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Doctoral Dissertation

Uncertainty dynamics in games and its implication to entertaining activity assessment

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Absract

A scientific theory's beauty is its ability to describe a vast area of knowledge from a few basic principles and make experimentally testable predictions. Such a notion aligned with employing game to access the world of the mind and established the law of motion in mind via game-playing, which formalized by considering the game progress model. A theoretical analysis using empirical data confirms such a model's effectiveness, where reasonable game length is determined based on the definition of a gamified experience. The discussion underlies correlations to physics in mind and physics in the real world as established by Newton, Einstein, and Hawking et al., which profoundly change how people perceive the world and how the players relate to games.

This dissertation focuses on the uncertainty dynamics in games and its implication to analyzing entertaining activity. (1) Firstly, the human player-versus-human player rules based card game is firstly discussed, which can be competed by multiple players at the same time but can maintain the same competition for each player. This study set experiments and simulate DouDiZhu self-play. The preliminary simulation results revealed that three-player DouDiZhu is perfectly refined with sophistication, entertainment, and fairness on the measurement of game refinement at $DouDiZhu[(3,1,2)(20,17,17)]$. Cooperation is essential to maintain the fairness and uncertainty of both sides. (2) It then explores single-player arcade games where human players simply compete against the rules of the game and can significantly observe individual player skill improvement through practice. Four popular arcade games are selected and analyzed as benchmark. The application scope of the theory of motion in mind is expanded by incorporating relative velocity and resultant force. A feasible scheme of potential growth rate is proposed to measure the single-agent arcade games that were unquantifiable before because these kinds of games have no definitive game length. It found jerk's dynamics would affect game's uncertainty to a great extent and is the essential factor to sustain the game's engagement. (3)Then we discuss idle games, which are not competitive and even encourage players not to participate in the progress of the game. Uncertainty is the expectation to closing the gap between income and cost in idle game domain. It also found that not only the equations of motion in mind model would help to analyze the idle games, but also the derivatives of the functions are also able to. Motives in mind (E_q) and predictive motion tendency ($\vec{p_2}$) are found to be the most important parameter when applied to the entertainment without much interaction. Moreover, long-term jerk was found able to maintain the freshness to the player. Synchronization can also help to maintain the engagement of the player.

From chapter to chapter, the model of motion in mind is also established, optimized its range, and evolved into a relative sophisticated shape. Taking different games and recreational activities as objects and using the construction of physical models in the mind as media, this study explores the effects of dynamic changes of uncertainty on the entertainment and attraction of games and recreational activities. Through the physical modeling of the motion in mind, this dissertation innovatively constructs the operation mode of the human mind world, provides a brand-new understanding and angle for the study of either human beings themselves or the nature of entertainment in different areas of the motion in mind model and is meaningful to establish entertainment science.

Keyword: *Game; Entertainment; Uncertainty; Information dynamics; Engagement; DouDiZhu; Arcade; Idle game*

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Chapter 1

Introduction

1.1 Background

As the global outbreak of COVID in 2019, people's lives all over the world have changed a lot out of necessity, gradually moving towards a post-epidemic era. As a direct result of the epidemic, which made people physically unable to contact each other, all sectors of the Internet have grown by leaps and bounds. The game industry, however, has been a blessing in disguise, with new users and explosive growth.

As Huizinga claims, games predate human culture [7]. With the emergence of games, human culture gradually began to take shape. One of the beauty of history is that there is always more and new to learn. Humans create games and, at the same time, learn from games. The shape and forms of game have obviously evolved along the history. Since ancient games such as Senet in ancient Egypt, Go in ancient China, Backgammon in ancient Rome, Patolli in ancient South America, games have shown the characteristics of fun, mystery, skill and chance. The core of gameplay is still what we are using today.

With the rapid development of hardware equipment, games are becoming more and more delicate in the performance of screen effects, and the application of 3D technology has also made a lot of achievements in the field of games. Many game sequels have attracted a lot of loyal players to continue to stick to the game because of the qualitative leap in visual performance, forming a very good user group backflow. The most remarkable thing about games is that the communication with the audience is often deeper, and games are the most immersive products.

Meanwhile, artificial intelligence (AI) advancement has driven games gradually evolving into an accessible way to simulate human society and explore more profound insights into human science. Some also claim that game is more considered “re-created learning”, as a form of ability [8] and beneficial to human beings. Many research efforts have investigated into reasons for a game’s attractiveness, from rules alteration to game setting. With the development of digital game technology, various new games gradually formed, becoming more sophisticated in either the types of rules, attracting more players, and initiating new research interests.

Scholars nowadays are focusing on games mostly from the perspective of entertainment technology such as optimization algorithms, principles of design or the perspective of interaction with human-being like communication studies, psychology, philosophy and so on to explore for the significant influence of human study, work, and life [9]. There are also emerging research trend that tend to explore the measurable sophistication of games that harmoniously address fairness, entertainment, and challenge [10]. But the essence of the engagement in game still needs further investigation.

The human mind, often considered synonymous with consciousness, has been called the last frontier of science [11]. The world of human mind is as mysterious as the universe, where the mind physics could be reasonably related to physics in nature. Current understanding of the mind physics has been discussed in psychology, cognitive science, and philosophy fields [12]. A scientific theory’s beauty is its ability to describe a vast area of knowledge from a few basic principles and make experimentally testable predictions [13]. Such a notion aligned with employing games to access the world of the mind and established the law of motion in mind via game-playing, which formalized by considering the game progress model [6]. The theoretical analysis using empirical data confirms such a model’s effectiveness, where reasonable game length is determined based on the definition of a gamified experience. The discussion underlies correlations to physics in mind and physics in the real world as established by Newton, Einstein, and Hawking et al., which preliminarily proved to be possible. It also profoundly change how people perceive the world and how the players relate to games [6].

1.2 Problem Statement

Most designers today design to provide a game experience. Designers look at game design in the same way. The ”simulation” approach can discover interesting design elements in the rules of a game [14]. When it comes to actual design, some higher dimensional reflections (even directly using psychological and anthropological conclusions) and extensive testing are needed to ensure that the experience is as consistent as possible.

In addition, in order to produce “experience” more intuitively, games can also borrow from other art forms, especially the lower sensory inputs that are the primary human inputs, such as sight and hearing (in this case, as opposed to concepts like “logic” and “thinking”). This is not hard to understand: in order for the player to “experience”, they need to “simulate” the situation in real life, which in itself includes the basic sensory input of a human being [15]. If technology can one day easily reproduce touch, smell and taste, these sensations will also be used by games to create a more complete “experience”.

However, there is still no study well and comprehensively exploring the game process in mind from the dimension of information dynamic. Various types of fun games and serious games have been explored from different aspects, which become closer to the nature of real entertainment perceived in mind.

Competitive games, or confrontational games are focusing on a single specialized player (Go, Chess, Mahjong, Olympic track and field events), while cooperation games that need teamwork (Bridge, Diablo 3, Olympic games except for track and field events). It has been known that creating confrontation is the fundamental means of a game, whether it is between players or between players and rules, where the goal is to give users timely feedback on wins and losses [16]. Game theory explains how and why cooperation emerged [17]. An essential condition for cooperation is that both sides will reciprocate, where cooperation can happen when both are equally profitable. However, how cooperation would affect a game, especially a competitive game is not well understood. Hence, it is interesting to observe cooperation in competitive games and find how they affect the engagement of the game. In addition, there are different competitive patterns and settings of different game-play. Therefore, it is also necessary to explore how the setting of a game would affect engagement in a game.

A great game will “take one minute to learn but a lifetime to master” [18]. In general, there are at least two kinds of games: finite and infinite. A finite game is played to win, whereas an infinite game involves continuing the play [19]. By Grossone, Louis D'Alotto presents a new model of infinite games on finite graphs. It provides some advantages, such as allowing for draws common in video games, and a more accurate and decisive way to determine winners [20]. Although many players seek coordination under imperfect information, infinite games are considered to be undecidable. Dietmar Berwanger et al. identify a class of games that can effectively construct joint winning strategies without limiting the direction of information flow [21]. Colcombet [22] presented a new breakthrough in the theory of infinite persistent games: the existence of a quasi-polynomial time algorithm for solving even and odd games by using two objects: automata games and universal graphs. William A. Donohue [23] argues that negotiations are usually a finite game, however, the activities and events leading to negotiations can be viewed as an infinite game. When the critical moment occurs, the parties can transit from an infinite game to a finite game. The study on engagement of arcade game would provide understanding of the model of motion in mind on gamified area. Overall, studies so far mainly focused games with a distinct goal or limited time. Currently, there is no general method to measure the entertainment properties of infinite games. Research into such games that without specific length requires a quantitative approach with scalable applications.

Game study never ignores the discussion on the interaction with game and player, which has been taken as the crucial part of game design [24]. There is a new game genre called idle games that are popular these year. It is fascinating and perplexing. Marked by minimal player agency and periods of inactivity, they seem to defy conventional logic about good game design, and yet nonetheless have attracted substantial players. Researchers have drawn many lines on idle games and measure the attractiveness to human player with ground theory and found a precise category for idle games. However, most of the studies only focus on the mechanics of the game. They try to explain the attractiveness gameplay mainly from a psychological point of view, seldom of which have mentioned about the game design or the information process during the game. Therefore, the systematic quantification is required on these kind of games to find out where the engagement lies in.

To sum up, different aspects of gaming engagement require further investigation as means to access the mind physics with the model of motion in mind.

1.3 Research Objectives

A scientific theory's beauty is its ability to describe a vast area of knowledge from a few basic principles and make experimentally testable predictions [13]. This dissertation aims to probe into a deeper sight of the mechanism of physics in mind using game and game-liked activities as the benchmark since games are embedded and grown with the human culture even before human society emerged [7]. Such a notion aligned with employing game to access the world of the mind and established the foundation of the law of motion in mind via game-playing [6].

Iida et al. have discussed the line between work and play [10]. Likewise, by understanding competitive zero-sum games as entertaining activities, we can have a better comprehension of the interpretation of entertainment and engagement. Physics-in-mind had been discussed in recent years [25]. Using the correlations to physics of the real world as established by Newton, Einstein, and Hawking et al., it will be possible to understand the physics in the mind [6].

Although sophistication of different games via the informational acceleration had been widely used and evaluated [26], there are still some room for investigating games with imperfect information, unfixed goal or time (e.g., arcade games), and long-term interaction-driven (e.g., idle games). Formalized by considering the game progress model [6], this thesis is about establishing the correlation between the psychological model and the information model, incorporating concept from physics, which can help people to better understand the human life. As such, possible connections between social behaviors such as Flow theory and addiction model can be discovered [1].

The main purpose of this study is to expand the theory of motion in mind and apply it to explore the internal mechanism of the popular games; thus, increases the possible utility range of the motion in mind. To this end, the main contributions of this dissertation are as follows:

- to conduct research on competitive games to find out how game setting would affect the game popularity and engagement;

- to further discuss and improve the model of motion in mind by defining the measurement of single player games without specific length;

- to discover the characteristics of each game or activity from the perspective of information dynamic, and reveal the internal laws behind engaged human activities by applying the theory of motion in mind to the entertainment field;

- to extend the range and further construct the model of motion in mind and to establish entertainment science by incorporating the nature of entertainment in different areas of the model.

1.4 Research Route and Thesis Structure

This dissertation firstly reviewed related works for the target research field and found unsolved directions. It then takes a famous card game in China as the test-bed to study the effectiveness of two sides setting of the game in Chapter 3. Result shows the game maintain competitiveness of two sides based on different features. It then comes to the arcade games to explore the continuity of the game in Chapter 4. Instead of focusing on the features of the game, Chapter 4 basically found that the continuity of arcade games can maintain the competitiveness as well. This dissertation digs into the idle games in Chapter 5, which changes how players perceive the continuity of the game without competitive sides. In general, we try to explain the game uncertainty in three particular directions. It reveled which feature is much more relevant or close to what we define entertainment in our daily life. The research route is shown in Figure 1.1.

This research comprises of 6 chapters:

- **Chapter 1: Introduction**

  The purpose of this chapter is to introduce the general situation and research background of the research. How each keyword is related in the study, and a brief historical overview of the areas involved. It explains the main problem to be solved in this study. It also explains the research problem, the purpose and significance of the research. The structure of the thesis is listed at the end of this chapter.

- **Chapter 2: Related Works.**

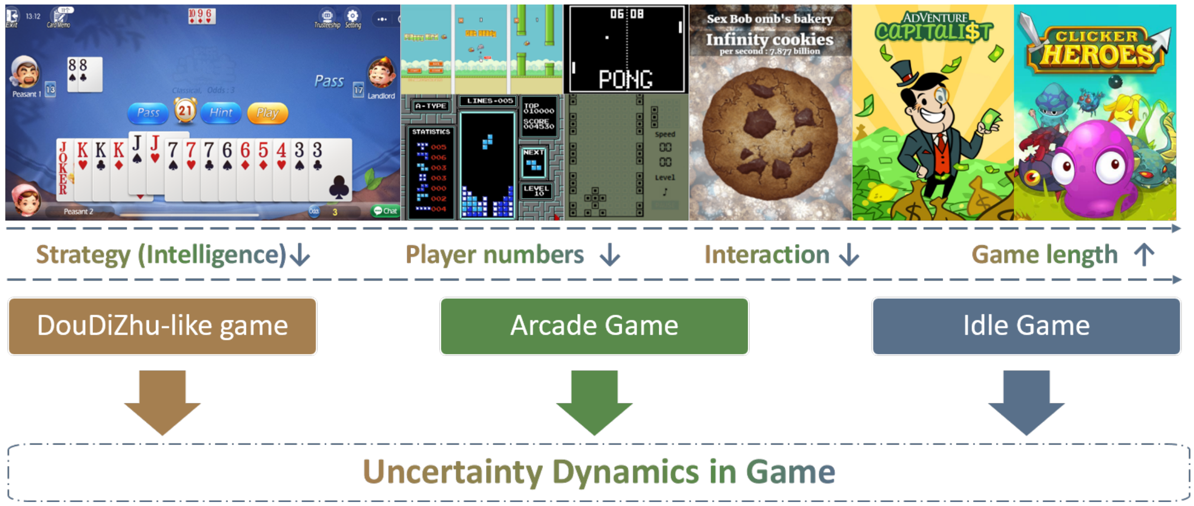


Figure 1.1: Research route of the thesis. The games from left to right are respectively the card game DouDiZhu; single player arcade game Flappy Bird, Tetris, Pong, and Brick car racing; idle game Cookie Clicker, Adventure Capitalist, and Clicker Hero.

This chapter reviews the theoretical background related to this research and introduces the latest research results in game field. And it also illustrate the basis of the study on motion in mind.

- **Chapter 3: Uncertainty under Different Settings.**

  Card games are typical imperfect dynamic information game which is taken as the next milestone for artificial intelligence. DouDiZhu, a multiplayer game with imperfect information, is the most popular card game in China. Although there are many similar card games in the world, the specific characteristics of classical DouDiZhu card games are a harmonious combination of player Numbers, player characters (landlords and peasants), deck Numbers, and scoring systems. However, research on the complexity and attractiveness of DouDiZhu has not been established. Therefore, in this chapter, artificial intelligence (AI) players of different levels of DouDiZhu game were constructed for research, self-play simulation was conducted for DouDiZhu AI players, and game refinement measures were used to evaluate and identify the best Settings of the game. The results show that classical DouDiZhu provides the most complex game setup for all types of DouDiZhu AI players, while also clarifying its popularity.

- **Chapter 4: Uncertainty Dynamics in Game Process.**

  Arcade game is one of the earliest forms in the history of modern video games.

The games have simple rules, but they keep people interested and enjoyable. This study selected four arcade games: Flappy Bird, Pong, Brick Car Racing and Tetris, collected data by simulation and tournament statistics of the games. From the perspective of physics in mind, the study analyzes the changes of physical quantities during the game process of arcade games to explain the reasons why arcade games are addictive, and gives the ideal range of risk frequency ratio of arcade games, as the essence of uncertainty in game. Meanwhile, we found that the dynamics of jerk is the most important nature for the appeal of arcade games. Moreover, concept of *relative velocity* and *resultant force* are raised for single agent games, taking a meaningful step in the exploration of the theory of motion in mind as well as the discussion on uncertainty.

- **Chapter 5: Long Term Uncertainty with Little Interaction.**
  Idle game is a relatively new game genre, which seems different from the traditional game design concept, lacks of uncertainty and is disputed by people as "whether this is a game or not". Existing studies have fully analyzed the historical changes of idle game and the game mechanism design to explore the reasons why it has attracted people's attention and obsession, but all of them are relatively abstract. There is very little literature on the nature of the idle game and the actual player experience. From the perspective of motion in mind, an innovative and profound game experience, this chapter discusses the causes of uncertainty in idle game and the real fun of the game. This study finds that idle games with m=*0.11* are affected mainly by E_q and $\vec{p_2}$. And it has a balance between interest and boredom. Idle game has a long-term jerk which can maintain the freshness to the game. With barely no time lag, players have the feeling of synchronization, i.e., the resonance with the game. It can also be applied to deliberate practice activities. This chapter extends the motion in mind model and propose the concept of law of attraction in mind. It provides a good perspective for people to understand the charm of idle game, and also proposes some starting points for the development and design of idle game in the future.

- **Chapter 6: Conclusion.**
  Chapter 6 gives the final conclusion of the whole thesis and illustrates the significance

of the findings both in theoretical and practical sense. Deficiency of the current study result and future direction are given at the end of the chapter.

Chapter 2

Related Works

2.1 Chapter Introduction

This chapter reviews the theoretical background related to this research and introduces the latest research results in game field, and illustrate the basis of the study on motion in mind.

2.2 Game and Uncertainty

With the emergence of games, human culture gradually began to take shape [7]. The information revolution of the past few decades has created a new virtual world, and today the virtual world has begun to profoundly influence the operation of the real life [16]. In current entertainment situation, game is increasingly becoming an ability, which can make people happy, repair themselves, or gain strength in the virtual world created by others. At the meantime, with the continuously upgrading of social informatization and increasingly sophisticated processing of daily affairs, people have an increasingly strong demand for information when playing games [27]. The process of a game is mutually influenced by information environment, information object, the user's cognition, psychological and irrational psychological factors [28].

A growing body of research has shown that the elements of a particular game can be fully aligned with the various "capacity requirements" of a particular knowledge. Everything in the game needs to be explored and played constantly in order to get a real connection with people. When people start to get tired of the rules they've learned, they start to create new ways to play, new values [29]. Therefore, by understanding game, we can understand the transcendent parts of life. More and more scholars begin to pay close attention to the game, mostly from the perspective of communication studies, psychology, philosophy and so on to explore for the significant influence of human study, work and life.

To "play a game" is to "willingly overcome unnecessary obstacles [30]". Understanding game mechanic is essential to create a good game. Game mechanic is a system of balances based on what exists. Jesse Schell, in his book The Art of Game Design, writes that game mechanics are the true core of a game [31]. They are the interactions and connections that remain after all aesthetic expression, technical implementation, and story setting have been stripped away. As Richard Rouse noted, game mechanics describe what the player can do in the game world and how they help improve the player's experience in the game, and are an expression of coordination and balance [32].

An argument like whether the universe governed by certainty had been addressed by the pre-Socratic scholars in the early days of Western reason. More than 2,000 years later, we still face these problems. The stars in the history of human video games, in addition to leaving players countless praise and moved, but also become the bottom elements of the next generation of new games. Recent advances in physics and mathematics related to chaos and instability, however, have opened up different avenues of study. As Brian Arthur mentioned in The Nature of Technology that technology must be a new combination of previously existing technologies to some extent [33].

As one of the earliest carriers of games, dice symbolizes chance and randomness and constantly remind people of the randomness of the universe. Researchers have done many work to illustrate that uncertainty is the most important factor of games. Howard Jones and Demetriou [34] investigated the effect of game uncertainty on learning. By measuring electrical skin activity, they showed that participants in the uncertain condition experienced higher arousal. This demonstrates that players do not like specific choices in a game, but rather prefer uncertainty itself. Uncertainty in a game can not only improve players' ability to code and recall, but also increase their willingness and interest to play the game [35] and increase their engagement. It is not easy for a game to change a person's basic preference for uncertainty, but it can be designed to include different mechanics and content to appeal to different levels of players [36]. The concept of a preference for uncertainty is also much more complex than it seems: it depends on factors such as an individual's ability to tolerate emotional discomfort, confidence in their ability to deal with uncertainty, and familiarity with their uncertain territory [36]. Costikyan's vision focuses on game mechanics and the designer's role in creating uncertainty, and analyzes the sources of uncertainty in games, from uncertainty about how to correctly solve puzzles to uncertainty about what other players will do [37].

But too much uncertainty may create a bad experience. Gu et. al [38] have reviewed the history of intolerance of uncertainty and proved that uncertainty about future events may lead to worry, anxiety, and even inability to function properly. Uncertainty fosters negative emotions, skewed expectations and inflexible reactions. Understandably, uncertainty is supposed to draw a line between what makes people feel good and what makes them feel bad.

Uncertainty usually is measured by entropy which is a widely used indicator. It is "the property of a system to develop to its most stable internal state when it is free from external disturbances", firstly proposed by the German physicist Clausius in 1865 [39]. In 1948, Claude Elwood Shannon introduced entropy from thermodynamics to information theory [40]. In information theory, entropy is the average amount of information contained in each received message as in Equation 2.1, also known as information entropy, source entropy, and average self-information.

$$H(X) = \sum_i P(x_i)I(x_i) = -\sum_i P(x_i)\log_b P(x_i) \tag{2.1}$$

Entropy measurements can predict the uncertainty of events. It reaches maximum when the probabilities of all events are equal. According to Christopher Fiorillo et al., uncertainty is determined by the probability P of the occurrence of the event, which is maximum at P = 0.5 and decreases at higher or lower probabilities [41]. Studies showed that information in game, i.e., uncertainty of game outcome, can also measured by the application of information entropy. And it is proved that most of the see-saw game with maximum uncertainty would be the most sophisticated [42].

