Abstract

This study deals with rep-cube at the frontiers of computational origami. Computational origami is a new field of study, and there are many unsolved problems in this field, including problems with rep-cube as in this study. By obtaining new results for rep-cube, we expect that future issues and unsolved problems will apply for future development in the research field.

Rep-cube is a new concept that was born in 2016 from the natural question "Is there any polyomino that folds to a cube, and its dissections fold to cubes?". In other words, it is a polyomino that is a net of a cube. This was inspired by two famous concepts: polyomino and rep-tile. Polyomino and rep-tile were proposed by Solomon W. Golomb. Since then, it has been researched extensively in the puzzle industry and has played an important role in recreational mathematics. Polyomino is a set of unit squares. Rep-tile is a polygon that can be divided into replicas congruent to one another and similar to the original.

A polyomino is a rep-cube of order k that means it can fold into a cube and it can be divided into k polyominoes of which each of them can fold into a cube. If all k polyominoes have the same area, the original polyomino is a regular rep-cube. Also if all k polyominoes are congruent nets, the original polyomino is a uniform rep-cube.

So far, several rep-cubes have been found by trial and error. Previous studies have shown that regular rep-cubes exist for order $k = 2, 4, 5, 8, 9, 16, 18, 25, 36, 50, 64$, by actually making rep-cubes. It was also found that for any positive integer k' and any element g of the set $\{2, 4, 5, 8, 9, 16, 18, 25, 36, 50, 64\}$, there exists a rep-cube of order $k = 36gk'^2$. This means that there are infinitely many regular rep-cubes. Moreover, an algorithm was designed to enumerate regular rep-cubes for order $k = 2, 4$ of area $6k$ with a check for polyominoes that can cover a cube of size $(\sqrt{k} \times \sqrt{k} \times \sqrt{k})$ without overlap. This algorithm applied to enumerate and analyze all regular rep-cubes of order $k = 2$ with area 12 and order $k = 4$ with area 24. However, rep-cubes have the same folding way and the same contour counted as different rep-cubes with a different division. Therefore, it needs to reconsider the rep-cube as a division of the surface of a cube.

The purpose of this study is to consider as based on the division patterns of a cube considered an approach based on the division of a cube. To achieve this, we designed the algorithm from the conventional method of fold a given polygon to unfolding the given polyhedron. It is also to discuss the value of the order k that can construct rep-cubes. For the case of a regular rep-cube, we consider the case for values of order k for which a regular rep-cube does not exist. For the case of a uniform rep-cube, we consider whether a uniform rep-cube can be constructed for every 11 nets of the unit cube.

First, we proposed an algorithm to divide the surface of a cube of $(\sqrt{k}\times\sqrt{k}\times\sqrt{k})$ by a unit square. We applied this algorithm to experiment with the program for cases unfolding a cube of $(\sqrt{2}\times\sqrt{2}\times\sqrt{2})$ into a polyomino of area 12 and unfolding a cube of $(\sqrt{5}\times\sqrt{5}\times\sqrt{5})$ into a polyomino of area 30.

Next, we proved the value of the order k for which regular rep-cubes do not exist as a general characterization of the numbers for which regular rep-cubes do not exist. Based on this, we explored the value of order k at which regular rep-cubes are expected to exist and found a new regular rep-cube of order $k = 10$ and a regular rep-cube of order $k = 13$.

Finally, for each of the 11 polyominoes of the unit cube, we proved the value of the order k at which a uniform rep-cube can be constructed. Although no uniform rep-cubes have been found so far for each two P_x and P_y of the 11 polyominoes of the unit cube, we showed that new uniform rep-cubes of order $k = 8$ can be constructed for each of the polyominoes P_x and P_y. We also showed that there does not exist a uniform rep-cube for order $k = 5$ for each of polyominoes P_x and P_y. By doing this, it indicated that a uniform rep-cube can construct for each of the 11 polyominoes of the unit cube, and the value of the smallest order k that constructs a uniform rep-cube was determined.