation source by particle filter represented by the yellow circle.

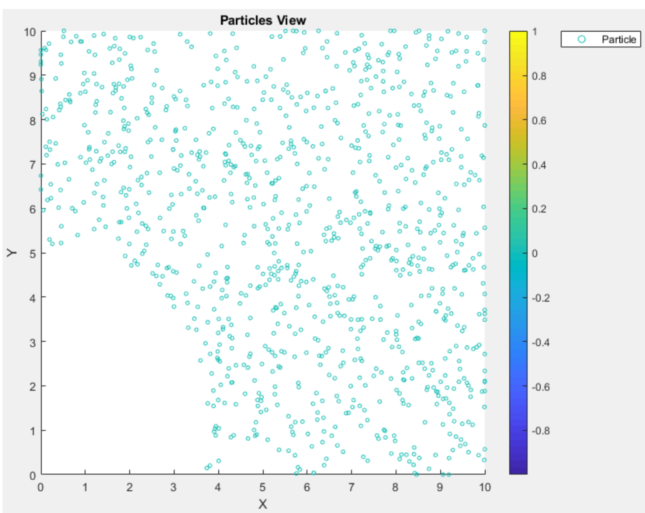

Fig. 2: The particles of the robot that represents the hypothesis of the source. The white area with no particle is the area that the robot already visited with low information, so the particles in the particular area are resampled elsewhere on the map.

radiation source, the particle filter estimation becomes more accurate. Thus, in order to get an accurate result, the robot should switch to use the particle filter estimation as the target, and takes the action that makes the robot get closer to the target.

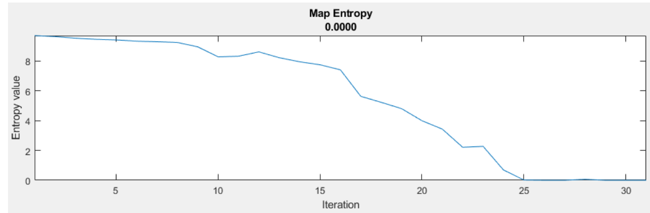

Fig. 3: The entropy value vs iteration when the robot explores the radiation field. The entropy value decreases as the iteration increases, which means the information that we gain from the map is lower after each iteration.

V. Experimental Results

A. Environment Settings

In the experiment, the environment is in a square shape, $20 \times 20\text{m}^2$ with no obstacle. We test each case with the scenario as in Fig.1, in which the radiation source spawns at the top right corner. The radioactive source has a random intensity between 30,000 to 50,000 counts. The background intensity is set to 1,000 counts [22].

B. Robots Settings

A robot in the test knows its location using a GPS with the assumption that the GPS reading has no error. The robot will always start at the furthest location away from the source at the bottom left of the map. The background intensity is known by the robot. There are 1,000 particles initialized. The map is divided into a small $1 \times 1\text{m}^2$ grid cell. The robot will measure the radiation only one time when they moved on the next exploration cell. It can move in 8 directions. When the robot is further away from the source, the information gain-based exploration will be used. The measurement threshold that the robot will switch to the source estimation seeking is 2,000 count.

C. Experimentation and Analysis

We run the experiment 100 times with different radiation intensity counts. We notice that the path of the robot of the proposed method is not the optimum path, which is to be expected because of the uncertainty of the measurement. We tested our algorithm versus the well-known methods such as the lawn mowing algorithm [23] and source estimation seeking method [8].

Table I shows the average travel distance and average error in intensity and position that the robot takes to finish the task. It clearly shows that the traditional method, such as the lawn mowing algorithm that does not rely on a sensor to guide the robot to the next possible best action as it gives the highest travel distance. The source estimation seeking method gives a competitive travel distance compared to the proposed method. However, the proposed method works better since it selects the next best action based on the maximum information gain.

In the accuracy aspect, the lawn mowing algorithm has a lot more measurement points that result in higher accuracy. The estimation seeking method and the proposed method have

TABLE I: Performance comparison of the proposed method vs traditional methods

	Travel distance (m)	Position Error (m)	Intensity Error (count)
Lawn mowing (row)	381.00	0.01	115.05
Lawn mowing (column)	381.00	0.03	152.90
Estimation seeking	39.67	0.23	1,661.40
Proposed method	36.16	0.20	1,283.00

a lower number of measurement points, but they give decent accuracy in terms of both intensity and position. Fig. 4 gives one example of the route that the robot takes using the proposed method and its corresponding entropy value.