However, in the field of game research, the measurement of uncertainty is still in the stage of trying to explore, and no consensus has been reached. The discussion of uncertainty still needs more practical discoveries.

2.3 Player Experience and Entertainment

Engagement is often associated with immersion, involvement, presence and flow, but engagement itself is murky. Bouvier et al. attempt to define and describe engagement in digital games [43]. They divided engagement behaviors into four categories: environmental, social, self and action. But the problems raised by the article itself have not been solved concretely.

O'Brien et al. define user engagement as "a quality of user experience with challenge, aesthetic and sensory appeal, feedback, novelty, interactivity, perceptual control and time, awareness, motivation, interest and influence" [4]. This definition is too realistic, mixing the factors and consequences of engagement, but it did not really describe engagement.

Players would easily get addicted to arcade games [44], involved in a kind of psychological state unconsciously, and become highly focused and engaged. Maslow defined such a phenomenon as the peak experience [45]. As shown in Figure 2.1[1], people at the peak experience have the highest degree of identification. They are able to reach the peak of their unique personality or traits, and to maximize their potentials.

[1] https://www.simplypsychology.org/maslow.html#gsc.tab=0

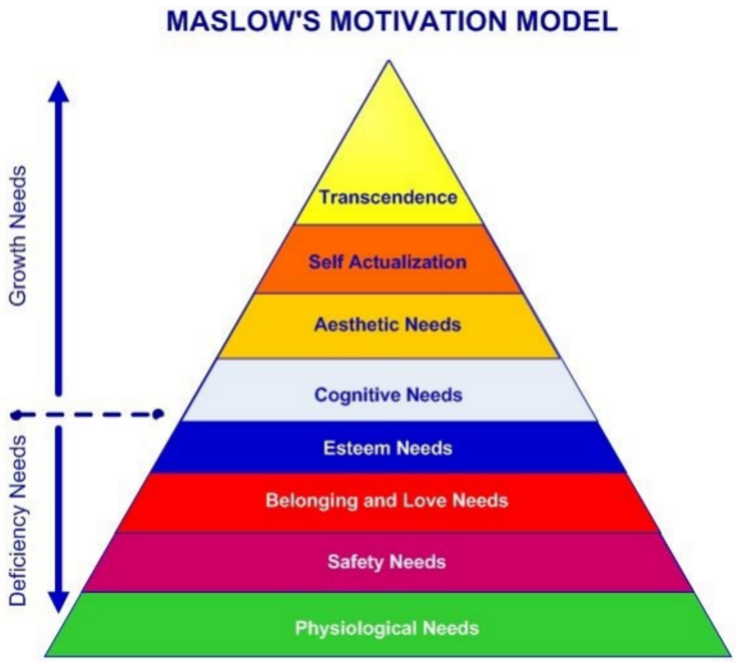


Figure 2.1: Maslow's motivation model

As Csikszentmihalyi points out, flow is the state of deep enjoyment of people, which can lead to a high quality of life [46]. Flow theory introduces a mechanism to get into the flow, through which people could find an approach to a better life. Playing games is a process of eliminating the uncertainty of game outcome, which is regarded as a typical flow activity.

Flow is originally derived from Csikszentmihalyi's observations of artists, chess players, climbers, and composers in the 1960s. He observed that when these people were engaged in their work with almost total concentration, often losing track of time and awareness of their surroundings [47]. They are engaged in their individual activities out of a common sense of fun. These pleasures are derived from the process of activity, and the external rewards are minimal or non-existent. This flow experience caused by concentration is considered by Csikszentmihalyi to be the best experience usually happening when skill people acquired and challenge they faced at a comparable level. The formula is shown in (2.2).

$$Flow = \frac{Skill}{Challenge}\text{(the goal is 1.0)} \tag{2.2}$$

As Csikszentmihalyi put it, immersing in the state of flow has specific general characteristics [47]:

- Total immersion. People totally immersed in the activity with enthusiasm.

- Ecstasy. People have the feeling of being in a space without any disturbance.

- Clear mind. People know what they are doing and what to do next.

- Sense of control. People realize there is challenge to finish their tasks, however they can reach the destination after some effort.

- Peace and calm. People forget about themselves and even lost their self awareness.

- Fleeting hours. Time flies without notice during the immersion.

- Inner motivation. People have the sense they spontaneously do it in the sake of their natural needs.

Flow theory offers a novel and feasible way to happiness. It has drawn a lot of attention of scholars from different fields, as the academic, education, business service. Many of the research are talking of the mechanism from the conceptual model [48–50]. Meanwhile most of which are focusing on the methods of getting into the flow [48, 51, 52]. Recent years, it has been widely applied to the field of game design or gamification design [53–55]. Realizing how flow is important for us to enjoy a happier life, scholars began to quantify the flow state. Most of the research are trying to explain the flow and the experience from the conceptual model proposed by Csikszentmihalyi. The main measurement methods to quantification the flow state are the flow questionnaire, the experience sampling method and the standardized scaling method. Giovanni B. Moneta analyzed these methods and suggested that all the methods should be improved in order to enable them to test the tenet that there is on and only one flow state [56]. Few of them had measure flow more concretely and quantified from the mind angle.

Theoretically, people would get into the flow when their skill is equivalent to the event challenge, as shown as a straight line in Figure 2.2. Nevertheless, in the real process, both skill and challenge dynamically interacting, since the challenge is inversely proportional to skills. As players become familiar with the game and become strong, the challenge

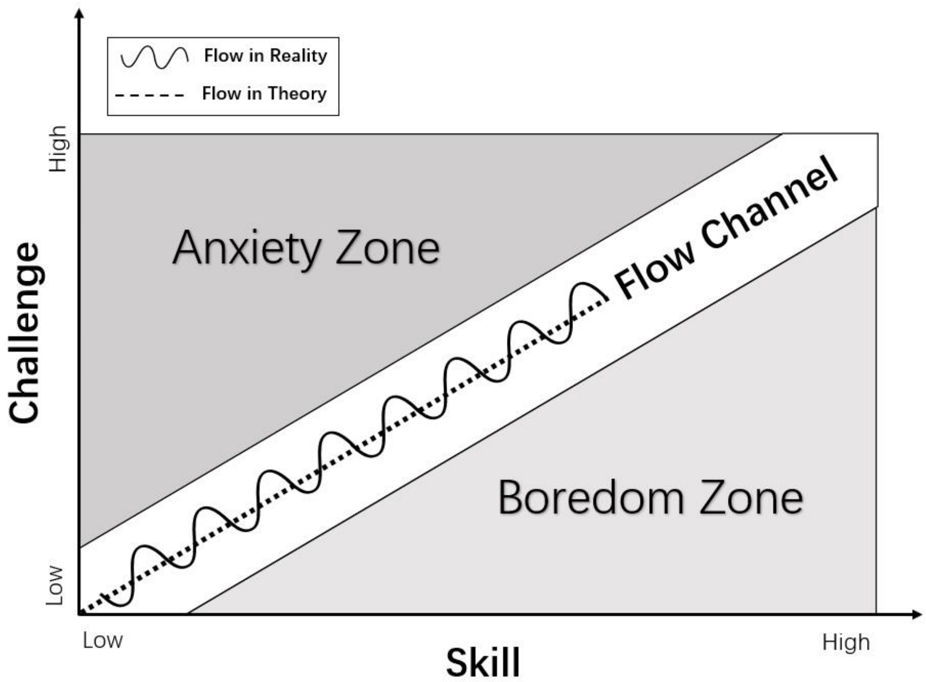


Figure 2.2: Paradigm of flow state in theory and in reality

would decrease, and the event would fall into the boredom zone. Hence, if we want to stay in a flow state, the challenge should be increased when players are getting stronger, by which we could reach the flow state (dashed line in Figure 2.2 [57]).

With the decreasing of challenge, people would feel boring since the game is no longer have much uncertainty for the player as at the beginning. They would come to the phase corresponding to the boredom zone in the flow theory when an event needs high skill but with low challenge.

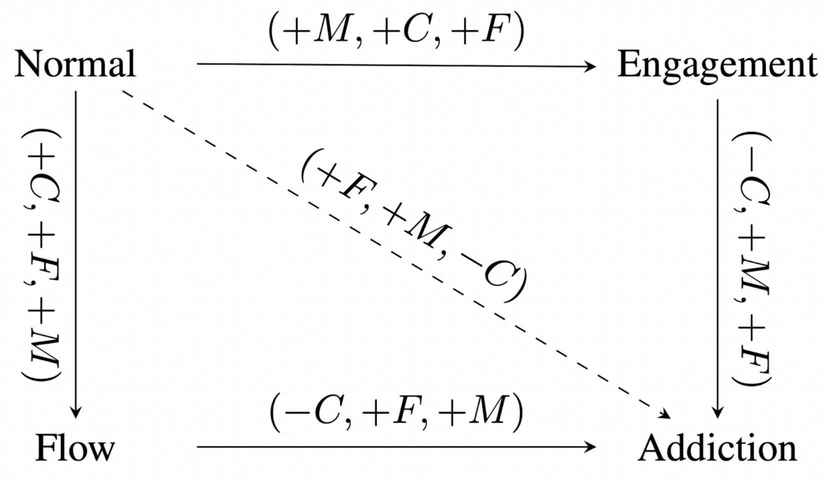


Figure 2.3: Conceptual model of engagement and addiction as a transition from baseline states (normal or flow states) in game-playing based on the prioritization (ordering, highest to lowest) and intensity (+ = high, - = low) of control, focus, and motives (C, F, M) [1]

2.4 Motion in Mind

As Perlovsky claims,"The current understanding of the mind has the status of a new field of physics: it is described mathematically, without mysticism on the one hand, and without reductionism on the other, in line with cognitive science, psychology and philosophy." The world is amenable to be understood on different levels. Understanding that searched by physicists is specific in certain ways: physics is a search for basic laws, a few universal "first principles" describing a wealth of observed phenomena [12].

A reason why many physicists are uncomfortable with the term "physics of the mind" is that the human mind is diverse and unpredictable, so how can it be reduced to a few fundamental laws? Newton saw nothing wrong with developing psychophysics, which he called spiritual substance [58]. Newton failed, however, and since then few physicists have dared to venture into the field. Lately, new data, new intuitions, new mathematical tools have emerged, and today we are trying something new. We try to find some basic principles of how the mind works, explain those principles mathematically, use them to explain the vast amount of known data, and make predictions that can be tested in the lab. The future will tell us just how close the theory of physics is to understanding the individual mind. Roger Penrose had mentioned, "It is our current lack of understanding of the fundamental laws of physics that prevents us from grasping the concept of 'mind' in physical or logical terms [59]." He also argued that Godel's results contained the uncomputability of thought processes and demonstrated the need for new physics [31]. Another counter-point of the review is that the uncomputability of logic does not mean the uncomputability of thought. Logic is not the basic mechanism of thought [60].

In both classical mechanics and quantum physics, the fundamental law now expresses probability. Not only do we need laws, but also we need to introduce completely novel elements into the events described by nature. This new element will enable us to obtain what Maxwell hoped would be a "new kind of knowledge"[2].

2.4.1 Game Refinement Theory

Game refinement (GR) theory proposes a logistical model of game information progress to quantify and evaluate the sophistication of different kinds of games [61]. It also provides a different perspective to understand better the design and direction of optimizing a game [62]. In the current context, the game information progress can be defined in twofold. One is speed or score rate of the game, while the other one is the game's information progress, which focused on the result, indicating the outcome certainty of the game. If we consider the information process in the human brain, which can be measured as in the physics, taking Newton's second law into comparison, we could get the game acceleration, which we denote as GR value [63].

The GR value is derived from the average scores (denoted as $x(t_k)$) over the average times of attempts (denoted as t_k). Game information certainty $x(t)$ is a function of time t corresponds to the average number of possible moves and game length in card games, respectively, where $0 \leq t \leq t_k$ and $0 \leq x(t) \leq x(t_k)$ [61,64]. In the continuous movement, games such as sports and video games can employ a game progress model to find the measure of game refinement. Different types of popular games had been analyzed, which is summarized in Table 2.1.

The GR values observed from the previous study, representing the acceleration of the game process, where popular games converge to a sophistication "zone" between the value of 0.07 to 0.08. This sophistication zone represents the balance region for a good game where the uncertainty of the game is appropriate to make the game enjoyable and entertaining. $GR \geq 0.08$ indicates that the outcome of the game becomes certain, requiring

[2] https://physicsworld.com/a/james-clerk-maxwell-a-force-for-physics/

Table 2.1: Measure (GR) of game refinement for some popular games [5]

Games	B	D	GR
Chess	35	80	0.074
Shogi	80	115	0.078
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22.00	0.073
Badminton	46.34	79.34	0.086
Table tennis	54.86	96.47	0.077
DOTA ver 6.8	68.60	106.20	0.078
StarCraft II Teran	1.64	16.00	0.081
Mafia(one of the setting)	6.25	46.90	0.074

$^*B$: average branching factor in card game,
or possible score chance in continuous movement game.
$^*D$: average game length.
$^*GR$: game refinement value.

skill to be exciting or the game become boring. Conversely, $GR \leq 0.07$ corresponds to the outcome of the game being too uncertain, which is on the chance to be enjoyable or the game becomes frustrating. Therefore, concerning the interpretation of GR value, a sophisticated game is a game that balances between chance and skill where players are expected to experience an appropriate enjoyment, entertainment, and challenge.

2.4.2 Physics in Mind

Iida has explored the relationship between acceleration-in-mind and force-in-mind [10]. He found that if identify the notion of non-game activities and gamified events, one would feel gamified experience with a certain degree of emotional impact when force-in-mind becomes equals to or larger than a certain threshold. We assume that force in mind is what changes the motion of the information in mind since force is what changes the motion of an object based on Newton's second law. From the standpoint of energy, we can take an activity or a game as a task, which is also a process of doing work, to eliminate the uncertainty. Hence we suppose that the work required during an activity can be measured by entropy in mind, i.e.,psychic entropy.

Velocity-in-mind and Acceleration-in-mind

Let us consider the progress of a task to accomplish within a given time framework. The 'progress' of a task is twofold. One is speed of the task, while the other one is the task's information progress which focuses on the result of the task. The task's information progress presents the degree of certainty of the task's results in time. Having full information of the task's progress, i.e., after its conclusion, the task's progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq T$ and $0 \leq x(t) \leq x(T)$. shown in (Equation 2.3).

$$x(t) = \frac{x(T)}{T}t \tag{2.3}$$

Here T and $x(T)$ corresponds to the end of the task and the goal to achieve in the task respectively. (Equation 2.3) indicates that the goal of the task under consideration can be achieved within the time T, meanwhile it may not be so within the shorter time than T. Under the given time constraint that the task should be completed within the time τ with $0 < \tau < T$ where $x(\tau) = x(T)$, the task's progress $x(t)$ is given as a linear function, as shown in (Equation 2.4).

$$x(t) = \frac{x(\tau)}{\tau}t \tag{2.4}$$

However, the task's information progress given by (Equation 2.4) is unknown during the in-task period since $\tau < T$. The presence of uncertainty during the task, often until the final moments of a task, reasonably renders the task's progress exponential. Hence, a realistic model of information progress of a given task is given by (Equation 2.5).

$$x(t) = x(\tau)(\frac{t}{\tau})^n \tag{2.5}$$

Here $n \in N$ stands for a parameter which is given based on the task's progress patterns or hardness of completing the task. Larger n corresponds to more hardness of the goal achievement. When $1 \leq n$, the first derivative of the task's information progress model $x(t)$ in (Equation 2.5) or velocity in the sense of dynamics is given in (Equation 2.6).

$$x'(t) = \frac{x(\tau)}{\tau^n} t^{n-1} n \tag{2.6}$$

The second derivative of the task's information progress or acceleration in the sense of dynamics is obtained by deriving (Equation 2.5) twice. Solving it at $t = \tau$, we have (Equation 2.7).

$$x''(\tau) = \frac{x(\tau)}{\tau^n} t^{n-2} n(n-1)|_{t=\tau} = \frac{x(\tau)}{\tau^2} n(n-1) \tag{2.7}$$

Remark Under the original condition that the uncertainty of a game outcome can be eliminated within the time T, the certainty of game outcome are obtained proportional as the time goes on, given in (Equation 2.3) thus the experience of these kind of games are more likely be boring since no surprise. Under the new condition when $1 \leq n$, uncertainty of game outcome would be like what happen in see-saw game, i.e.,the experience in of these kind of games are excited.

For continuous video game, the total length of the game is significantly different for players with different level. Theoretically there will be no end for the perfect player. Therefore, in continuous video game case, the total uncertainty of a game initially is regarded as 100 percent, i.e. $x(T) = 1$, the velocity of a game outcome certainty obtained at time t in (Equation 2.3) is shown as $x(t) = \frac{t}{T}$, while in (Equation 2.5) is shown as $x(t) = (\frac{t}{\tau})^n$. So from the perspective of achievement in game process, solve (Equation 2.5) we can get (Equation 2.8).

$$x''(\tau) = \frac{t^{n-2}}{\tau^n} n(n-1)|_{t=\tau} = \frac{n(n-1)}{\tau^2} \tag{2.8}$$

Definition We call the first and second derivative given in (Equation 2.6) and (Equation 2.8) *velocity-in-mind* and *acceleration-in-mind* in arcade games.

(Equation 2.8) is divided into two parts, where the term $\frac{1}{\tau^2}$ can be used as the measurement of game refinement to examine the balance between deterministic and stochastic

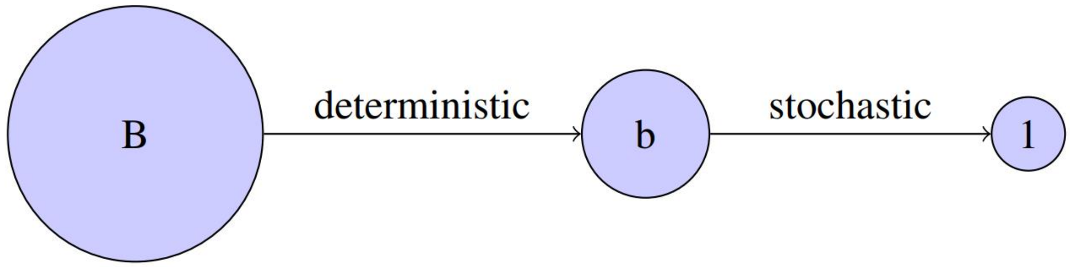


Figure 2.4: A model of move selection in most games [2]

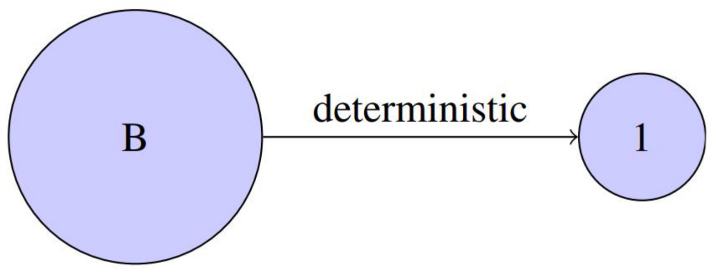


Figure 2.5: A model of move selection for arcade games experts

aspects of the arcade game outcome [10]. It has been found that sophisticated games have a similar value of game refinement located at the zone between 0.07 to 0.08 [65] [66] [61] using the term in (Equation 2.7). Another term $n(n-1)$ may correspond to the game progress patterns such as one-side game and seesaw game, or the difficulty of completing the task. The game progress pattern or its difficulty is important with respect to engagement. Former research have found that the most exciting setting , either for boardgame with a focus on average branching factor, or the progress approach with a focus on the average scoring points in a scoring sport game, is given by $n = 2$ [67] [66] in the sense that the outcome is highly uncertain. People would feel gamified experience if GR is a zone value and $n = 2$.

The model of move selection process in game playing is shown in Figure (2.4). A move to play would be selected first in a deterministic way, which is followed by a stochastic way. Usually, amateurs are not skilled at making a move selection. They are likely to eliminate choices based on instinction, which is inefficient and unreliable, hence their b would be relatively large. On the other hand, experts can always reduce choices from B to relatively small b based on their skills and make high quality of decision. In boardgame case, $b = \sqrt{B}$ according to $\alpha - \beta$ pruning method.

Gamified Experience

Let p be the probability of selecting the best choice among n plausible candidate choices. For this case, $p = \frac{1}{n}$ holds in which the probability of such choice is equally likely to occur (equiprobability). As such, the definition of gamified experience is based on the notion of the risk frequency ratio.

Definition (Gamified Experience):
Risk frequency ratio m (risk frequency over the whole game length) is defined as $m = 1 - p = \frac{n-1}{n}$. Then, gamified experience is gained if and only if the risk of failure occurs with $m \geq \frac{1}{2}$, which implies $n \geq 2$.

Let t be the time or length of a given game, and $y(t)$ be the function for the uncertainty solved at time t . As such, a player who needs to solve such uncertainty can be measured using the average ratio of v , given by (Equation 2.9).

$$y(t) = vt \tag{2.9}$$

On the other hands, information acceleration in player's mind is given by (Equation 2.10). The function of $y(t)$ for informational acceleration is formulated by adopting analogous to the physical kinematic formulation of $y(t) = v_0 t + \frac{1}{2}at^2$. Since $v_0 = 0$ at t=0 , then (Equation 2.10) is obtained. The cross point can be found at t=D , where the relation in (Equation 2.11) is identified.

$$y(t) = \frac{1}{2}at^2 \tag{2.10}$$

$$a = \frac{2v}{D} \tag{2.11}$$

The cross point $t = D$ indicates the right balance between skill and chance concerning the gamified experience and comfortable thrill given by the informational acceleration of the game under consideration. Moreover, if the game length or the total score is too long (or too short), it would be boring (or unfair).

