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Efficient Enumeration of Non-isomorphic Ptolemaic Graphs

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In Euclidean geometry, Ptolemaic inequality relates six distances by four points in the plane. For any four points A, B, C, D, Ptolemaic inequality is represented as $\overline{AC} \cdot \overline{BD} \leq \overline{AB} \cdot \overline{CD} + \overline{BC} \cdot \overline{DA}$. By Ptolemaic inequality, the characterization of Ptolemaic graphs is easy to understand. A Ptolemaic graph is a connected graph which for any four vertices u, v, w, x of G, $d(u, v)d(w, x) \leq d(u, w)d(v, x) + d(u, x)d(v, w)$ holds.

Howorka shows that the class of Ptolemaic graphs is the intersection of the classes of distance hereditary graphs and chordal graphs. Hence the Ptolemaic graphs also hold the properties of both the chordal graphs and the distance hereditary graphs. A graph is said to be chordal if every cycle of length at least 4 has a chord. The class of chordal graphs is well investigated with related massive research. On the other hand, a graph G is distance hereditary if it is connected and every induced path is isometric; that is, if the distance function in every induced subgraph of G is the same as that in G itself. The vertex incremental description is one of the ways of the characterizations of a graph class, which means, by applying vertex incremental rules that add one or several vertices each time, all graphs of a certain graph class can be obtained. The vertex incremental descriptions of the classes for both distance hereditary graphs and Ptolemaic graphs are proposed by Bandelt and Mulder in 1986.

Uehara and Uno give the clique laminar tree (CL-tree) to represent a Ptolemaic graph as a tree structure. The clique laminar tree represents laminar structure on cliques in a Ptolemaic graph. Using CL-tree, Tran and Uehara propose an enumeration algorithm of Ptolemaic graphs in 2020. However, it only shows the two phases of the algorithm and gives the polynomial upper bound between the enumeration of two Ptolemaic graphs. In 2009, the DH-tree is proposed as the tree representation of the distance hereditary graph by Nakano et al. As the class of Ptolemaic graphs is the subset of the class of distance hereditary graphs, the DH-tree can also be applied to the representation of Ptolemaic graphs. By converting the DH-tree to a string representation, the graph isomorphism of distance hereditary graphs can be solved efficiently. As one of the applications for the DH-tree, Nakano et al. also give the theoretical time complexity for enumerating distance hereditary graphs by using DH-trees, whereas no specific algorithms are given. In 2018, Yamazaki et al. proposed an enumeration framework for the graph classes, which uses reverse search as the technique to avoid duplicates and solve the graph isomorphism of the graph class efficiently. Using the framework, Yamazaki et al. proposed a new enumeration algorithm for distance hereditary graphs, which uses the vertex incremental characterization of distance hereditary graphs. Since the class of Ptolemaic graphs holds a similar vertex incremental characterization with the class of distance hereditary graphs, we can modify the algorithm to enumerate Ptolemaic graphs.

In this paper, we focus on the enumeration algorithm for Ptolemaic graphs. We first introduce the related work. Next, we give the preliminaries, which include the vertex incremental characterizations of both distance hereditary graphs and Ptolemaic graphs, then we give the notion of the DH-tree and reverse search. By proposing the notion of a function, we give an efficient way to compute if a vertex is simplicial. Then, by modifying the enumeration algorithm from distance hereditary graphs, we give the enumeration algorithm for Ptolemaic graphs, which enumerates all Ptolemaic graphs with at most n vertices in $O(n^3)$ time for each.