VI. Conclusion

In this paper, we presented a strategy for a radiation source localization using robotic systems. The particle filter is employed to deal with the measurement uncertainty of a radioactive measuring instrument. We utilize the information gain-based exploration to solve the problem of navigation in low-intensity areas and measurement uncertainty by using the information from the particle filter to determine the best possible action for the robot. We compare the proposed method to the traditional methods such as lawn mowing and the source estimation seeking methods. The proposed method clearly shows that it surpasses all of the mentioned methods in terms of the average traveling distance, which means the robot takes less time to finish the task. It also gives a decent accuracy (0.1949m positional error and 1,283 counts intensity error) compares to the other methods.

Future work of this research may include real environment testing using mobile robots, such as UAVs or ground robots. The increment in uncertainty, such as low GPS accuracy, and false or loss in information, can be highly challenging. The mapping of radiation sources with disturbances from natural factors is also an important research topic.

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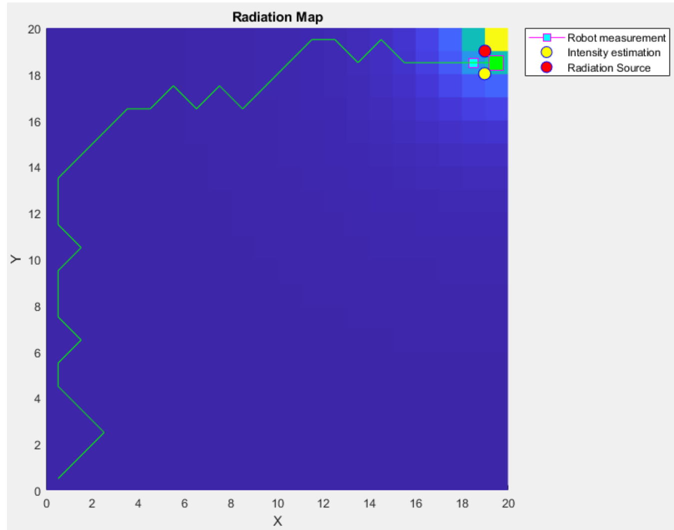
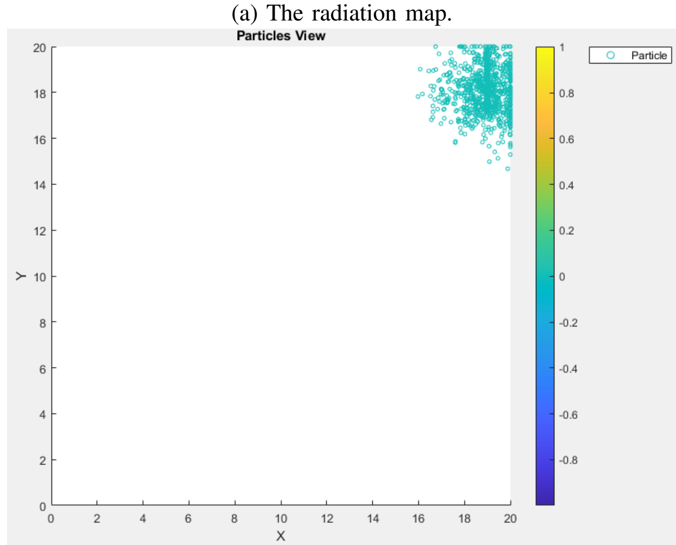
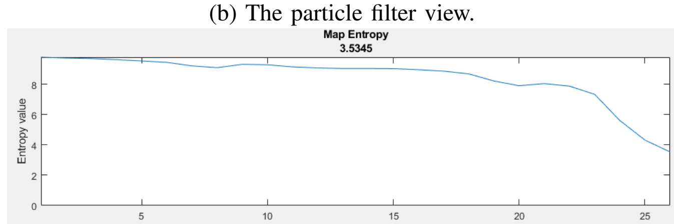


(a) The radiation map.



(b) The particle filter view.



(c) The map entropy value.

Fig. 4: The green line in the radiation map (a) shows the route that the robot takes to localize the radiation source. The particles in the particles view (b) converges to the source. The map entropy decreases as the iteration increases as expected.

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