The game information dynamics were studied by taking the game process to solve the game outcome's uncertainty. The idea originated from the definition of gamified experience, which is based on the notion of the risk frequency ratio [6]. If the probability of selecting the best choice p among n plausible candidate choices is equally likely (equiprobability), then $p = \frac{1}{n}$ holds. Such a situation leads to a better understanding of the 'speed' of a game's progress. Such a concept's gradual development leads to the analogy of Newtonian physic (called a motion in mind).

Better understanding of the human mind was ascertained where the foundation of motion in mind has been gradually established [6, 61] and verified in various domains [65, 66, 68, 69]. It has been proved that measure of game refinement ($GR = \frac{\sqrt{B}}{D}$) for a sophisticated game would come to the zone value of $GR \in [0.07, 0.08]$, which corresponds to the lower limit (fairness) and the upper limit (engagement), respectively. Moreover, the game progress pattern or its difficulty is important concerning the player's engagement.

In this chapter, the notion of motion in mind is expanded in evaluating and investigating the arcade games' underlying mechanisms and mechanistically determine the characteristics or factors that make such games engaging and potentially addictive.

Mass in Mind and Risk Frequency Ratio

Descriptions of motion establish the foundation of modern physics. In physics, work is a measure of energy change, and various forms of energy can be transformed into each other. Mass is the property of a body and its inertia, commonly taken as a measure of the amount of material content and causes it to weight due to a gravitational field[3]. In physics, mass-energy equivalence means that any object with mass has the same amount of energy.

An object is characterized by its mass in Newton's mechanics, whereas agent, by its intelligence [6]. Relative to uncertainty in the game, a task requires a certain amount of intelligence. A stronger player has a greater skill to solve uncertainty and vice versa. Such a situation implies stronger (weaker) players would be faced with lower (higher) uncertainty; thus, a small (large) value of mass is given to them.

The *Time Dilation Effect and Length Contraction Effect* [70] in special relativity have implied the subjective relativity of the world to some extent. Based on the same thought

[3]mass: www.britannica.com/science/mass-physics

with the subjectivity embodied in human species [71], there also high likely exists mass-velocity relation in the human mind [72]. Such a notion leads to the interpretation of mass in game-playing (m), which corresponds with a game's challenge (relates to the frequency of risk). According to the game progress model, the slope (v) of a linear game progress model ($y(t) = vt$) has an opposing relationship to m. In the current context, v is generally meant the rate of solving uncertainty, whereas m implies the difficulty of solving such uncertainty, as given by Equation 2.12.

$$m = 1 - v \tag{2.12}$$

To this end, formulations have been established around the analogy of motions, and the most prominent ones were the measure of the force, momentum, and potential energy, which were established based on the game progress model. Table 2.2 shows the analogical connection between the physics model and the game progress model.

Table 2.2: Analogical link between physics and game (adopted from [6])

Notation	**Physics**	**Game**
y	displacement	solved uncertainty
t	time	total score/game length
v	velocity	win rate
M	mass	difficulty rate, m
a	gravitational acceleration	informational acceleration
$\vec{p}$	momentum	momentum in mind
U	potential energy	potential energy in mind, E_p

Momentum and Energy in Game

Energy and momentum are combined to form a four-dimensional vector from the Special Theory of Relativity perspective. This four-dimensional vector is called four-dimensional momentum, and its modulus is equal to $E^2 - P^2 = m_0^2$ ($c = 1$), which does not change from a fixed frame of reference of an observer. The force of gravity on an object and gravity force on an object is determined by the dynamic tensor defined by such four-dimensional momentum.

In Special Theory of Relativity [73], energy and momentum are hyperbolic. Both momentum and mass can be considered as the manifestation of energy. Under certain circumstances, the mass of a substance can be derived from the energy of interactions

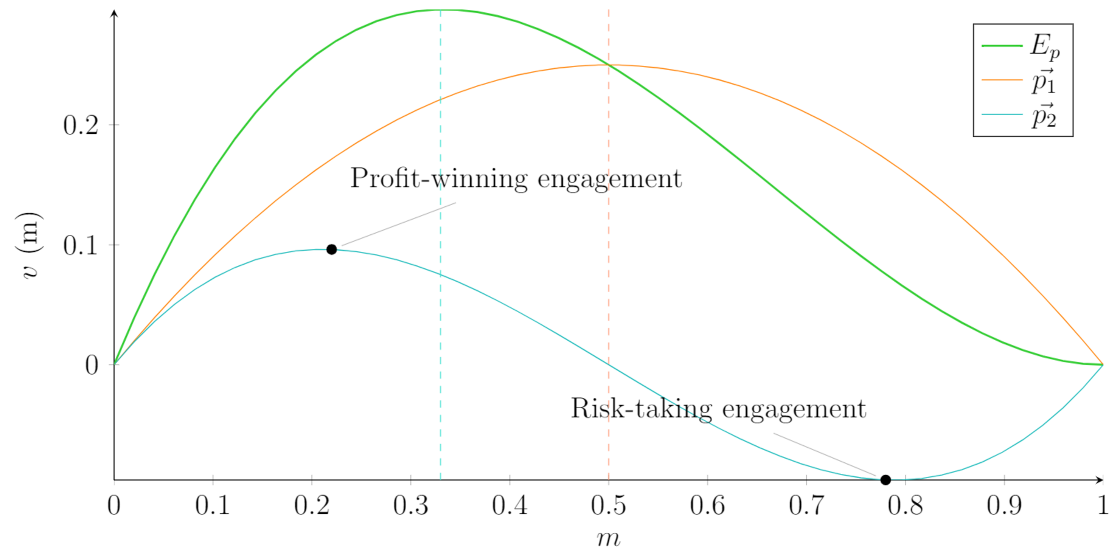


Figure 2.6: Illustration of law of motion in mind over various mass. $\vec{p_2}$ is derived based on the conservation of E_p, which provided two peaks of profit-winning engagement ($m = \frac{3+\sqrt{3}}{6}$) and risk-taking engagement ($m = \frac{3-\sqrt{3}}{6}$)

between substances, and the mass of other substances can also provide the energy of a substance. Hence, energy, momentum, and mass should be considered as a whole.

A notion of momentum in game playing is defined as Equation 2.13, focusing on the two factors: hardness to move in a game (m) and game progress ratio (v). Thus, momentum in the game playing implies the player's tendency to continue playing the game. Meanwhile, a game's energy is defined as the amount of the required game information a player needs in the game process, which describes the player's expectation in finishing the game or anticipation the game give to the player [6]. Concerning the gravitational potential, the game itself is attractive to players (like gravity), and the potential energy of this attraction is the energy of the game. Solving Equation 2.11 and $d = \frac{1}{2}at^2$, E_p is given in Equation 2.14

$$\vec{p} = mv = m(1-m) \tag{2.13}$$

$$E_p = mgh = ma(\frac{1}{2}at^2) = \frac{1}{2}ma^2t^2 = 2mv^2 = 2m(1-m)^2 \tag{2.14}$$

In the study by Khalid and Iida [1], the potential energy (E_p) is regarded as conserved over time which is transformed into the momentum of the game's motion ($\vec{p_1}$) and momentum of mind's motion ($\vec{p_2}$). E_p is expected to be conserved where the momentum of the game playing motions in the game contains both the objective (in-game) and subjective recognition (in mind), analogous to the law of conservation of energy in classical physic.

Hence, conservation of energy is given as Equation 2.15, where Equation 2.16 is acquired by applying Equation 2.13 and Equation 2.14. Then, Equation 2.17 is the first derivation of Equation 2.16. Solving Equation 2.16 obtain $m = \frac{3 \pm \sqrt{3}}{6}$ which indicates positive ($m \approx 0.28$) and negative ($m \approx 0.72$) peaks which implies that engagement is maximized.

$$E_p = \vec{p_1} + \vec{p_2} \tag{2.15}$$

$$\vec{p_2} = E_p - \vec{p_1} = 2m^3 - 3m^2 + m \tag{2.16}$$

$$\vec{p_2}'(m) = 6m^2 - 6m + 1 \tag{2.17}$$

2.5 Chapter Conclusion

This chapter summarizes relevant concepts and studies to the topic, and illustrate the formation of motion in mind theory, which serves to give the fundamental understanding of the latter research topics.

Chapter 3

Uncertainty under Different Settings

This chapter is based on the integration, update, and abridgment of the following publications:

- Yuexian Gao, Wanxiang Li, Yuhao Xiao, Mohd Nor Akmal Khalid, Hiroyuki Iida, Nature of Attractive Multiplayer Games: Case Study on China's Most Popular Card Game—DouDiZhu. Information, 11(3), p.141, 2020.

- Yuexian Gao, Wanxiang Li, Mohd Nor Akmal Khalid, Hiroyuki Iida, Quantifying attractiveness of incomplete-information multi-player game: case study using DouDiZhu, In Computational Science and Technology, Springer, Singapore, pp.301-310, 2020.

3.1 Chapter Introduction

Card games have a long history where its form is easy to simulate, and the rules are relatively simple, while it is capable of explaining the sophistication of the game structure. Moreover, card games are popular with all ages. In this chapter, we consider using a shedding-type card game, called DouDiZhu, in which the primary purpose is to empty one's hand of all cards before all other players[1]. DouDiZhu is used as the benchmark

[1] https://en.wikipedia.org/wiki/List_of_shedding-type_games

to explore how settings changing in multi-player games would affect the engagement of players. Among all the settings, we further studied the importance of cooperation in the gameplay.

DouDiZhu [74], one of the most popular games in China, also known as 'Fighting Landlord,' '2 against 1'. It has a massive amount of users and generally regarded as the most sought-after card game in China (Figure 3.1). The DouDiZhu's mobile application was downloaded 1.13 billion times in 2017 alone[2]. The classic DouDiZhu game involves three players, two of whom, called "the peasants," and needs to cooperate against one another, called "the landlord." The game is short, usually lasted around one to three minutes. This situation allowed people to play the game anytime and anywhere. The peasants win if one of their hands were played first; otherwise, the landlord wins. The profits or losses in the game are shared between the peasants while the landlord carries himself alone, which means that the game is a zero-sum game satisfying Nash Equilibrium. As in most card games, the starting hand of DouDiZhu can primarily affect the outcome of the game. The rules of DouDiZhu are not complicated; however, the two essential aspects of winning the game required strategies and skills.

"Happy DouDiZhu", one of the game platforms, has 200 million players playing over 8 hours online per day.[3] At the end of 2017, "Happy DouDiZhu" was awarded as "Best mobile casual game of the year" on China's game rankings[4], which also indicates the popularity of the DouDiZhu game.

The conventional wisdom is that a good game should have both strategic challenges and player skills, but there should also be an element of chance (or rather, uncertainty). This chapter attempts to explore the optimal parameter setting of DouDiZhu, and analyze the game from the perspective of game progression through the game refinement theory [61], then pry into the uncertainty of competitive game.

Analysis of the rules of DouDiZhu is conducted where its respective game length and branching factor is estimated. Then, a simulation experiment is conducted by changing the setting of the game in several variants of the game. The bidding session to be a

[2] http://youxiputao.com/articles/13003/

[3] QuestMobile Mobile big data research institute of China: WeChat small program depth insight research report, questmobile.com.cn/research/report-new/32

[4] 2017 Chinese game billboard awards ceremony has come to a successful conclusion, games.qq.com/a/20180118/028654

Figure 3.1: A screenshot of the Happy DouDiZhu game

landlord is skipped for simplicity. Then the optimal setting of DouDiZhu is adopted as the benchmark to consider game cooperation issues. Mainly, the investigation of this chapter is concerned with the following questions:

- Can the DouDiZhu game be considered a good game? As an competitive card game, how well sophisticated is the game for different level players? What is the uncertainty in this game?

- What would happen if the settings of DouDiZhu change? Does the game have the most enjoyable settings already? How cooperation affects engagement among these types of card game?

- There are many card games similar to DouDiZhu with different hands setting. Moreover, there are also other well-refined games in China. Whether having cooperation is the most distinct feature. Is cooperation the essential factor that makes DouDiZhu game becoming so popular and exciting with only about 30 years' development?

The contents of this chapter are as follows. Firstly we studied the related works of multi-player card games and cooperation in games, and also previous study on DouDiZhu, giving in Section 3.2. Section 3.3 discusses the methodology and assessment method to analyze the game for game process view. Then, an illustration of the simulation experiment is described where the data are collected and analyzed in Section 3.4. After discussing the findings in Section 3.5, Section 3.6 concludes the chapter.

3.2 Game Test-beds

Invented in China's Hubei province around the 1990s, the name DouDiZhu comes from the land reform movement in the early days of the founding of the People's Republic of China, which is a typical imperfect information game for the Chinese people. Formerly known as the champion (known as "paodekuai"), it was renamed as DouDiZhu in 1995. The DouDiZhu game to be recognized as an athletic event in China[5] is officiated on September 3rd, 2016 by the Chess and Card Sports Department of the General Administration of Sport of China, held the national two-in-one chess and card championship.

Due to the distinctive Chinese characteristics of DouDiZhu, most domestic studies focus on the historical reasons for the popularity of DouDiZhu in China, as seen in [75] [76]. Also, many books about the winning strategies of the DouDiZhu game are being published, mainly derived from the author's experience [77]. As the field of AI is conquering more and more games, researchers have now begun to study the aspects of DouDiZhu AI. Lin [78] analyzed the characteristics of the DouDiZhu game, and believes that DouDiZhu game is a two-person zero-sum game. On the premise that computer players know the information of opponents' hand cards, an AI engine of the DouDiZhu game system was designed by using efficient search mechanism, historical inspiration, transposition table, iterative deepening, and other search techniques. By analyzing the reasoning and learning of the dynamic fuzzy logic, the game of "three against one" applied and the learning function of the game system was realized theoretically [79]. The concept of equal hand quality is proposed to solve the fairness problem of the compound DouDiZhu competition [80]. The classification method based on the probability distribution has been proven to provide a better classification effect than the statistical method.

In the previous study, a performance comparison between the cheating UCT agent against a determined UCT agent has been conducted, which is called Mini DouDiZhu [81]. By studying the performance of the precise algorithms in the simplified version of DouDiZhu, they presented evidence that the advantages of cheating are lesser than

[5] General Administration of Sport of China, http://www.sport.gov.cn

obtaining hidden information and overcoming the inherent disadvantages of determined one. Nevertheless, narrowing the gap effectively between complete and incomplete information games has small consequences for the AI agent's strength. Fairness in the compound DouDiZhu competition is addressed through the concept of equal hand quality [80]. There are two kinds of methods: (1) classify the two hands according to the mean and standard deviation of Landlord players' scores in-game data, and (2) classify the two hands according to the probability distribution of Landlord players' scores and dimensional reduction is applied in each stage. Better classification impact has been justified from the probability distribution compared to the statistical method. Monte Carlo Tree Search (MCTS) was employed for training its agents by self-playing and successfully achieved a human level [82].

To this end, more and more scholars have begun to pay attention to this field, mostly focusing on the development of the game from technical or historical point-of-view. To the best of our knowledge, this study is the first attempt to rationalize the popularity of DouDiZhu from the perspective of game processes and settings. Therefore, this study proposes a method to measure how the DouDiZhu game becomes refined or sophisticated, in which different settings are tested to determine the most suitable setting.

3.3 Assessment Methods Employed for DouDiZhu

If we take a game as a process of constantly acquiring information about the outcome of a game, then the game ends when the uncertainty of the outcome is finally eliminated. The information processing model is shown in Figure 3.2.

From the player's point-of-view, with increases of the obtained information, the more predictable the game outcome becomes when approaching the end. As such, the outcome of game information or certainty can be approximated as a linear function of time t, denoted as $x(t)$ in (3.1).

$$x'(t) = \frac{n}{t}x(t) \tag{3.1}$$

The variable n is a constant parameter based on the skill differences among the players of the game. Let D be the total moves of the game. Assume $x(0) = 0$ and $x(D) = B$,

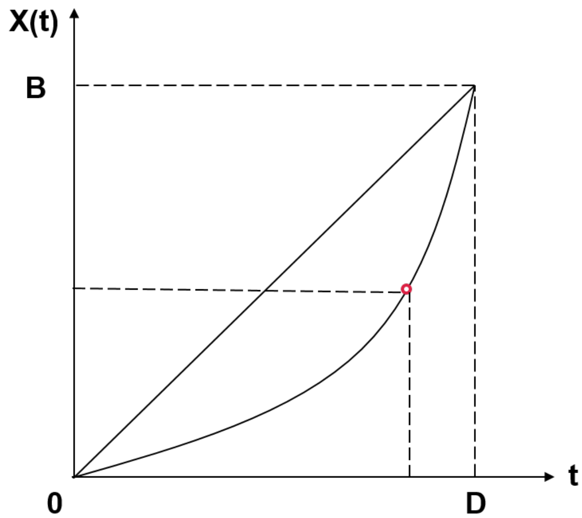


Figure 3.2: Game information process: certainty of outcome

noting that $0 \leq t \leq D$, $0 \leq x(t) \leq B$. Hence, determining the rate of outcome uncertainty of the game $x'(t)$ is ensured, which is directly proportional to $x(t)$ (3.2).

$$x(t) = B\left(\frac{t}{D}\right)^n \tag{3.2}$$

The accelerated velocity of the solved uncertainty along the game process can be achieved by derived (Equation 3.2) twice. Assuming the game last until $t = D$ (In DouDiZhu, one of the player plays out all his hands), (Equation 3.3) is obtained.

$$x''(t) = \frac{B}{D^n} n(n-1)t^{n-2} \mid_{t=D} = \frac{B}{D^2} n(n-1) \tag{3.3}$$

The measurement (GR) of game refinement for DouDiZhu is given in (Equation 3.4), where $\frac{\sqrt{B}}{D}$ could reflect some attractive characteristic of the DouDiZhu, where B stands for the average number of possible moves at each hand and D stands for the average game length.

$$GR = \frac{\sqrt{B}}{D} \tag{3.4}$$

3.4 Game Simulation and Data Collection

Usually DouDiZhu is played as 4 stages, as shown in Figure 3.3. The simulation experiment is conducted by utilizing a simulation program (namely as DouDiZhu AI) with a fixed strategy. Such a strategy is similar to the strategy used by the best human best player, which involves calculating the numerical setting of each card category [83]. The parameter settings used in this study are the number of players (two types of players: landlord and peasants, where only one landlord is present, and peasant can be more than one) and their card distribution. The simulation was run 10,000 times for each of the game settings. The average possible options (B) and the average game length (D) were calculated, which then analyzed using the GR measure.

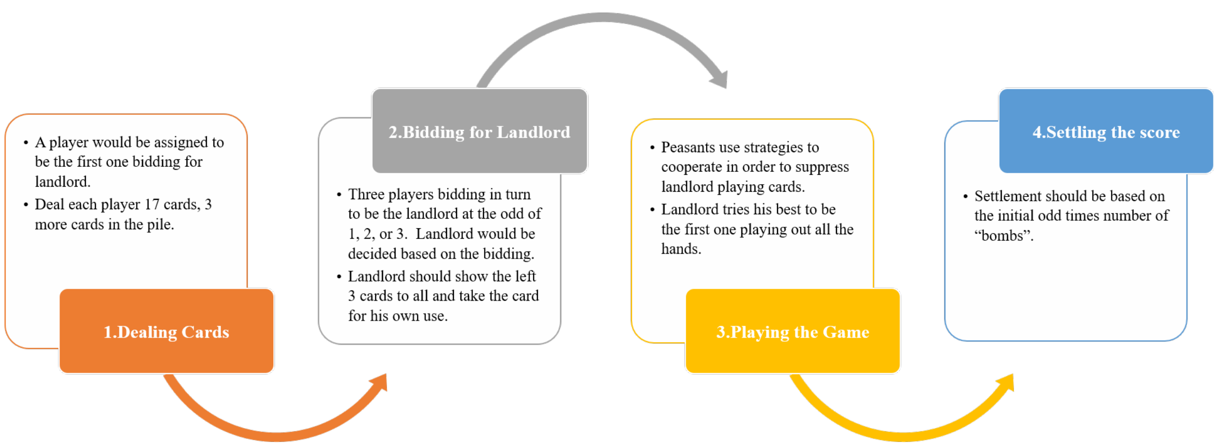


Figure 3.3: Stages of classical DouDiZhu

The experiments were conducted as follows. Firstly, the simulation is first set up to accommodate different settings, annotations of the experiment, and strategy adoption, while considering three different DouDiZhu AI levels (weak, fair, and strong) to simulate the presence of cooperation among the players. Secondly, the simulation experiment is conducted on Classical DouDiZhu, which then compared with different DouDiZhu AI levels. Thirdly, another simulation experiment is conducted on known variants of the DouDiZhu in China to highlight the impact of the cooperation in the Classical DouDiZhu game.

3.4.1 Simulation Setups

In this study, no bidding phase is conducted where landlord assignment is random. Fifty-four cards are split randomly among three players in the classical DouDiZhu, where the cards number is (20, 17, 17), where the one with 20 cards corresponds to the landlord, which is denoted as (L, $P1$, $P2$). From one side, the landlord (L) is fighting on his/her own. On the other side, peasant 1 ($P1$) and peasant 2 ($P2$) cooperate against L. The game is won by the side that first plays out all the card hands.

There are two categories of gameplay, initiative play or passive play. After dealing with the cards, the landlord will play the first card or combination (initiative play). Alternatively, $P1, P2, L$, can play a bigger-force card to follow the last player's cards one by one (passive play). If they do not have bigger cards or decided to skip, it is called a *pass*. When *pass* is chosen by two players consecutively, the round will end. Then, the third player can initiate play with any card and begin the next round. For instance, $L, P1, P2$ play card in turns as $L(33)$, $P1(55)$, $P2(PASS)$, $L(AA)$, $P1(PASS)$, $P2(22)$, $L(PASS)$, $P1(PASS)$, which then $P2$ will start a new round and initiate the card plays. The game ends when any one of the three players has played all the cards.

In this study, the game length (D) of DouDiZhu is the number of the total moves of the three players. Assuming the number of game players as n, number of landlord as l, number of the peasants as p, hands allocation will be shown as $(h_1, h_2, \ldots, h_n)$, a game setting will be denoted as $DDZ[(n, l, p)(h_1, h_2, \ldots, h_n)]$. In the case of classical DouDiZhu, a setting denoted as $DDZ[(3, 1, 2)(20, 17, 17)]$. The average number of possible moves at each hand (B) is counted as follows:

- When a player begins a game phase, the number of possible conventional combinations on the player's hand is the estimation for the number of options. For example, $P1$'s deck is "2221K999888633", $P1$ can play a card with the possible options of 3, 6, 8, 9, k, 1, 2; Thus, a total of 7 cards; Possible options $P1$ can play two cards is 33 88 99 22, a total of 4 cards; And so on until the possible option $P1$ can play ten cards is "8889992233", this is just one card. Add all the options together, $7 + 4 + 3 + 18 + 9 + 1 + 4 + 12 + 4 + 1$, the total for $P1 = 63$.

- When a player plays a phase passively, the number of choices is limited to the last played card of the same type. For example, $P2$ plays "66633", L's remaining card is “22”, so L’s option is only to “pass”, which is just a single option; $P2$ deck is “21kkkjj8876544”, $P2$ can play “kkk44,” “kk88,” “jjj88,” “JJJKK,” and “pass”; thus, totaling to 7.

* Notation: peasants need to cooperate, sometimes they will choose “pass” even when they have cards to play. Sometimes they do not want to split bigger-force combinations; thus, they may also “pass” anytime when they passively play cards.

The implementation of the DouDiZhu AI program involves dealing cards and playing cards during which cooperation between peasants should work as intelligent as possible. For functional convenience, different card categories are provided with a different numeric value according to the game rules as shown in Table 3.1 [84].

Table 3.1: Numerical value of the considered card categories in the simulation

Categories	**Description**	**Numeric values**	**Examples**
Single	3 (Joker)	1-15	‘3’ $= 1$,‘K’ $= 11$
Straight	Sum of single	15-78	‘456789’ $= 2+3+4+5+6 = 20$
Pair	single * 2	2-30	‘66’ $= 4 \times 2 = 8$
Wings	sum pair	12-110	‘66778899’ = 8+10+12+14 = 44
Triplet	single*3 + length	6-44	‘KKK55’= $11 \times (3+5) = 38$
Plane	sum triplet	17-191	‘JJJQQQKKK854’ $= 31 + 34 + 37 = 72$
Bomb	single *200	200-3000	‘5555’ $= 3 \times 200 = 600$
Rocket		5000	‘JOKER+joker’ $= 5000$

- The classical DouDiZhu has $n = 3$ players, where the player is randomly assigned to be the landlord at the beginning of the game.
- Shuffled cards are dealt with the landlord, peasant 1, and peasant 2, respectively, where 20, 17, 17 cards are given as the initial state.
- As mentioned in Section 3.3, the round can be an initiative play or a passive play. Initiative play means that the player can decide to play any combination, while passive play means that the player can only follow the last player with the same card combination or with a bigger force. Such play is defined as follows:
  - If the player is the first one to play cards (i.e., the landlord), then, that player initiates the play;

- If two players in a row choose a pass, the third player initiates the play;
- Otherwise, the player plays passively.

- On each player's turn, the option that the player can choose will be counted as a branching factor (B).
- The landlord always starts the game in the first hand, then peasant 1, peasant 2 and so on.
- All players can pass even if they have a greater power card (this rule is the opposite of the Winner); see Table 3.13).
- When there are two "Pass" in a row, a new round is declared.
- Once there is a player who plays out all the hands, the game is over.

The adopted optimal strategy for the simulation experiments is according to the strategy mentioned by the practical technique books of DouDiZhu [77], which corresponds to the *strong* DouDiZhu AI level. For initiative play, the DouDiZhu AI will prioritize playing a sequence of planes, wings, lines, triples, pairs, and singletons. However, smaller cards are used when playing against an opponent or of the same class. Passive play is one of the alternative strategies that use smaller cards. The landlord is unscrupulous. At all costs, the peasant plays a bigger card than the landlord. Finally, when the card force is less than the lower bound, the first peasant plays more cards than the second peasant to ensure the success of both. The algorithm 1 describes the optimal core strategy.

- For initiative playing, preferentially play categories as sequence of plane, wings, straight, triplet, pairs and single. Among the same category, firstly play the smaller-force cards.
- When passive playing, among the optional strategies, firstly play the smaller-force cards.
- The landlord plays his/her cards at all costs. Peasants play cards bigger than landlord at all costs.

Algorithm 1 Optimal simulation strategy for DouDiZhu

Initialize player size N_p and card size N_c
Initialize the team t_n for each player p_n, $t_n \in [Landlord, Peasants]$
Initialize initial card number i_n for each player p_n, and $N_c = \sum_{n=1}^{N_c} i_n$
Initialize each player p_n's cards c_n with card number i_n
Initialize each player p_n's policy P_n
while TRUE **do**
  for $n = 1 \leftarrow N_p$ **do**
    if player p_n in passive state **then**
      player p_n play valid cards C from c_n or play pass according to policy P_n
    end if
    if player p_n not in passive state **then**
      player p_n play valid cards C from c_n according to policy P_n
    end if
    if c_n is empty **then**
      Team t_n win. End game.
    end if
  end for
end while

- Peasant 1 will play card bigger than peasant 2 when card force is smaller than 50; Otherwise, he/she will *pass*. Likewise for peasant 2.

The simulation uses three levels of DouDiZhu AI: *strong*, *fair* and *weak*. The *strong* DouDiZhu AI always follows the above strategy. For the *fair* DouDiZhu AI, peasants cooperate, and peasants always choose *pass* for any card, while landlords can play as long as they have a larger card. For the *weak* DouDiZhu AI, all three players played bigger cards or passed at random.

Note:*The AI we are talking about in this study is based on generalized artificial intelligence, which refers using machines to simulate a given algorithm. Though divided into three categories according to the corresponding human hierarchy, they can only execute a given strategy and are not capable of learning.*

3.4.2 Experiment on the Classic DouDiZhu

The classic DouDiZhu is the most popular version in China, played by three players. Valid cards include single card, pair card, straight card, triple card, double wing card, plane card, bomb card, rocket card, and 300 cards, which can also be played by kickers (see Table 3.2).

Table 3.2: Categories of Cards

Category	Description	Examples	
		Smallest rank	largest rank
Single	one card	3	JOKER
Straight	at least 5 consecutive single cards	34567	34567890JQKA
Pair	2 same cards	33	22
Wings	at least 3 consecutive pairs	334455	556677889900JJQQ KKAA
Triplet	3 same cards	333	222
	3 same cards and a single kiker	3334	222A
	3 same cards and a pair kiker	33344	222AA
Plane	at least 2 triplets	333444	JJJQQQKKKAAA 88990022
Bomb	4 of a kind	3333	2222
Rocket	JOKER and joker		

* *0* represents for card *10*.

DouDiZhu game has a unique score system. The basic score is set as 3 point in this study where the score is doubled when the player played a *bomb* or *rocket*[6]. For example, the landlord obtains a total of 24 points while the peasant loses 12 points if there are three *bombs*, and the landlord wins the game. The conducted simulation considers a fair level of DouDiZhu AI, which corresponds to an average level of a human player. The average possible option during each move (B) is 8.197 and the average game length (D) is 40.032 as in Table 3.3.

Table 3.3: Measures of game refinement for classical DouDiZhu

B	D	GR
8.197	40.032	0.072

In a real gameplay situation, however, emotion tends to overtake human players where changes in decision and strategies are expected corresponding to their experiences and preferences; thus, producing an unexpected outcome. Hence, simulating such a situation is crucial through the consideration of the computer peasants' cooperation. Additionally, DouDiZhu has millions of players which composed of novice to master level of players. In this simulation experiment, weak DouDiZhu AI represents a novice with little skills and intentions to cooperate, while fair AI represents an average human level with a weak cooperation strategy. A strong DouDiZhu AI stands for a specific kind of professional players with tournament-level performance.

Since cooperation and competition are important aspects of the game, the expected fairness of the game setting can be determined based on the average score for each set. Table 3.4 provides different settings of the landlord and peasants with their respective possible scores. The scores are collected using DouDiZhu AI with the optimal strategy that prioritizes the best sense of cooperation. As a zero-sum game, the peasants' score and the landlord's score should be the same from the perspective of fairness.

The result indicates that setting $DDZ[(3,1,2)(20,17,17)]$ is relatively fair based on their respective winning rate. However, observing the score setting of $DDZ[(3,1,2)(18,18,18)]$, it is perceived to be less fair for the peasants' side, which implies that it is hard for the landlord to win equivalent scores. Interestingly, for $DDZ[(3,1,2)(24,15,15)]$ and $DDZ[(3,1,2)(22,16,16)]$, while also perceived to be less fair to the peasant side, actually implies the opposite where one of the peasants tends to win at the cost of the other. This

[6] `https://en.wikipedia.org/wiki/Dou_dizhu`

Table 3.4: Possible scores of landlord and peasant with different card distribution settings

Setting	L_{win}	$P1_{win}$	$P2_{win}$	$Avg.rocket\&bomb$	$Ratio$	L_{score}	P_{score}
DDZ[(3,1,2)(20,17,17)]	0.317	0.388	0.295	0.222	1.222	2.32	2.5
DDZ[(3,1,2)(18,18,18)]	0.491	0.309	0.2	0.215	1.215	3.58	1.85
DDZ[(3,1,2)(22,16,16)]	0.078	0.55	0.372	0.273	1.273	0.6	3.5
DDZ[(3,1,2)(24,15,15)]	0.019	0.632	0.349	0.459	1.459	0.2	4.3

*L score $= 6 \times$ L win $\times$ score ratio, P score $= 3 \times$ (P1 win + P2 win)

*In 4 person case, "P2 win" represents total wining rate of P3 and P4.

setting implies that one of the peasants, while co-operating with another, played selfishly; thus, promoting less cooperation.

The simulation results performed with different settings through different DouDiZhu AI levels are given in Table 3.5 which suggests that both weak DouDiZhu AI with little cooperation and fair DouDiZhu AI with lower level cooperation show almost the same performance quality. However, when come to the strong DouDiZhu AI with optimal cooperation strategy, GR value dropped radically.

Table 3.5: The results of simulation performed using different DouDiZhu AIs for various game settings

Setting	**DouDiZhu AI level**	B	D	GR
$DDZ[(3,0,3)(18,18,18)]$	Strong	6.6570	53.2120	0.0485
	Fair	7.3440	45.418	0.060
$DDZ[(3,1,2)(20,17,17)]$	Strong	6.6130	53.2115	0.048
	Fair	8.1980	40.0320	0.072
	Weak	7.6670	38.4860	0.072
$DDZ[(3,1,2)(18,18,18)]$	Strong	6.0210	53.0745	0.046
	Fair	7.6440	40.6960	0.068
	Weak	7.2320	38.9640	0.069
$DDZ[(3,1,2)(22,16,16)]$	Strong	8.0370	50.9290	0.056
	Fair	10.0150	39.0700	0.081
	Weak	9.3120	37.6050	0.081
$DDZ[(3,1,2)(24,15,15)]$	Strong	11.7610	48.0260	0.071
	Fair	14.3730	37.8100	0.100
	Weak	13.4380	36.5200	0.100

Numbers of hands also affect the experience of a card game. DDZ [(3,1,2) (20,17,17)] is found to be the most sophisticated one for novice and average players, while the setting of DDZ [(3,1,2) (24,15,15)] is ideally exciting and challenging for professional players.

It might reveal that for a typical case, we should play as the setting of DDZ [(3,1,2) (20,17,17)]; however, in the tournament, it should adjust the hands to the more significant disparity.

Different levels of the players also affect the possible outcome and attractiveness of the game [85]. By analyzing the performance of the classic game setting *DDZ[(3,1,2)(20,17,17)]*, a possible score of different DouDiZhu AI levels was collected and summarized in Table 3.6. While achieving a fairest winning ratio, weak DouDiZhu AI has no cooperation which implies that they fought among themselves. Comparing the strong DouDiZhu AI with fair DouDiZhu AI, it could be observed that strong DouDiZhu AI with a high level of cooperation can also maintain balanced benefits of two sides (landlord versus peasants).

Table 3.6: Scores using different DouDiZhu AIs of DDZ[(3,1,2) (20,17,17)]

AI level	L_{win}	$P1_{win}$	$P2_{win}$	*Avg.rocket&bomb*	*Ratio*	L_{score}	P_{score}
Strong	0.317	0.388	0.295	0.222	1.222	2.324	2.500
Fair	0.261	0.584	0.295	0.513	1.513	2.369	3.354
Weak	0.342	0.325	0.333	0.486	1.486	3.049	2.933

*L score = 6 × L win × score ratio, P score = 3 × (P1 win + P2 win) × score ratio

In multiplayer cooperative games, cooperative strategies can keep DouDiZhu relatively fair, which may have a significant impact on game complexity. When the cooperative strategy is prioritized, the GR measurement drops, meaning that the game is challenging for most players. In other words, when considering a strong cooperative strategy, the game is less fun for the novice. Summarizing the results from Table 3.5 and Table 3.6, it can be conjectured that DouDiZhu is profoundly refined with both luck and skill under high level of cooperation.

3.4.3 Comparison with the Variants of DouDiZhu Game in China

The rules of DouDiZhu has not changed much, although its development is about 30 years. Still, its popularity showed that if the game changes, it will do so for the better. Table 3.7 provides the variants of the DouDiZhu game analyzed in this section.

For two-player DouDiZhu, while the rules are similar to the classic DouDiZhu, one player plays as the peasant while the other plays as the landlord. At first, each player is randomly dealt with 17 hand cards. After the bidding, the landlord will have three

Table 3.7: Versions of DouDiZhu games

Version Name	N Players	Adversarial Setting	Cards	Initial Hands
Classical DouDiZhu	3	2 vs. 1	54	(20,17,17)
2-person DouDiZhu	2	1 vs. 1	54	(20,17)
4-person DouDiZhu	4	3 vs. 1	108	(33,25,25,25)

When 2 players, initial hands will be 17-25 cards for both players such as (20,20); when 3 players, initial hands will be (18,18,18); when 4 players, get rid of two jokers, initial hands will be (13,13,13,13); more people will deal 2 decks of card in this way.

additional cards. As such, a total of 17 cards will be unknown during the game. On the other hand, the rules for the four-player DouDiZhu is similar to the classic DouDiZhu but utilizes two decks of cards. Instead of two versus one, DouDiZhu of four players are three versus one (a single player is the landlord while the remaining players are peasants). At first, each player is dealt with 25 hand cards. After the bidding, the landlord will have 8 more cards.

The simulation experiment in this section utilizes similar rules as the classical DouDiZhu to simulate other versions of the DouDiZhu game considered for this study. The first simulations were conducted for the versions of two-players and four-players setting with fair level DouDiZhu AI. The simulation result is given in Table 3.8.

Table 3.8: Measures of game refinement for two variants of DouDiZhu

n	B	D	GR
2 players	10.817	23.185	0.142
4 players	20.276	69.398	0.065

According to the GR theory, sophisticated games is a game that harmoniously balances challenge and skill as they changed over time [61]. Since the sophisticated zone of game refinement value for most popular games has been verified to be $GR \in [0.07, 0.08]$, it was found that the GR measure is different in the traditional settings. The data might imply that the two-player DouDiZhu game ($GR = 0.142$) is more likely based on chance, while the four-player DouDiZhu ($GR = 0.0649$) with too many cards to play is complicated, more likely skill based.

The traditional settings of DouDiZhu have a different number of cards and players, which means that we could not yet find the exact influence of players and hands categories. Therefore, we conduct the simulation of other settings to examine further into the nature of this multiplayer game.

The two-player DouDiZhu is too easy to finish, making it GR value higher than the sophistication zone. The four-player DouDiZhu, on the other hand, is not equivalently fair for every player, and too challenging for the landlord to compete and for peasants to cooperate; thus, GR value is lower than the GR zone. This distinction of the player number probably offered a numerical interpretation of the popularity of the classic DouDiZhu game in China (three-player setting).

Table 3.9 provides the possible scores of landlord and peasants in different settings. The scores are collected using DouDiZhu AI with the optimal strategy that prioritizes the best sense of cooperation. As a zero-sum game, the peasants' score and the landlord's score should be the same from the perspective of fairness. Table 3.9 indicates that the settings $DDZ[(2,0,2)(21,17)]$ is relatively fair based on their respective winning rate. However, it can be observed from the score that the settings $DDZ[(4,1,3)(15,13,13,13)]$ and $DDZ[(4,1,3)(33,25,25,25)]$ are perceived to be less fair for the landlord side, which implies that it is hard for the landlord to achieve equivalent scores. These findings further justify that the four-players setting, sacrifice both the expected enjoyment and fairness from the game.

Table 3.9: Possible scores of landlord and peasant in different game settings

Setting	L_{win}	$p1_{win}$	$p2_{win}$	**Ratio**	L_{score}	p_{score}	$R(\%)$
DDZ[(2,0,2)(21,17)]	0.508	0.492	-	1.089	1.69	1.64	0.03
DDZ[(4,1,3)(15,13,13,13)]	0.195	0.415	0.39	2.288	4.02	5.53	-0.37
DDZ[(4,1,3)(33,25,25,25)]	0.201	0.39	0.409	1	1.809	2.397	-0.33

*Score ratio,Ratio $= 2^{\text{average bombs} + 1}$
*L score $= 6 \times$ L win $\times$ score ratio, P score $= 3 \times$ (P1 win + P2 win)
*In 4 person case, "P2 win" represents total wining rate of P3 and P4.
*R is represent for the deviation rate of P score to L score, negative numbers represent
*unfairness to landlords

The simulation results performed with different settings through different DouDiZhu AI levels are given in Table 3.10. Similar to the experiment for the classical DouDiZhu, Table 3.10 implies that both weak DouDiZhu AI with little cooperation had almost similar performance quality to the fair DouDiZhu AI with lower-level cooperation (except for strong DouDiZhu AI). Additional insights also can be observed from the distribution of the cards between the landlord and the peasants. Increases in the number of cards the landlord have, the lower the GR value. This situation implies that the game is more challenging since the landlord has more hands at the start of the game. With more cards on hand, the landlord has to deal with more combinations and strategies (high branching factors). Also, peasants have to deal with higher risks that the landlord might have a higher chance of getting high-value card categories. Such a situation demands the landlord to play out all the cards skillfully. Thus, more cards are nonequivalent to being advantageous in the DouDiZhu game.

Table 3.10: The results of simulation performed using different DouDiZhu AIs for various game settings

Setting	**AI**	B	D	GR
$DDZ[(2,0,2)(20,17)]$	Fair	10.8172	23.1845	0.142
$DDZ[(4,0,4)(27,27,27,27)]$	Strong	9.432	114.069	0.027
	Fair	15.299	75.115	0.052
	Weak	13.207	69.425	0.052
$DDZ[(4,1,3)(15,13,13,13)]$	Strong	4.6520	50.0260	0.042
	Fair	5.3465	41.3415	0.056
	Weak	5.1588	40.1325	0.057
$DDZ[(4,1,3)(33,25,25,25)]$	Strong	15.110	112.881	0.034
	Fair	20.276	69.398	0.065
	Weak	22.930	67.667	0.071

The most popular action in this game is the "pass". Peasants can choose "pass" even when they have bigger cards to play to let their teammates win. Hence, based on the simulation strategy, the frequency of "pass" in a game could be taken as a parameter to estimate the cooperation between peasants. Table 3.11 shows that peasants with weak DouDiZhu AI represent human novice, with no intention of cooperating, passes less than a fair DouDiZhu AI representing average players with a weak level of cooperation skills. In other words, after the novices get some sense of cooperation, they begin to reserve cards and choose to deliberately "pass" to let more cards to be played by their teammate. It can be inferred that the existence of cooperation increases the engagement of the game. By having an awareness of cooperation in playing this game, they become more cautious at every step.

To justify the reliability of simulation policy given in algorithm 1, we collected comparison data about 100 human-play games with one of the most famous application

Table 3.11: Pass frequency of different level DouDiZhu AIs: comparing classical DoudiZhu and four-players DouDiZhu setting

Settings	**DouDiZhu AI Level**	**Peasant Pass**	**Landlord Pass**
$DDZ[(3,1,2)(20,17,17)]$	Strong	4.947	4.349
	Fair	12.457	8.641
	Weak	10.0	9.993
$DDZ[(4,1,3)(33,25,25,25)]$	Strong	20.199	8.352
	Fair	39.771	14.201
	Weak	34.343	17.176

"Happy DouDiZhu[7]". And the settings of DDZ(2,0,2)[20,17], DDZ(3,1,2)[20,17,17] and DDZ(4,1,3)[33,25,25,25] are selected, since these settings are widely played in the game platforms. In addition, 9 DouDiZhu players with different levels (novice, normal and expert) are volunteered to play with the simulation program (each one for 10 times) to testify every level's intelligence, where winning rate is recorded. Results listed in Table 3.12.

Table 3.12: Justification results of different levels of AI and human players

Setting	**Level**	B	D	GR	L_{win}	P_{win}
DDZ(2,0,2) [20,17]	Strong	7.7028	35.867	0.0774	0.501	0.499
	Fair	10.8172	23.1845	0.1419	0.529	0.471
	Human	5.789	17.89	0.1345	0.7	0.3
DDZ(3,1,2) [20,17,17]	Strong	6.6130	53.2115	0.0483	0.317	0.683
	Fair	8.1980	40.0320	0.0715	0.342	0.658
	Weak	7.6670	38.4860	0.0719	0.261	0.739
	Human	5.166	31.71	0.0717	0.35	0.65
DDZ(4,1,3) [33,25,25,25]	Strong	15.110	112.881	0.0344	0.201	0.799
	Fair	20.276	69.398	0.0649	0.277	0.723
	Weak	22.930	67.667	0.0708	0.316	0.684
	Human	15.670	59.23	0.0668	0.195	0.805

*B, D, GR of human are collected from the human vs. human competition.
*Winning rate of landlord (L_{win}) and winning rate of peasants (P_{win}) are collected from human vs. simulation program.

It shows that there is an obvious advantage with human player at the setting of DDZ(2,0,2)[20,17]. Human average data of DDZ(3,1,2)[20,17,17] is nearly equivalent with the fair level policy. Whereas for the setting of DDZ(4,1,3)[33,25,25], human's perfor-

[7] Homepage: syzs.qq.com/game/tbg/hlddz

mance is close to the strong policy. Some of the volunteers claim that, the simulation program played some "weird moves" or "stupid moves", but it on the contrary unexpectedly creates difficult situations for the human player. Generally, though there is obvious shortage of fixed policy, the simulation results still can provide meaningful insights to understand the game and the process.

3.5 Comparisons with other game

In this section, a comparison between different card games similar to DouDiZhu is conducted where different variations of similar popular card games from other countries were analyzed relative to the DouDiZhu game and cooperation factor from the perspective of GR measure.

3.5.1 Other Popular Shedding-type Card Games

Analysis of different DouDiZhu and similar games all over the world is conducted to test if DouDiZhu has the most advantages among them. In general, they have similar rules. However, they are quite different in the card categories and their cooperation aspects. To understand the difference of all the card games mentioned, their main categories are summarized in Table 3.13.

1. $Winner$ shared similarity to the DouDiZhu. The main differences between *Winner* and classic DouDiZhu are found in two aspects. Firstly, the number of players where *Winner* can play at least by two players and usually not more than eight, using two decks of cards when more than four players[8]. Secondly, each player of *Winner* fights on their own. *Winner* plays out all the hands, while other players continued until there is just one player and they score by ranks of sequence to play out cards. Thirdly, hands in *Winner* have a different weight of the suit, where spades are biggest followed by hearts, clubs, and diamonds. Suit discrimination gives *Winner* a lot of card combination possibilities.

2. $Daifug\bar{o}$ (Grand Millionaire) or Daihinmin (Grand Pauper) is a Japanese card game played by three or more players using standard 52 cards.The goal of the game is to

[8] https://en.wikipedia.org/wiki/Winner_(card_game)

get rid of all hands as quickly as possible.The method is to gradually play a stronger card than the previous player's card. The major characteristic of Daifugō is that Daifugō only has the category where straight works with the same color and with more than 3 consecutive cards.

3. *Big two* is popular among the southern Asia and East-southern Asia, commonly played by four players. Valid card combinations is the primary differences between *Big Two* and DouDiZhu. *Big Two* can play flushes, while DouDiZhu does not care about the suit. In *Big Two* the straight can only consist of exactly five cards, while DouDiZhu's straight can consist of five to 12 cards.

4. *Tiên Lên*, also known as Vietnamese cards, Thirteen, "Bomb", is a Vietnamese shedding-type card game devised in Southern China and Vietnam. *Tien Lên* has many variants. The rules are similar to *Big Two* with no boom and the game starts with spade 3.

5. *Killer* is a variant from *Tien Lên* which is famous among Hawaii district. It is an updated version of *Tien Lên*, with a complicated bomb category.

Table 3.13: Popular shedding-type card games from different regions

Name	**Districts**	N **Players**	**deck**	**Initial Hands**	**Valid Categories**
Winner	China	2-6	54	Divided Equally	Sg,Pr,Tr,Bm,Str, FH,FL,SF
Big Two	South East Asia	4	52	13	Sg,Pr,Tr
Daifugō	Japan	2-8	52	Divided Equally	Sg,Pr,Tr
Tiên Lên	Vietnam	2-4	52	Divided Equally	Sg,Pr,Tr
Killer	Hawaii	2-5	36-52	13	Sg,Pr,Tr

* Sg: single, Pr: pair, Tr:triplet, Bm: bomb, Str: straight, FH: fullhouse, FL: Flush, SF: straight flush

Theoretically, the characteristics of local games can reflect humans' culture from a side profile. The results of simulation using DouDiZhu AI performed on these DouDiZhu and similar card games are shown in Table 3.14 while finding their game refinement values. It can be inferred that the Chinese are overall tend to explore the unknowns by playing games with more stochastic elements, which prefer "entertainment". Likewise, it can also be conjectured that the Japanese are likely to challenge their limits by conquering more difficult games, which prefer "accomplishment". More importantly, the results of Table 3.14 suggest that popular similar to DouDiZhu card games over the world were outside of the GR zone. We believe that the existence of cooperation is the most important distinction in DouDiZhu compared to other card games that make it situated in the most sophisticated zone of the GR measure.

Table 3.14: DouDiZhu similar card games over the world

Games (players)	B	D	GR
DouDiZhu (3)	8.197	40.032	0.072
DouDiZhu (4)	22.930	67.667	0.065
Tien Len (4)	9.779	49.460	0.063
Killer (4)	9.770	49.553	0.063
Winner (3)	19.303	44.125	0.100
Winner (4)	15.599	48.822	0.081
Big two (4)	17.353	51.849	0.080
Daifugo (4)	13.033	87.366	0.041

3.5.2 Why DouDiZhu is the Most Popular Card Game in China?

An early psychological study by Csikszentmihalyi had pointed out that flow is the state of deep enjoyment of people [46]. Human getting fully engaged and immersed in a game may be caused by the games' mechanism, which contains a clear purpose, instant feedback, and the feeling of control. Based on this study, it was found that DouDiZhu provides players with an exciting play experience and perceived to be fair in a particular setting while promoting engagement and cooperation in its play mechanism. Additionally, the DouDiZhu game has a similar sophistication level compared to other popular games in China, which is justified by being the highest sought after the game just next to Mahjong online (Figure 3.4[9]). Hence, revisited the question "Why DouDizhu has the largest amount of users among all the card and chess games?", several reasons were found based on the simulation experiments conducted in this study.

Firstly, it is reasonable to justify the reason for DouDiZhu's popularity relative to the Mahjong, Chinese chess, and Go games. According to the GR value (Table 3.15), these games were well-refined to have a level of sophistication similar to DouDiZhu. However, the differences were based on their respective game length and rival mode. The game

[9] www.youxiputao.com/articles/13003

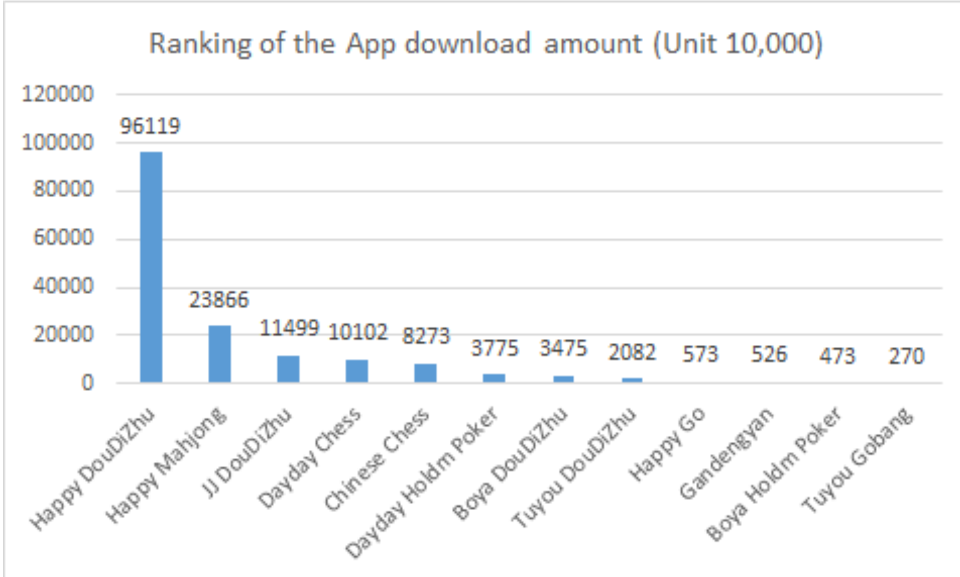


Figure 3.4: Download ranking adopted from the web article

length alone may not be meaningful in this context since different game mechanisms, adopted by each game, may affect it differently. For example, short game length of the Mahjong and DouDiZhu typically associated with stochastic and chance-based gameplay [86]. However, their rival mode may hint on the importance of balancing cooperation and competition[10]. Explicit cooperation through the role of the peasants in DouDiZhu makes the game superior to Mahjong, although the game length is shorter than Mahjong. Hence, it can be speculated that with some level of cooperation, the more exciting the game becomes to the players as the shorter the duration of the game.

Table 3.15: Comparison of some well-refined board and card games in China (ordered by decreasing GR value)

Game	**Ranking****	**Game Length***	**Rival Mode**	GR
Go	4th	Casual: 20–90 minutes Professional: 1–6 hours	1 *vs* 1	0.076
Chinese Chess	3rd	20 min to several hours	1 *vs* 1	0.074
DouDizhu	1st	around 5 min	2 *vs* 1	0.0715
Mahjong	2nd	10 min	1 *vs* others	0.065

*Game length from `http//www.en.wikipedia.org/wiki`
**Based on Figure 3.4

[10]For example, it is typical for Mahjong to be played individually or cooperatively. While the former may be right most of the time, the latter could potentially make the game more interesting.

Secondly, compared to Chess and Go, the DouDiZhu is an imperfect information game that usually has more complicated solutions. In Mahjong, players have complete information about their situation but not their opponents. However, in DouDiZhu, for the peasants, they have both incomplete information of the rival's and their own. This situation adds more uncertainty to the game and makes this game unique. Hence, there is a high degree of confidence DouDiZhu is situated in the zone of the game sophistication, where attractiveness and fun of the game strongly affected by the degree of cooperation.

Finally, Figure 3.5 showed various board and card games and possible score opportunities in continuous sports games, which are depicted by the game length (D) versus the average branching factors (B). It can be observed from the figure that DouDiZhu, both three-players, and four-players versions, were located close to the "zone" of sophisticated game. Also, the DouDiZhu provides the best setting concerning the aspects highlighted in the Flow theory [46], where the game incorporated a level of sophistication that includes instant feedback and much uncertainty as well as fun.

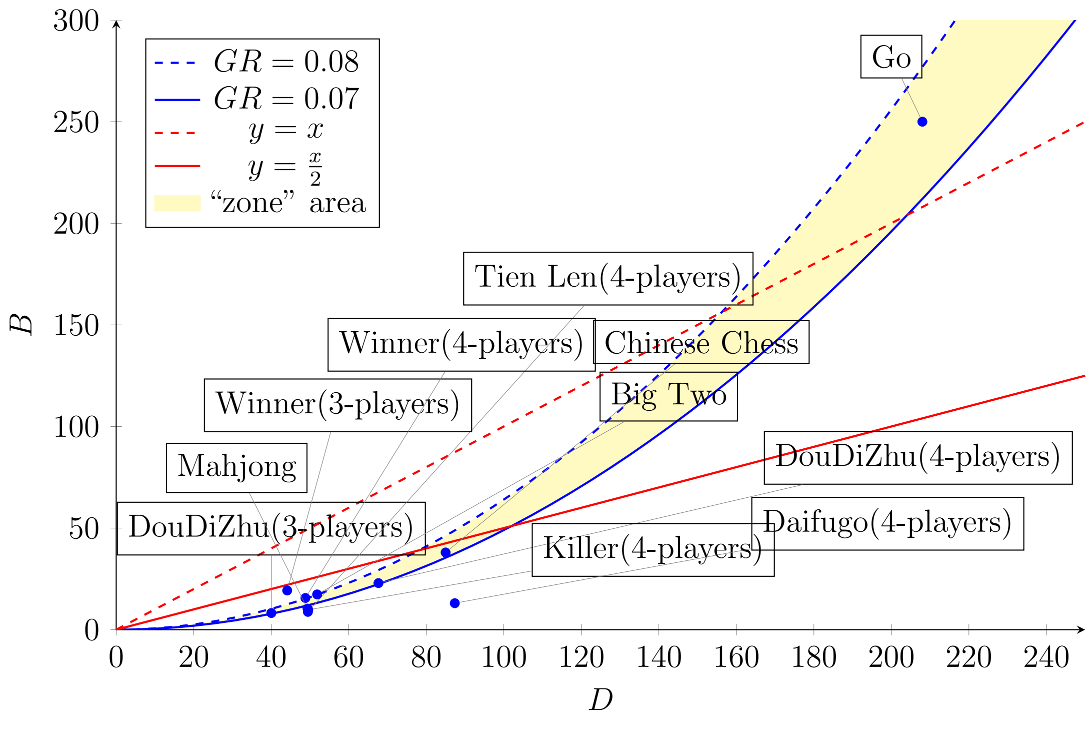


Figure 3.5: The depiction of game refinement values of the well-refined games mentioned in the study

In essence, DouDiZhu is a unique game that emphasizes both competition and cooperation. The occurrence of cooperation in the game is affected by two factors: the number of players and their respective roles (landlord vs. peasants) and the game mechanisms which allow players to cooperate or compete against one another based on the usage of "pass". These factors encourage cooperation between peasants while balances the expected competition between the landlord and the peasants, which provide insights into the nature of the DouDiZhu game as well as being sophisticated enough to be an attractive multiplayer game. Hence, such a setting is not surprising when the DouDizhu is nominated as the most popular card game in China within less than 30 years of development.

3.6 Chapter Conclusion

This study had tested different aspects of the Chinese most popular card game, DouDiZhu. Simulation results indicate that the multi-person incomplete-information games like DouDiZhu, having both cooperated behavior and competitive behavior, follow the principle of a seesaw game. The preliminary simulation results revealed that three-player DouDiZhu is perfectly refined with sophistication, entertainment, and fairness on the measurement of game refinement at the value of 0.0715.

It was found that the number of cards effectively influences the player's experience. A small disparity is needed for novice and fair level players, while significant disparity is reasonable for a professional level. The most sophisticated game setting for DouDiZhu is found as $DDZ[(3,1,2)(20,17,17)]$. Lastly, we compared DouDiZhu with a similar version over the world and other well-refined card and games in China, where the DouDiZhu is numerically found to be the most exciting game. Also, a cooperation strategy is required to balance the expected game fairness among the players, which is measured using the scoring system.

Nevertheless, the DouDiZhu game was somewhat simplified in term of its score system compared to a real-world DouDiZhu game scenario. Further study can be conducted to investigate how different scoring mechanism will affect the sophistication of the game. Additionally, it can be hypothesized that the player's characteristics could hugely affect the game result. Our simulation just tests one specific type of strong players. Future studies can focus on the various characteristic of players to determine the influence of such characters on the game outcome.

Chapter 4

Uncertainty Dynamics in Game Process

This chapter is based on the integration, update, and abridgment of the following publication:

- Yuexian Gao, Naying Gao, Mohd Nor Akmal Khalid, Hiroyuki Iida, Finding Flow in Training Activities by Exploring Single-Agent Arcade Game Information Dynamics. In International Conference on Entertainment Computing, Springer, pp.126-133, 2020.
- Yuexian Gao, Chang Liu, Naying Gao, Mohd Nor Akmal Khalid, Hiroyuki Iida, Nature of arcade games, Entertainment Computing, Volume 41, 2022.

4.1 Chapter Introduction

Arcade games, firstly referred to entertainment machines installed in public businesses, focusing on the user's reflexes, while usually featuring very little puzzle-solving, complex thinking, or strategy skills[1], has been chosen as the target of this study. Since Apple launched the arcade games project in 2019[2], arcade games have entered the limelight with a new look. The term "arcade game" nowadays refers to action video games designed to

[1] Arcade Game, http://en.wikipedia.org/wiki/Arcade_game

[2] www.apple.com/newsroom/2019/09/apple-arcade-invites-you-to-play-something-extraordinary/

play similarly to an arcade game in the game center, where the gameplay is frantic and addictive. Although arcade games have been greatly improved in both form and content over the decades, their core feature is still "simple but fun to play."

Kim and Jang illustrated that arcade games up until early 2000 had been highly influential and profitable, which had occupied most of the market share for the Korean domestic game industry [87]. Furthermore, they discussed the current arcade game industry in Korea and the best practices in other countries to develop propositions for the domestic arcade game industry's comeback and competitive improvement. Some researchers focus on specific arcade games to re-design the game, tested advanced algorithms, or as a predictor for human activities [88–90].

More recently, some studies explored the practicalities of using games for learning. Researches have shown that the potential usefulness of arcade games for higher education is promising. The use of arcade game elements in other serious game can reach a positive efficiency [91–95]. In essence, scholars have been using arcade games as their research target, with main focus on aspects such as assisting psychological experiments, improving the efficiency of algorithms, and using it as a learning medium.

Nevertheless, uncovering the arcade games process's dynamic relative to optimal flow experience in game-playing is limited. Therefore, the following questions arise:

- What measures can be used to quantify the process dynamic of an infinite arcade games?
- What are the significant factors that were possessed by an infinite arcade game?
- What contributes to a successful infinite arcade game, and take a step further, can be learned from the mechanism of addiction occurred in an infinite arcade game?

4.2 Game Test-beds

For this study, four arcade games are selected as the benchmark: Flappy Bird, Tetris, Pong, and Brick Car Racing as in Figure 4.1. These games were selected due to their distinct characteristics. For example, Pong is considered the first traditional arcade game in history with limited features. Meanwhile, Flappy Bird represents the types of games

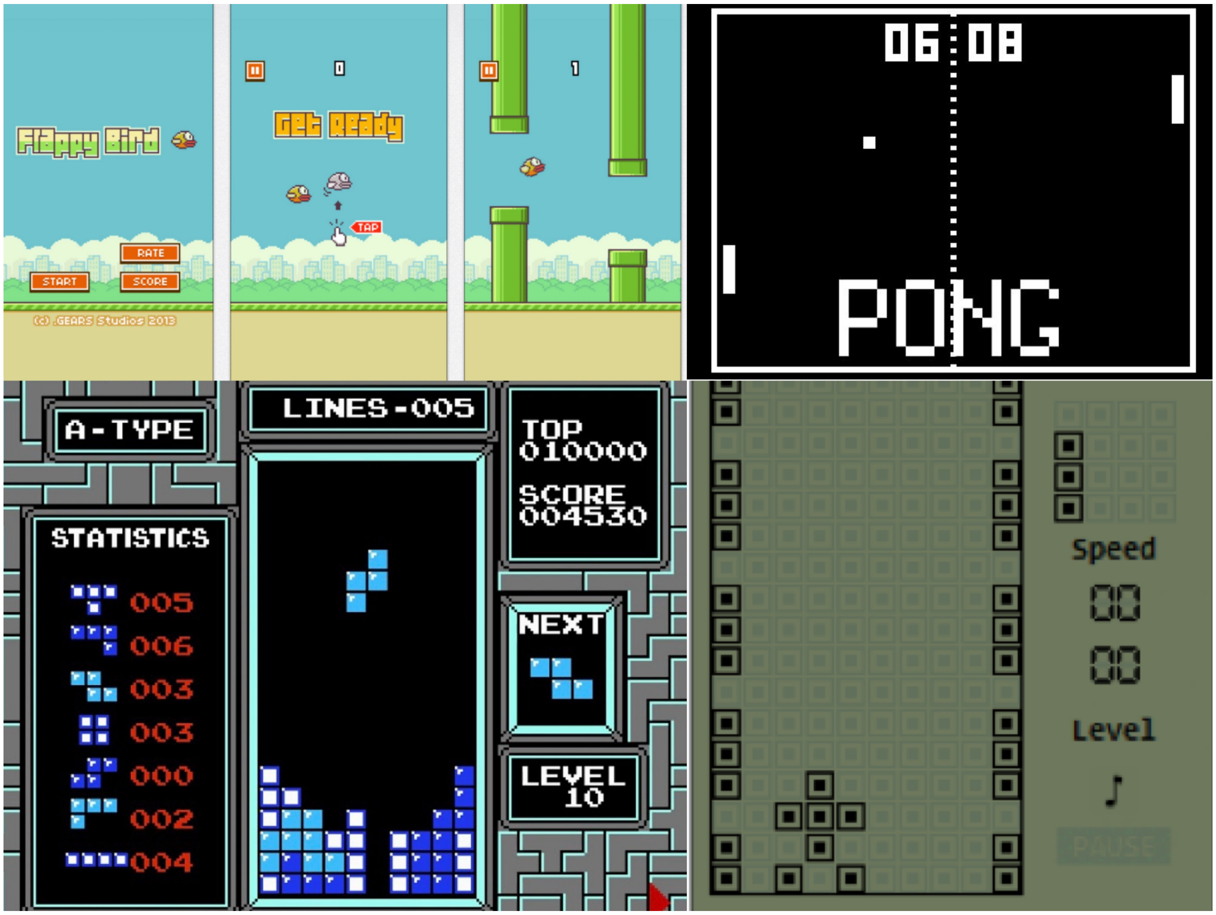


Figure 4.1: Screenshot of target arcade games

with no difficulty change with the game's progression. In contrast, Brick Car Racing could be regarded as the prototype version of running games (i.e., Temple Run and car racing games) where the difficulty changes with the game's progression. Finally, Classic Tetris is one of the most popular infinite arcade games that incorporate simple features but remain popular over the past decades. This study investigates a more in-depth mechanism of the infinite games by comparing them while establishing the correlation between the psychological model and the information model to better understand the nature of human's enjoyment via the law of motion in the mind.

4.2.1 Flappy Bird

Flappy Bird was released in 2013 developed by Vietnamese video game artist Dong Nguyen. It was the most downloaded mobile in the app store for IOS, with a download amount of 50 million per month, during which the developer earned 50,000 dollars a day from the in-app advertisements. The game was removed from all the platforms as of February 10th, 2014, due to guilt over what the programmer considered to be its addictive nature and overuse[3]. Players only need to use a finger to control, click touch the screen, the bird will fly up, continuous click will continue to fly to the heights. Relaxation of the finger will drop quickly. So the player has to control the bird to fly forward, and then pay attention to avoid the uneven pipe on the way.

Many researchers have studied the game and trying to find out why the game is so attractive. Connor Sauve[4] collected data by a local sever and logged 419,000 attempts of the game. He found that the skill is proportional to playing times, whereas difficulty is proportional to game length. Besides, there is a distribution for players from the data, as shown in Figure 4.3[5]. The plot reveals statistics of the player based on players' best score against the time of best score that was obtained on the arcade game. The green, yellow, and red areas are defined as the zone of skill, casuals, and colossal failure. Data showed that 50 percent of the players play ten times or less, and 75 percent try less than 25 attempts[6]. With training over a longer time, these activities would quickly become tedious. Flappy Bird is a game that quickly becomes addictive with a low retention period as the trend interest indicated in Figure 4.2.

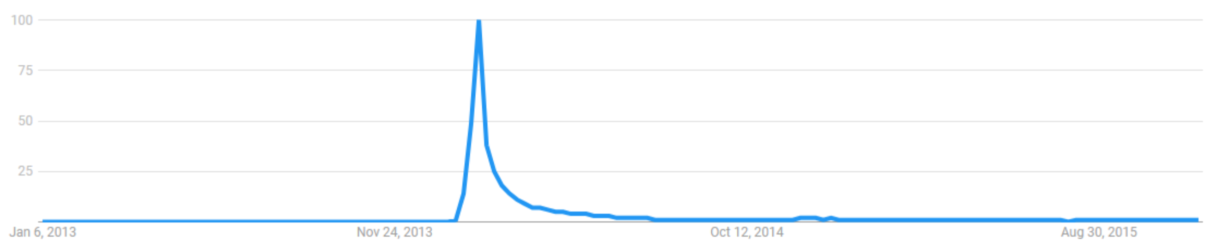


Figure 4.2: Google trend interest of "Flappy Bird" over time

[3]Flappy bird: https://en.wikipedia.org/wiki/FlappyBird

[4]Analysis of 419,000 FlapMMO attempts: https://t3hz0r.com/post/analysis-flapmmo-attempts/

[5]https://t3hz0r.com/post/analysis-flapmmo-attempts/

[6]https://ahstat.github.io/Flap-mmo/

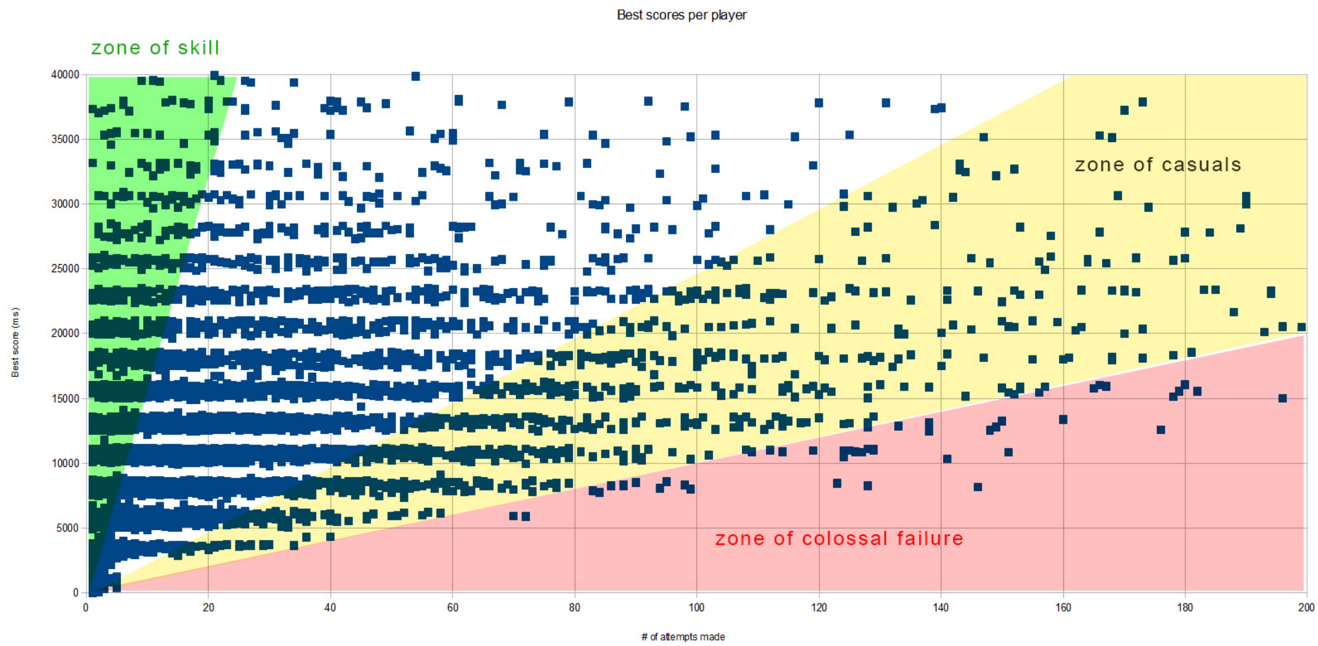


Figure 4.3: How practice affected score of Flappy Bird

Flappy bird is a good sample to study and explore machine learning, reinforcement learning [96–98] method and other artificial algorithms for its simple gameplay. Deep reinforcement learning already reached an efficient way for conquering the game [99]. Aaron Isaksen et. al use the metrics version of Flappy Bird, combined with automated playtesting, Monte Carlo simulations, player models based on human motor skills, and survival analysis, to effectively predict game difficulty variations [100]. The game variant is playable and significantly different from the original game in terms of challenge, sense, and theme [101].

4.2.2 Classic Tetris

Tetris is a tile-matching video game created by Russian software engineer Alexey Pajitnov in 1984. The game was published by several companies and causes dispute over the appropriation of the game's rights in the late 1980s. Tetris holds many world records as one of the most successful games to play[7]. One of the four squares that make up a Tetrimino, the basic pieces in Tetris. When a Tetromino locks and fills all the empty space in one or more rows of the game domain, these full rows are cleared. In classical

[7] https://tetris.wiki/

Tetris, as the level increases, drop velocity of blocks will increase. The game ends when the number of non-eliminated rows reaches the top of the matrix board.

Study of Tetris touches on not only game but also many aspects of academic research. Brzustowski [102] raised the question "Can you win at Tetris" and built a model to analyse the worst-case scenario for Tetris, claiming that there would be a winning strategy only when player can make the game last indefinitely. Burgiel [103] conjectured and proved that the Tetris game consisting of only Z-tetrominoes alternating orientation will always end before 70,000 Tetrominoes have been played. Demaine [104] showed that in the offline version of Tetris, maximizing the number of rows cleared, Tetris, occupied blocks, or blocks placed before the end of the game is NP-complete. Correspondingly, Breukelaar [105] offered "a much simpler way" to solve the problem. There are also many studies focusing on the application of Tetris artificial intelligence, algorithm and other technologies [106–110]. In addition, Tetris also has been widely treated as the tools for psychological experiment, skill training and cognitive study [111–113]. Experiments even find a way that offers a promising new low-intensity psychiatric intervention that could prevent debilitating intrusive memories following trauma [114].

4.2.3 Pong

Pong was invented by Atari in the early 1970s. It is one of the earliest video games, taking two-person table tennis as the prototype, featuring original graphic interface with a ball and two paddle on opposite sides [115]. The in-game paddles move vertically across the screen's left or right side. A player can compete against another player by controlling the second paddle on the opposing side, where the paddles were used to hit a ball back and forth. The game's original goal is to reach eleven points by either player before its opponent does, and points are earned when one fails to return the ball to the other [116]. "Pong" was the first to appear successfully in the commercial market of the public in 1972 and lead the arcade game gradually into industrialization. It is one of the earliest arcade video games and table tennis-themed games that feature simple two-dimensional graphics and is initially manufactured by Atari [87]. Henry Lowood et al. evaluate Pong occasionally depicted as a product of the computer age or even as a computer artifact. They intertwined histories of Nolan Bushnell's Computer Space and Pong to illustrate the transition from these "university games" to accessible entertainment and educational games as well as the complicated historical relationship among the arcade, computer, and videogames [117]. Allan Alcorn created it as a training exercise, which later Bushnell and Ted Dabney (Atari co-founder) decided to manufacture the game [118]. Double Pongs game is also implemented to study learning coordinate with deep reinforcement learning [119].

4.2.4 Brick Car Racing

The game Brick Car Racing is a dedicated handheld game console popular in the early 1990s[8]. Introduced in China, it was initially intended as a clone variant of the original Tetris that dates back between 1984 to 1985 in the former Soviet Union, developed by Russian programmer Alexey Pazhitnov for the Soviet ELEKTRONIKA-60 home computer system.

There are a lot of variant games such as Race (1978), Monte Carlo (1980), S-Racer (1980), Enduro (1983), PC-DOS (included game) (1981)[9]. Most of the racing games also contain the other operation and factors like collision, surpass and attack etc. The test-bed we choose is the one of the simplest in which the player switches between left and right lanes to avoid other cars passing through. The player loses if the car hits one of them. Game speed increases with different level.

Luigi Cardamone et al. use the neural evolution approach of augmented topology to optimize competitive non-player characters for racing games [120]. [121] developed two exercise-based interfaces for a racing game, presented an evaluation of a new sports interface in a racing game, and provided general guidelines for transforming a common type of sedentary game into a sports game. Car racing is quite a general scope. But the core of Car Racing is "move" and "avoid", which is classified as "DRIVER" MetaBrick by Damien et al. [122, 123].

[8] https://retroconsoles.fandom.com/wiki/Brick_Game
[9] http://www.gameclassification.com

4.3 Motion in Mind for Single Agent Games

As illustrated in Chapter 2, $\vec{p_1}$ represents for objective motion and $\vec{p_2}$ for subjective motion. Correspondingly, since $\vec{p_2} = m \cdot v_2$, the velocity v_2 can be obtained (Equation 4.1). The acceleration is relatively the derivation of the velocity v_2 which was given by Equation 4.2. Hence, the subjective force (F_2) is denoted as Equation 4.3.

$$v_2 = 2m^2 - 3m + 1 \tag{4.1}$$

$$a_2 = 4m - 3 \tag{4.2}$$

$$F_2 = 4m^2 - 3m \tag{4.3}$$

Subjective motion indicates the information that player perceived , therefore, v_2 defines the utility of losses that was distorted and regarded as higher utility compared to gains of the equivalent value [124]. Likewise, subjective acceleration a_2 is a measure of how fast the subjective velocity (v_2) of change, or how flexible the mind is with the amount of uncertain information, also can be used as an indicator of the player's skill. F_2 therefore, is the resistance from the game-play.

4.3.1 Relative Velocity

Correspondingly, the difference between objective velocity in the game (v_1) and subjective velocity in mind (v_2) was also defined, denoted as v_R, namely as relative velocity, given by Equation 4.4. Since v_2 is always lower than v_1, there were instances where $v_2 < 0$ indicates that the game motion is much faster than our mind, which requires adequate skill or ability by the player to catch-up.

$$v_R = v_1 - v_2 = 2m - 2m^2 = 2m(m-1) = 2mv \tag{4.4}$$

Conjecture 1 *Relative velocity represents the game information process's sensory-accurate speed, where players experience constant exertions when v_R is the greatest due to the player's mental state always catching up with the game's physical progression.*

Because there is no opponent in single player games, compared to discussing the results of v_1 and v_2 changes separately, discussing v_R is more representative of the operation of game-play in single agent games due to its integrity. Thus, v_R can more effectively reveal the impact of uncertainty in games.

4.3.2 Resultant Force

The force of gravity ($G = Mg$) is defined as the product of the object's mass (M) and gravitational acceleration (g). That is the reason why we can stay on Earth instead of being flung into space. When the gravitational acceleration of an object is constant, the gravitational force's magnitude depends on the object's mass—the greater the mass, the greater the gravity. An anti-gravity system is one that puts a counter-force on the object. When this reaction is greater than the object's gravity, the object can escape the Earth's gravity. When a balance is reached between gravity and anti-gravity, objects can be suspended.

According to the third law of thermodynamics, everything goes from order to disorder spontaneously, which is well known as entropy. The mind, too, accelerates in the direction of mind gravity. People like to play games because they provide a kind of anti-gravity in mind, a short break from the original state of motion, which is pleasurable for players. Therefore, force in mind is generated as the game starts and moves in the opposite direction of gravity.

As previously deduced, objective force is a negative mass ($F_1 = -m$) which obtained by deriving the Equation 2.12 where v is the subject, Equation 4.5 and Equation 4.6 were obtained. This situation implies that once a game begins, the game's gravity would be increasing from the opposite side as the gravity of mind (F_2). Thus, considering both systems as a whole, the resultant force can be given as Equation 4.7. Figure 4.4 demonstrates the motion in mind concepts adopted from H.Iida [6] and its expansion from this study.

$$a_1 = -1 \tag{4.5}$$

$$F_1 = -m \tag{4.6}$$

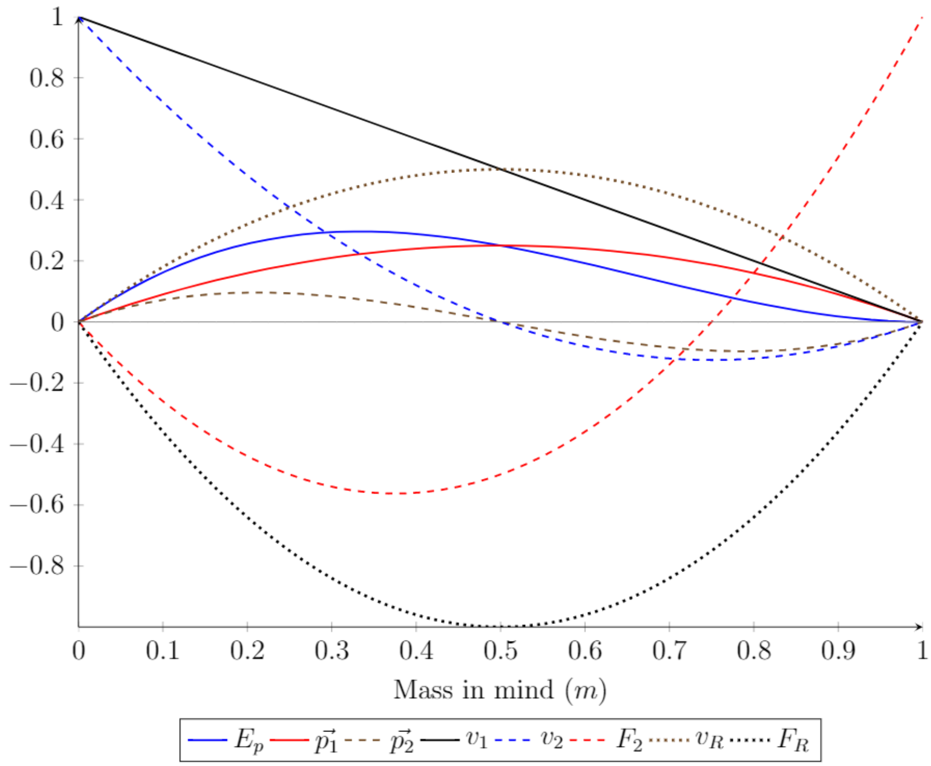


Figure 4.4: Various measures of motion in mind, where the objective (in-game) is depicted based on solid lines and subjective recognition (in mind) is depicted based on dashed line. Also, the dotted line represent the relative velocity and resultant force measures

$$F_R = 4m^2 - 3m - m = 4m^2 - 4m \tag{4.7}$$

Conjecture 2 *The resultant force's negative peak is reached when* $m = 0.5$*, which implies the most significant magnitude of the game excitement and attractiveness. Also, the player faces constant anxiety in game-playing due to the total uncertainty and frequent seesaw.*

4.3.3 Jerk as the measure of "Suprise"

Jerk is the measurement of a sudden change of motion, widely used in vehicle design [125]. To avoid injury caused by passengers losing control of body movement, the acceleration, and Jerk during the process should be paid special attention to since for humans, it takes time to adjust muscle for adapting to limited changes in stress. Sudden changes in acceleration can lead to injuries such as sprains and lead to an uncomfortable ride experience. In mechanical physics, Jerk is referred to as the first time derivative of acceleration, the second time derivative of velocity and third time derivative of position [125], as given by Equation 4.8.

$$j(t) = \frac{d\vec{a}(t)}{dt} = \frac{d^2\vec{v}(t)}{dt^2} = \frac{d^3\vec{r}(t)}{dt^3} \tag{4.8}$$

It was previously found that an appropriate amount of risk and uneasiness were necessary for entertaining design [16], such as the phenomenon of weightlessness in the roller coaster. Since roller coaster is thrilling, it neurologically triggers the fight or flight response within the human nervous system, releasing adrenaline and dopamine in the brain since it was perceived to be dangerous. Such a situation is regarded as a rewarding experience; thus, the greater the rush, the more dangerous the ride. In another aspect, jerk brings excitement and thrill for players in games, and it has been regarded as a parameter that indicates possible addiction and retention of motivation in games [126].

In this study, instantaneous jerk shows the dynamic for different arcade games related to the acceleration during the game process, as formulated by Equation 4.9. Jerk implies that some frequency of changeover (seesaw turnover) during a game may happen, which produces entertaining unexpectedness, upset (failure) possibility in two (one) person context. The dynamics will be maximized at around $m = 0.5$ since the subjective velocity will be switched from positive to negative, and vice versa.

$$j_{ins} = \frac{a_n - a_{n-1}}{t} \tag{4.9}$$

4.4 Data Collection and Analysis

The experiment was conducted twofold. Firstly, the game's overall progress is analyzed based on the motion in mind measure. Secondly, the game process is further analyzed where an expanded analysis of motion in mind is conducted. Depending on the game, data was collected by the player ranking, statistics from the game competition, and self-play simulation from the game agent, adopted from the open-source code repository (i.e., Github). All the experiments were conducted or collected with an Intel i9-9900 processor running at 3.1 GHz using 16 GB of RAM, running with Windows 10, on a 64-bit machine.

4.4.1 Experiment and Environment Setting

Open-source code of Flappy Bird is adopted[10] and the simulation environment is conducted using PyGame[11]. In the simulated Flappy bird game, the player gains a score whenever successfully crossing through a pipe. The game simulates for 10,000 times where the play was optimized using Cartesian Genetic Programming (CGP) algorithm [127,128].

At the same time, the deep reinforcement learning method of learning to play Pong from original pixels is used to simulate the Pong game process by using general strategy gradient algorithm [12]. A self-play game was simulated for 10,000 times where the data were recorded.

Brick car racing and Tetris belong to the arcade games that have difficulty change with the progression of the game-playing. For brick car racing, the updated version on iOS game center[13] was chosen, in which data from 8,196 players were collected from the game application. Meanwhile, Tetris was analyzed based on top players' data in the Tetris world championship[14].

4.4.2 Evaluation Metric via Growth Rate

Theoretically, if the player is skillful enough, there would be no end to the game. This study measures the game process ratio by incorporating the concept of *Milestone*, which is often regard as the records. Take the Olympic sprint as an example: ever since Thomas Burke set the world record of 11.8 seconds at the first Modern Olympic Games in Athens, Greece in 1896, athletes from all over the world have broken the world record 20 times, taking the limit of the human sprint to new heights one after another[15]. It is hard to define or predict when the next record occurs and how much potential human players can achieve. Therefore, the game process ratio for the arcade game is defined as *growth rate*, which relates to the gap between subsequent milestones of different games as illustrated in Figure 4.5.

[10] Learn to Play Flappy Bird using Cartesian Genetic Programming, https://github.com/ShuhuaGao/gpFlappyBird

[11] https://www.pygame.org

[12] Deep Reinforcement Learning: Pong from Pixels, http://karpathy.github.io/2016/05/31/rl/

[13] Brick Game Car Racing, https://appadvice.com/app/brick-game-car-race/951438662

[14] Classic Tetris World Championships, https://thectwc.com/

[15] https://olympics.com/en/olympic-games/olympic-results

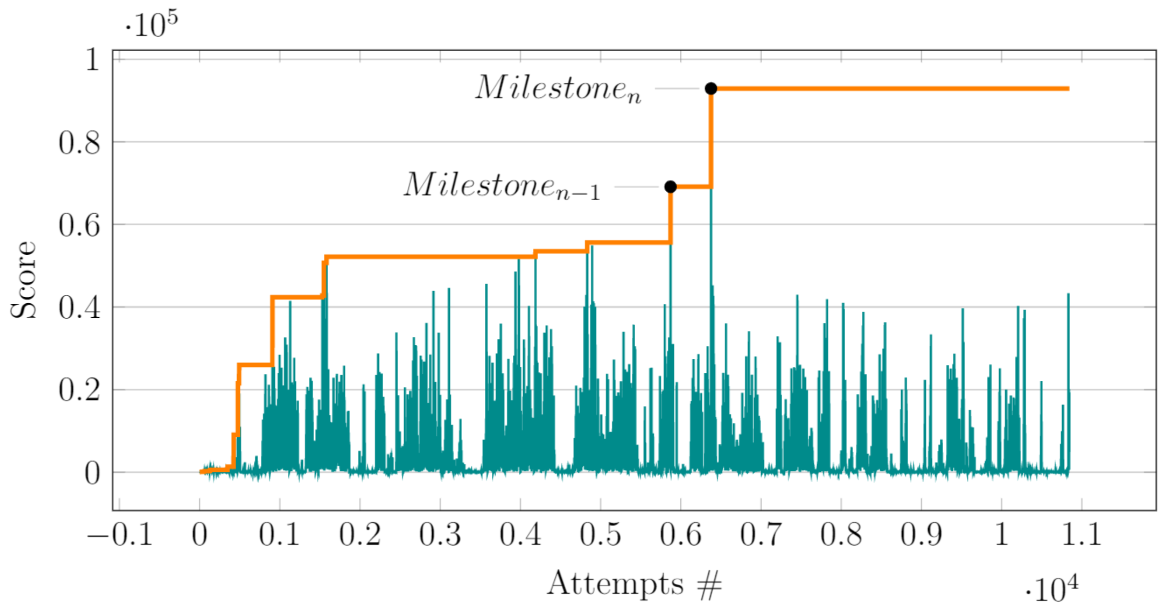


Figure 4.5: Schematic diagram of milestone measure in the arcade games context

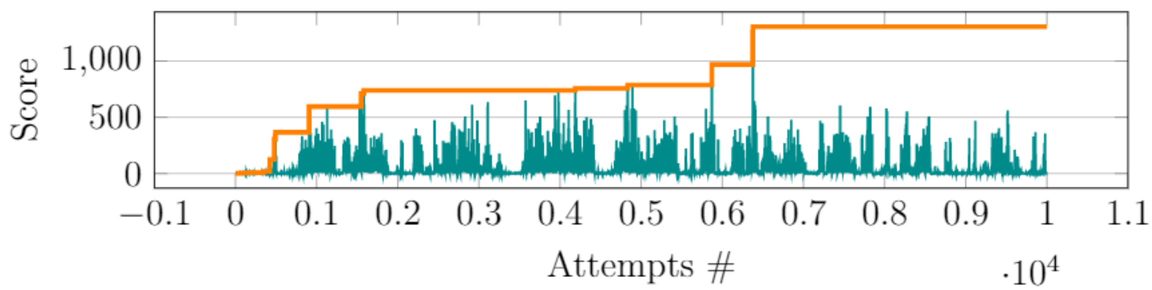
Calculation of the growth rate (v) is by the average of the growth rates between adjacent milestones and is given by Equation 4.10, where the average growth rate with attempts to break the milestone. Furthermore, the instantaneous growth rate given by Equation 4.11 was also applied to observe the variation in the game processes. It neglects the whole process and just focus on instantaneous velocity with in each milestone.

$$v = \frac{\frac{(\mathrm{Milestone_{n+1}} - \mathrm{Milestone_n})}{\mathrm{Milestone_n}} + ... + \frac{(\mathrm{Milestone_2} - \mathrm{Milestone_1})}{\mathrm{Milestone_1}}}{\mathrm{Number\,of\,Milestones}} \tag{4.10}$$

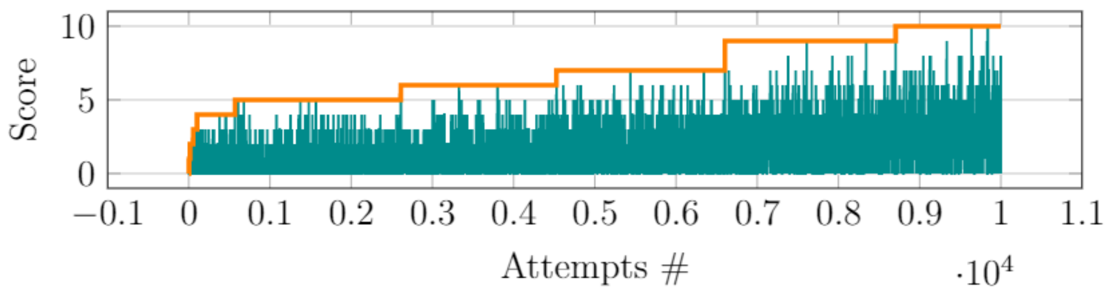
$$v_{ins} = \frac{1}{T}\int_0^T f(t)d_t \tag{4.11}$$

4.4.3 Analysis of General Game Process Dynamic

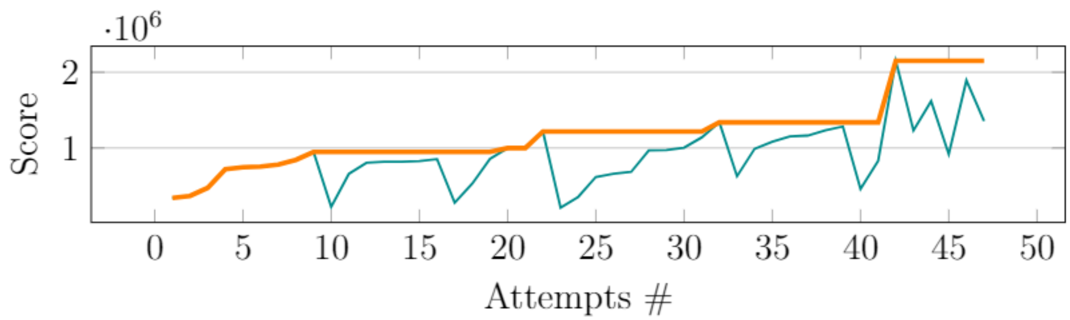
The simulation results of Flappy bird (FB), Pong (PN), Brick Car Racing (BC), and Tetris (TR) were illustrated by Figure 4.6. Based on the evaluation metric of growth rate, FB and PN demonstrate the training process. It can be observed that the challenge is decreasing as the player undergoes further practice (i.e., high score improvement). It is sensible since as players become skillful, they will have an upper level of control, which results in risk reduction. However, when a player's skill becomes strong, the challenge will correspondingly decrease, which would probably bore the player quicker.



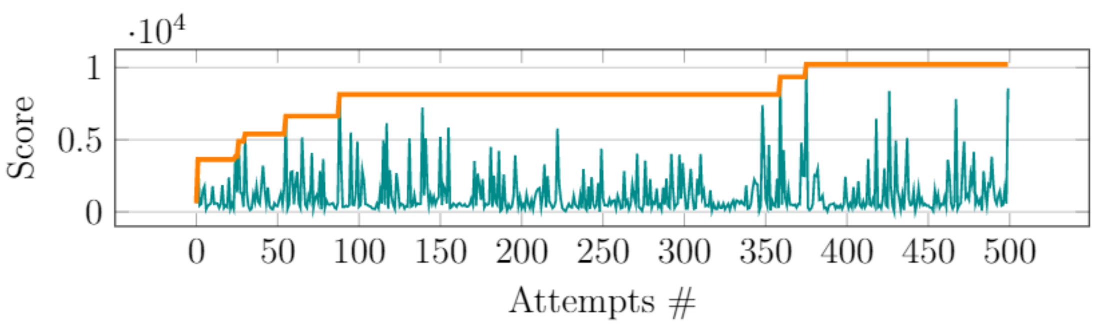
(a) Training growth of Flappy Bird agent



(b) Training growth of Pong agent



(c) Skill growth in Tetris CTWC2019 Ranking



(d) Skill growth in Brick Car Racing

Figure 4.6: Growth rate of four different arcade games (a) Flappy bird, (b) Pong, (c) Tetris, and (d) Brick Car Racing. Green lines designates the record score for each game, while orange lines designates the milestone reached so far.

TR and BC demonstrate skill acquisition. In TR, the top world-ranking players' leaderboard was considered as milestones for players where the score from the last place to the first place is seen as a necessary distance for a player to grow. In BC, the process of a player transitioned from amateur to expert player was demonstrated where the player gains score whenever the car successfully avoids a collision. Unlike the FB and PN, the game would level up for every specific number of passes, and the difficulty would increase by sudden speed changes. Although the four arcade games showed similar growth rate patterns, the growth trend was inconsistent between different games, which requires further analysis in the aspect of the magnitude, frequency, and change the speed of differences between the milestones of each game.

Applying Equation 2.13, Equation 2.14, Equation 2.16, Equation 4.1, Equation 4.2, and Equation 4.3, the motion in mind measures for the four considered arcade games were plotted as in Figure 4.7. Previous work had verified that a game would be perfectly balanced between skill and chance at about $m = 0.5$ since m indicates the difficulty to move the game or mind forward [6]. It can be observed that FB has the lowest m, followed by BC, TR, and PN. This condition indicates that FB is friendly to the novice players but boring for the expert players. In contrast, PN is considered exciting for professional players, while casual players consider it to be challenging.

Moreover, FB had the highest $\vec{p_2}$ while PN had the lowest. The objective momentum ($\vec{p_2}$) showed the motion tendency of the game, while subjective momentum (p_2) indicates the willpower to remain in moving the mind. As such, FB is highly engaged and potentially addictive due to high-win expectations. Such a situation is also further supported by the E_p of FB, which indicates the game had high potential (high expectation) but did not necessarily remain interesting when the player acquired the skill to play the game efficiently. In contrast, PN is engaging due to high-tension situations and the need to minimize error or mistake. While E_p of PN is lower than $\vec{p_1}$ (implying more effort required), the game had some potential (some expectation) but emphasizes planning (not easy to acquire the skill).

In the arcade game domain, v_1 represents the efficiency of uncertainty solving of a game. When the v_2 reaches a negative value ($m > 0.5$), it implies the situation where subjective motion in mind hindering the information process (i.e., requires some thinking

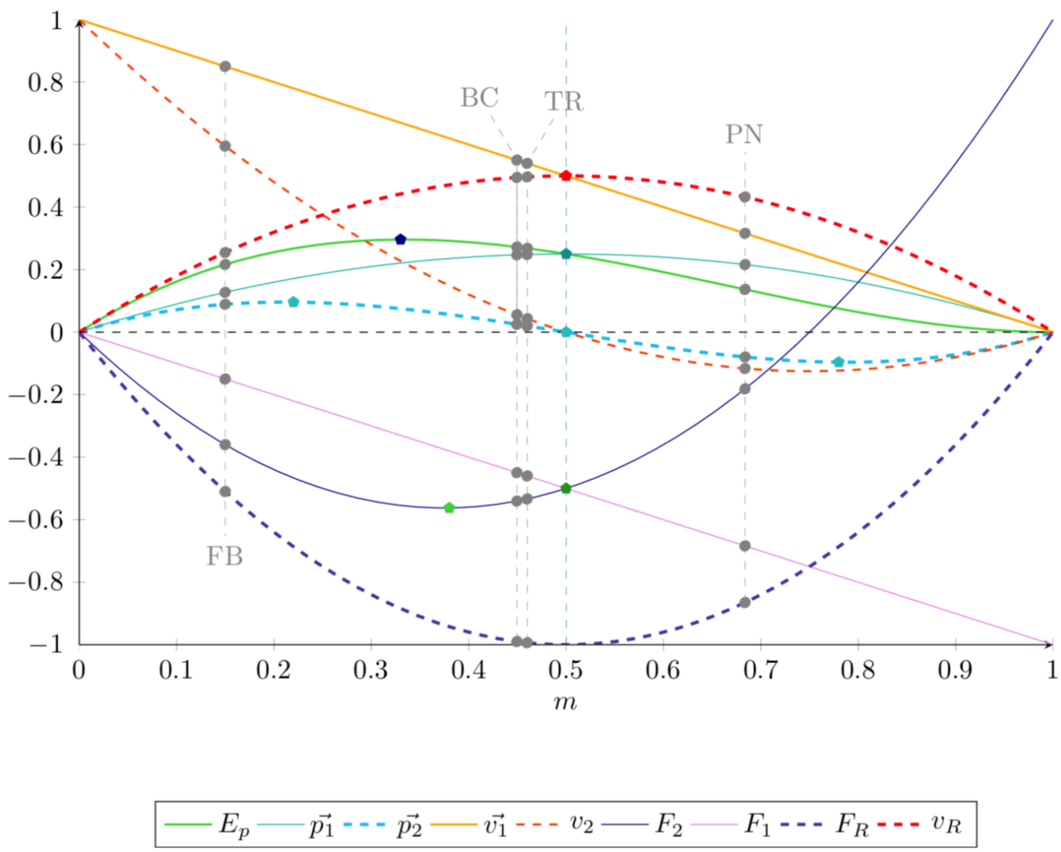


Figure 4.7: Motion in mind for arcade games

or planning to make a move) while the objective motion of the game perpetuates the game forward. In those context, FB has the highest v_1 and v_2, which makes v_R low. Such a condition is likely why FB quickly faded from the limelight since the game can easily and quickly become dull and tedious because it lacks motives to continue the play. On the other hand, PN remains attractive much longer due to intense play experience but ultimately lacks depth in game-playing, making it insufficient to challenge the player mentally. Meanwhile, TR and BC provide the greatest v_R, which implies that these two games would be more excited when playing due to the high turnover of uncertainty and constant exertion in game-playing.

Furthermore, measures of motion in mind in relations to the proposed milestones measures were analyzed by the rate of the increasing milestones of each game's process, which regarded as equivalent to the velocity in the context of game progress model, computed by Equation 4.12. After standardized all of the milestone growth process $\in [0, 1]$, Figure 4.8 shows the milestone process of each game.

$$V_{Milestone_n} = \frac{T_n}{\sum TotalScore} \tag{4.12}$$

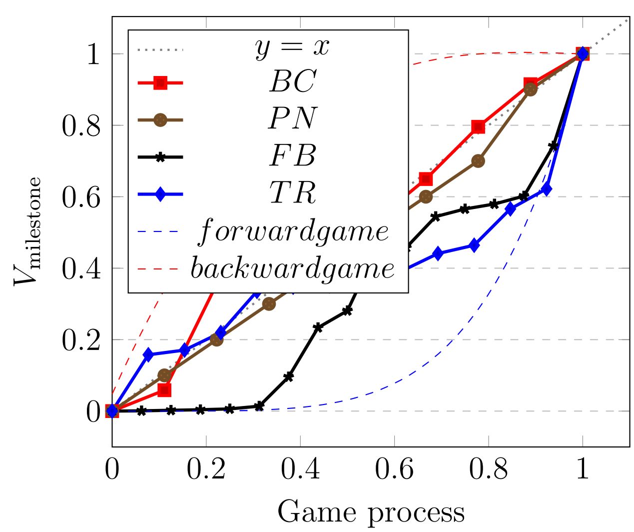


Figure 4.8: Milestone process for game brick car racing (BC), Pong (PN), Flappy Bird (FB), and Tetris (TR). Forward game and backward game concepts are proposed by Kita et al. focusing on the logistics model of game-outcome uncertainty [3].

The figure shows how to player's skill grows as practicing. The process shows that uncertainty solved for a game is consistent with the game progress model of the previously proposed game refinement theory [61]. Three masters model was proposed to explore human intelligence and investigate the sophistication of games [2]. Combined with the backward game and forward game concepts proposed by the logistics model of game-outcome uncertainty [3], arcade games followed the pattern of forward information growth. It offers a hint that while two-person games can be well-balanced with challenge and skill, they follow the master of winning. Single-agent arcade games would be most entertaining when they follow the master of playing [2].

Finally, the velocity, acceleration, and jerk of the target games were computed and illustrated in Figure 4.9. All games showed the informational acceleration (a), and informational jerk (j) fluctuates throughout the game progression. The difference lies in that $a \geq 0$ while $j < 0$ or $j > 0$. Such a condition implies that each game has sufficient attractiveness while providing refreshing or surprising events (i.e., new high score).

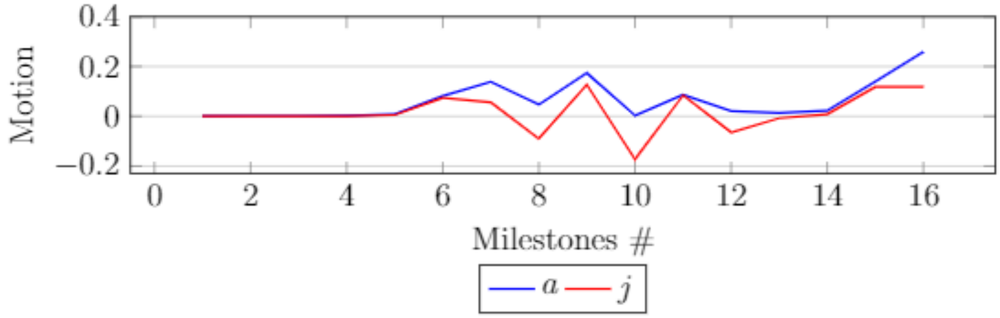
Remark 1 *The dynamics of jerk is the essential characteristic that constitutes the entertainment aspect of arcade games. The amount of jerkiness (or vibration) above and below zero would maintain the feeling of freshness and unexpectedness of the players' play experience.*

4.4.4 Analysis of In-Game Process

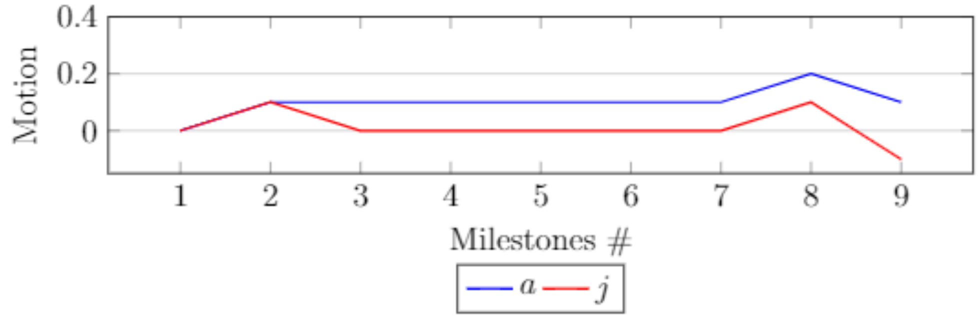
FB and PN have no change in difficulty during the game process. Therefore, no matter what stages the player during the game, they face the same risk and velocity as described in Figure 4.10. In contrast, BC and TR have different levels, and with the game progression, they gradually speed up until players make a mistake and fail. As such, the velocity of the game process for BC and TR were investigated. Previous research had attempted to illustrate the importance of difficulty dynamic adjustment to the player experience when sudden change into a new level [84, 129]. This study will explore the deeper connections and patterns between information dynamic change and player performance.

The velocity of different levels for BC is based on the ranking statistic from the game center[16]. For this purpose, the velocity for BC's in-game process is given by Equation

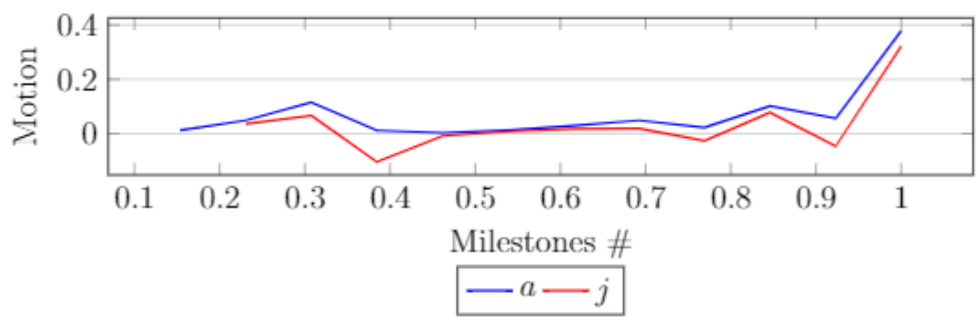
[16]For example, if there are 100 people participate the level 1, 70% of them passed where the v at level 1 is 70%. Among the 70 people, 35 (50%) of them passed level 2. Then, v at level 2 is 50%, not 35% since the calculation is not included in the previous stages (30% percent of the players excluded from the latter risk)



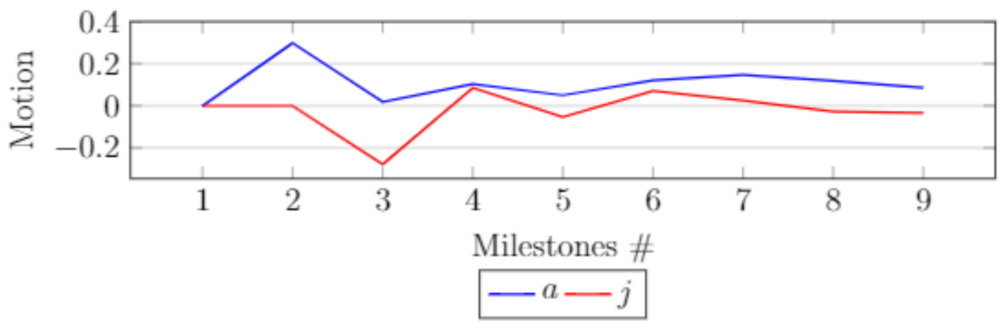
(a) Training growth of Flappy Bird agent



(b) Training growth of Pong agent



(c) Skill growth in Tetris CTWC2019 Ranking



(d) Skill growth in Brick Car Racing

Figure 4.9: The acceleration (a) and jerkiness (j) of four different arcade games (a)Flappy bird, (b)Pong, (c)Tetris, and (d) Brick Car Racing

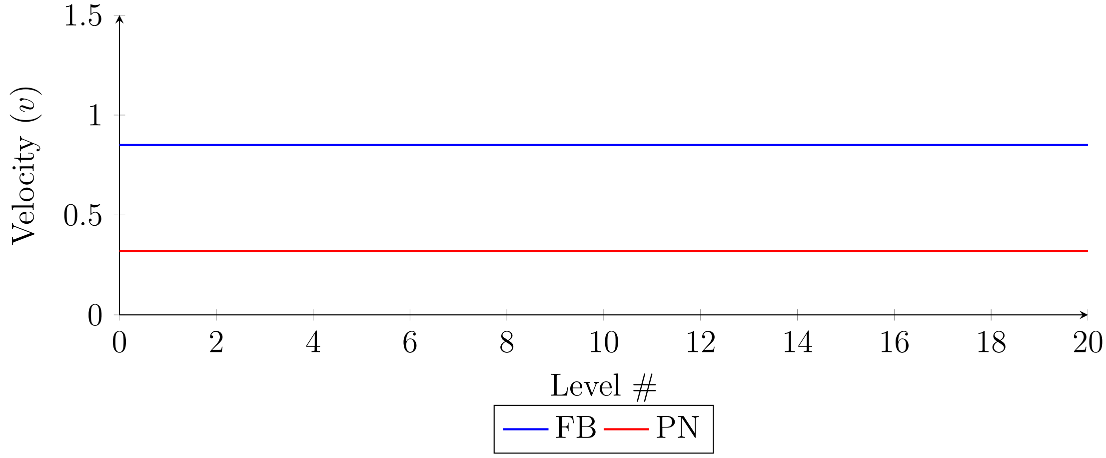


Figure 4.10: The dynamic of velocity (v) in Flappy Bird (FB) and Pong (PN) game

4.13. Meanwhile, TR's velocity is based on the percentage of lines cleared, called Tetris rate (TRT). TRT is crucial for the Tetris competition since Tetris lines have significantly higher point effectiveness than other lines, and it is necessary to keep the higher point in competition before both players reach killscreen[17] in the competition. TRT is the parameter of winning ability, and it also implies the speed of solving the game uncertainty, which is equivalent to the v, as defined by Equation 4.14. The dynamics of in-game process of TR and BC can be observed as in Figure 4.11. Both velocity of them are showing the tendency of dynamic change. The velocity of BC fluctuates around 0.5 while TR shows the dynamic mostly higher than 0.5 and at a relatively stable pattern. Acceleration of BC undulates wildly and irregularly around 0, while undulating period of positive and negative changes of TR is longer and the magnitude of the change is smaller. Such that points to the frequency of jerk dynamics could be another indicator for the sophisticated attractive pattern.

$$v = \frac{\text{Players passed this level}}{\text{Players passed previous level}} \tag{4.13}$$

$$v = \frac{\text{Lines Cleared via Tetris Rate}}{\text{Lines Cleared Total}} \tag{4.14}$$

Therefore, to demonstrate the measure of jerk dynamics of the in-game process of the game, an example data was manually collected from a high-profile player for the classic

[17]https://gamicus.gamepedia.com/Kill screen

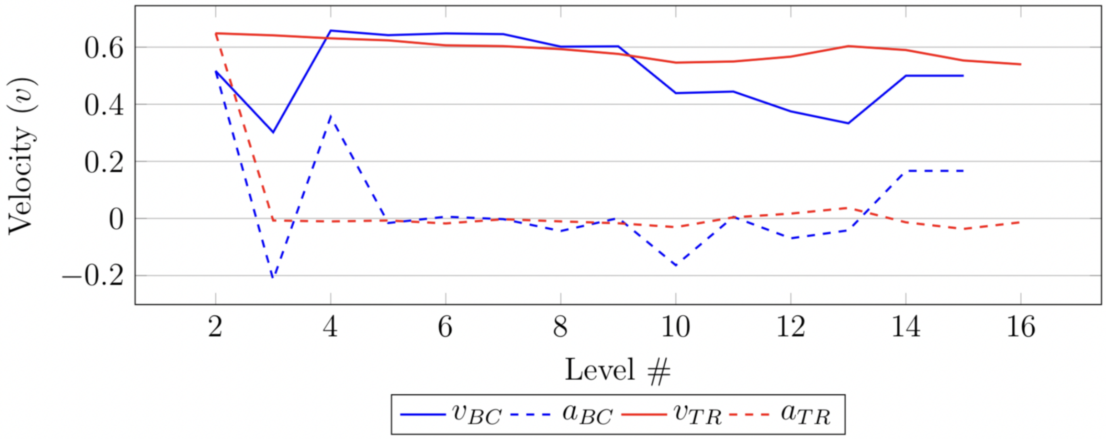


Figure 4.11: The dynamic of velocity (v) of Brick Car Racing (BC) and Tetris (TR) game

Tetris competition on YouTube (with the view over 14 million) and the Final of latest world championship 2020[18]. Table 4.1 shows the statistic of the 4 top players' performance chosen for such purpose, where the game between the two pairs, Koryan versus Joseph and Dog versus Andy, were illustrated in Figure 4.12.

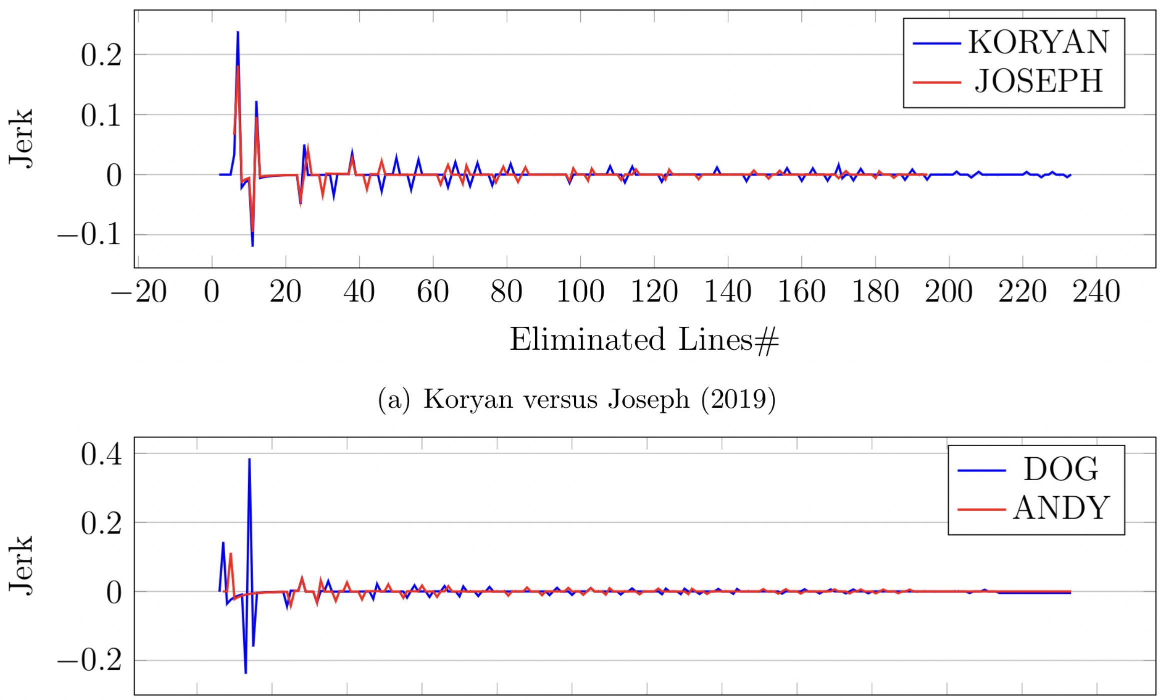
Table 4.1: Information for the chosen players

Player	Year	Result	TRT
KORYAN	2019	Lose	0.58
JOSEPH	2019	Win	0.68
DOG	2020	Win	0.63
ANDY	2020	Lose	0.54

It can be observed that the two final matches appear to have the highest jerk at the early stages of the game. Besides, the amplitude and frequency of the jerk are rapidly decreasing towards the end-game. While jerk relates to vehicle design discomfort, the in-game process corresponds to the feeling of surprise in a game. Moreover, this surprise can maintain players' interest in the game, leading to high engagement and high retention. Comments and evaluations under the video confirmed audiences' enjoyable attitude towards the game. The game process leaves audiences panting for more.

Furthermore, small peaks of the jerks in Figure 4.12 is further analyzed where the

[18] https://www.youtube.com/c/ClassicTetris



(a) Koryan versus Joseph (2019)

(b) Dog versus Andy (2020)

Figure 4.12: The jerk dynamics of the players in (a) 2019 and (b) 2020 Final for Classic World Championship

average jerk amplitude, the positive jerk frequencies, negative jerk frequencies, the average frequency of the jerk, as well as the proportion of jerk occurrences in percentage, were calculated and provided as in Table 4.2. It can be observed that champion players for TR maintain the jerk proportion of about 18% to 20%. In game-playing, considering that jerk itself means subversion and disharmony, the proportion of jerk should be present to some extent while not too excessive. This situation may be associated with the levels of "surprise" experienced from the game that players could associate with pleasure before becoming too excited and potentially frustrating to the players instead.

Table 4.2: Analysis of Jerk for the chosen players

Player	Ave. jerk	jerk+	jerk-	f_{jerk}	j%
KORYAN	9.34E-4	18	20	6.64	17.8%
JOSEPH	1.03E-3	17	18	6.12	18%
DOG	6.81E-6	23	24	6.16	20.2%
ANDY	-1.07E-5	22	24	5.78	19.3%

Ave.: average; f_{jerk}: frequency of jerk;

j%: proportion of jerk occurrences;

Although there is no conclusive evidence, the championship matches' results provided the intuition that a higher proportion of jerks is likely being less preferred. Meanwhile, higher frequencies of jerks would likely stimulate the player to be more engaged and retain in the state of flow since the player is continually being "surprised" resulting from the jerk occurrences. Interestingly, the amplitude of jerk does not consistently support such claims, which drives future investigation.

4.5 Chapter Conclusion

This chapter expands the application scope of the theory of motion in mind while proposed a feasible scheme of potential growth rate to measure the single-agent arcade games that were unquantifiable before because these kinds of games have no definitive game length. Furthermore, relative velocity (V_R) and resultant force (F_R) are proposed to illustrate the physics game. Besides, the inference that arcade games would be the most exciting if the magnitude is in the zone of [0.38$\sim$0.5] is generated.

Four typical and popular single-agent arcade games are selected for testing. After simulating and collecting the game data, Tetris and Brick Car Racing, with the difficulty change that was built-in the game mechanism, elicits a bigger attraction for players since the feeling of high anti-force in mind. Moreover, jerk dynamics during the process of Tetris provides a sophisticated pattern for an infinite single-agent arcade game that can maintain enduring popularity. Furthermore, jerk's dynamics would be the essential factor to sustain the game's engagement.

This study offers a potentially fascinating game mode, exploring the core of attractive infinite single-agent arcade games intuitively. However, more samples of verification are needed in the future. The exploration of scientific theories is never accomplished overnight. Little progress in the exploration of the motion in mind using games as the experimental field will eventually open the door to the real appearance of the world of physics in mind.

Chapter 5

Long Term Uncertainty with Little Interaction

5.1 Chapter Introduction

Idle games are a minimalist gaming phenomenon that has become popular in recent years. They are also called self-play games, or incremental games, are computer or mobile games that involve simple actions such as mouse clicks or screen clicks [130]. By doing so, the player receives rewards or so-called in-game currency, which in turn can be spent on in-game items, increasing the speed of reception or the amount of rewards. The goal of idle game is to unlock as much as possible generators and multipliers to accelerate their income velocity. Though the achievements are reachable to some extent, it would take player years to technically "finish" the game. With the purchase of various game improvements, it is possible to earn rewards, even without the participation of the player. This game logic means that the player's profit increases through an exponential model. However, the price of in-game purchases is also increasing exponentially. Therefore, the time it takes the player to receive the third-level object equals or exceeds the time it takes the player to receive the second-level object. Some games are based on resetting the progress system, like Cookie Clicker, Adventure Capitalists. When resetting, the player gets some currency that can be used to buy improvements that increase the speed at which they get the reward or the reward itself.

"Human behavior is based on progressive rewards.[1]". Some people have compared Cookie Clicker's addiction to the real world: a small business owner who spends years baking cookies by hand is eventually able to pay someone else to do the heavy lifting[2]. The Marxist interpretation defines it as a person's class transition from labor to capital.

In general idle games are fascinating and perplexing. Marked by minimal player agency and periods of inactivity, they seem to defy conventional logic about good game design, and yet nonetheless have attracted a substantial player base. Researchers have drawn many lines on idle games and measure the attractiveness to human player with ground theory and found a precise category for idle games [131].

Idle games are usually played on a web browser, mobile device or PC. The player progresses with little or no interaction. Idle games often involve repeating a simple action to accumulate resources as a core mechanic [15]. It is also called environment games, incremental games, click games, background games, and zero player games [130]. Most of these also include automatic game mechanics so that the game is played automatically. While the interaction is simple, players find these games valuable.

Idle Game was questioned by many players because of its special innovative gameplay and was considered not a game. Scholars have also studied the definition, classification, addiction mechanism and other aspects of idle game. Purkiss at el. conducted questionnaire among 103 people and discussed about the importance of interaction and player's perception to a game, claiming that interaction is the option for extending periods of gameplay [130]. They focus on discussion of the concept of the game, where the rating and definition of interactivity is instructive. But there was no convincing basis for the evaluation. There is not much discussion of the core of Idle Game, nor much in-depth discussion of ZPG (zero-player-game). By adopting the term "zero-player" game, this study mainly intends to address idle game's feature that it can have little interaction with human players.

A survey of 1,972 samples has been conducted by focusing on the problem of "will the game mode of 'Process while gone' attract the player?" [132]. Players are found to be "highly engaged" in the game, interacting in four different ways: Time Spent Playing;

[1] by Griffiths, the director of the psychology division of the International Gaming Research Unit at Nottingham Trent University, and affiliating British Psychological Society's cyberpsychology

[2] dailydot.com/parsec/cookie-clicker-julien-thiennot-incremental-games/

direct sociability; social media sociability and checking frequency. However, the research findings are not consistent with the existing engagement model. They propose that idle game engagement is more of a habit that players acquire and maintain. Alharthi et al. analyze 66 idle games based on grounded theory, including gameplay, game mechanics, rewards, interactivity, progress, and user interface [131]. Ten non-idle games were analyzed in the same way. They discussed how players move from playing games to planning in games, how they question mainstream game assumptions and how they allocate resources. This is basically the most accurate evaluation and extension of idle games.

However, most of the studies only focus on the mechanics of the game. They try to explain the attractiveness gameplay mainly from a psychological point of view, seldom of which have mentioned about the game design or the information process during the game.

Therefore, the research interests are raised by this chapter, we try to figure out:

- How to define and quantify the uncertainty in idle games?
- What features might contribute to the engagement and popularity of idle games?
- How to maintain players interest for idle-game-like activities?

5.2 Measurement of Engagement with Motion in Mind

Motion in mind aims to explore the intangible physics in human mind during activities, which is more stemed from microphysics perspective. According with the former study [1], this study assumes that potential energy (E_p) is conserved over time and converted into game-motion momentum and mind-motion momentum, thereby proposing a new approach to measuring engagement. Similar to the law of conservation of energy in classical physics, E_p is also thought to be conserved, i.e. the momentum of game motion in a game contains both objective (in game) and subjective recognition (in the mind), although at different levels. Motion in games refers to the change of in-game performance such as scores accumulated with time. Whereas motion in mind stands for the change of player's estimation or expectation for the game outcome like it will get excited and agog to carry forward the game.

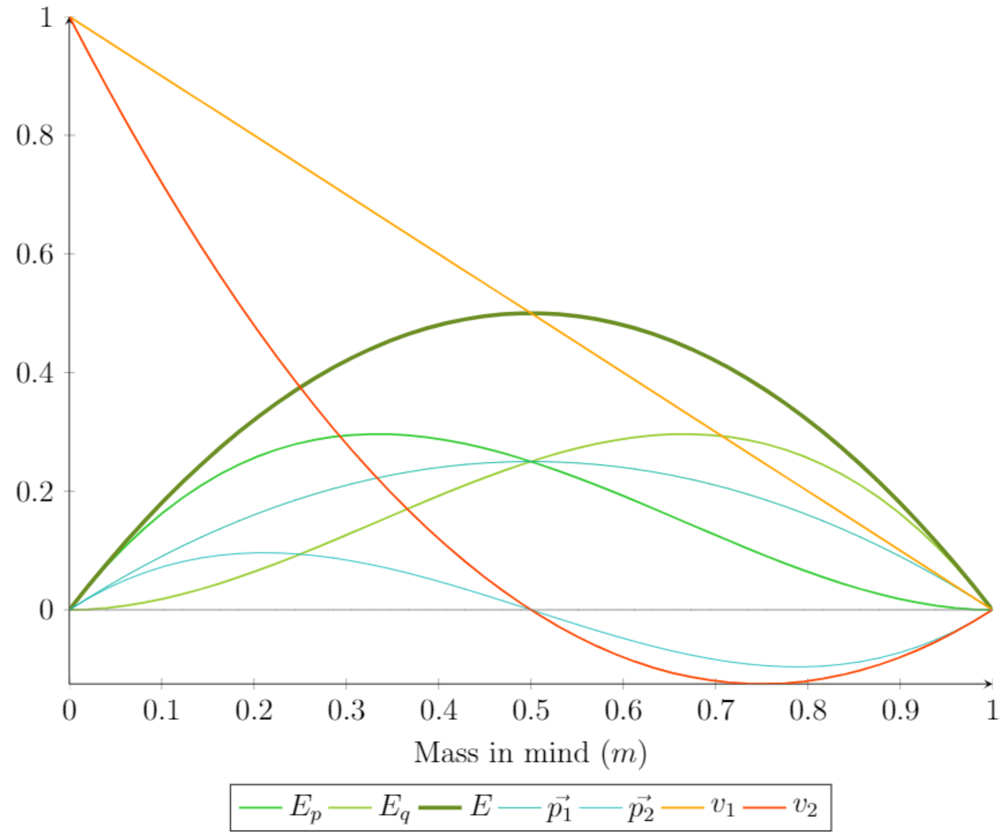


Figure 5.1: Various measures of motion in mind, where the objective (in-game) is depicted based on solid lines and subjective recognition (in mind) is depicted based on dashed line. When m=0.5, the point $\vec{p_2} = 0$, $E_p = E_q = \vec{p_1}$, and E reaches its peak. It indicates at this state, the game-playing would process without any influence from the engagement experience thus expected to have the greatest attractiveness.

Therefore, in this chapter, we conceptualize the model (Figure 5.1) and verified with various activities. Where total Energy in mind is proposed as (5.1) by accumulating $Ep = p_1 + p_2$ (2.15) and (5.2). Total energy(E) comes from law of conservation during the information motion in mind and equally refer to the sum of E_p and E_q. E_p stands for game's motivational potential, albeit amount of information perceived. Whereas E_q is generated as the difference between $\vec{p_1}$ and $\vec{p_2}$ and represents the minds motivational potential, albeit the amount of information expected. Since total energy (E) is expected as the measurement of motivational potential in a game, freedom expected from a game is defined.

$$E = E_p + E_q = 2m - 2m^2 \tag{5.1}$$

$$E_q = \vec{p_1} - \vec{p_2} = -2m^3 + 2m^2 \tag{5.2}$$

As Figure 5.1 shows, when m=0.5, the point $\vec{p_2} = 0$, $E_p = E_q = \vec{p_1}$, and E reaches its peak. It indicates at this state, the game-playing would process without any influence from the engagement experience.

Interestingly, the v_R (Section 4.3.1) and E (Section 5.2) coincide, i.e., $E = v_1 - v_2 = v_R$, the two functions are same. This coincidence gives us evidence that total energy generated is equivalent with the real velocity of the whole game-play process. In another word, the freedom of the game is consistent with the real velocity of the game, all relate to the ability to solve uncertainty.

5.3 Game Test-beds

Focused on the game design, Cookie Clicker, Adventure Capitalists and Clicker Heroes are selected to be the benchmark for studying engagement of idle games. Cookie Clicker is regarded the first idle game to form the click mechanic as well as one the first addictive idle game play that bring this game genre into the public stage[3]. Adventure Capitalist is known as one of the most famous and successful monetization case in idle game market

[3]Sankin, Aaron (12 February 2014). "The most addictive new game on the Internet is actually a joke". The Daily Dot. Retrieved 17 June 2014.

and in idle game history. It is also one of the earliest idle game that can played offline[4]. Clicker Heroes merged with RPG elements and has been rewarded as the best mobile game in 2015[5]. Though these three games cannot include entire range of the idle game genre, they are typical and the preliminary finding will also reveal the core attractiveness of idle games. In addition, the birth of these three games has made the gameplay of idle games complete, and each of them has its own distinct characteristics and improvements. In addition, seeing different quantitative results for different gameplay, we can also see changes in idle games development.

5.3.1 Cookie Clicker

Cookie Clicker (Figure 5.2[6]) is an incremental game created by French programmer Julien "Orteil" Thiennot in 2013. The user initially clicks on a big cookie on the screen, earning a single cookie per click. They can then spend their earned cookies upon purchasing assets such as "cursors" and other "buildings" that automatically produce cookies. Upgrades are also available and can improve the efficiency of clicks and buildings, among many other mechanics that allow the user to earn cookies in different ways. Though the game has no ending, it has hundreds of achievements, and users may aim to reach milestone numbers of cookies.

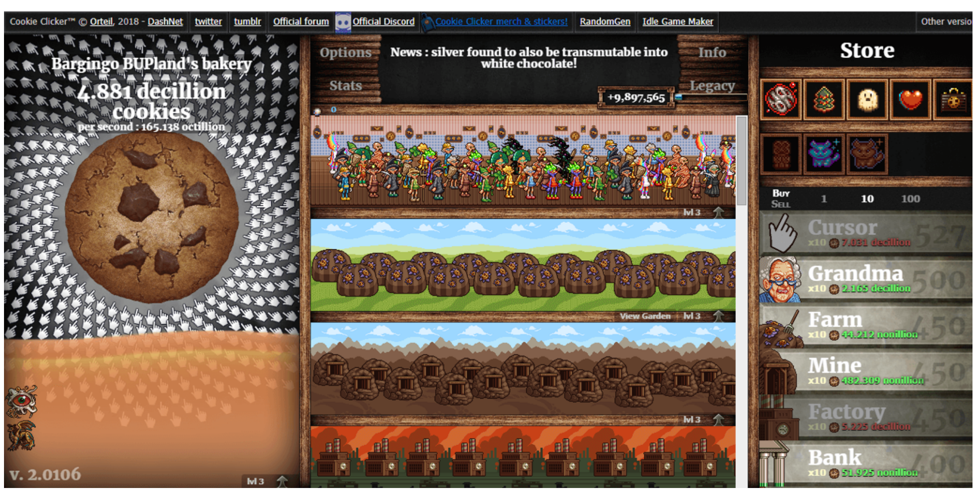


Figure 5.2: Screenshot of Cookie Clicker

[4]Anthony Pecorella (February 2015). Idle Games: The Mechanics and Monetization of Self-Playing Games (Recorded presentation with slides.). Game Developer Conference (GDC) 2015.

[5]Tassi, Paul (14 October 2014). "Why 'Clicker Heroes' Could Be The Top Mobile Game Of 2015". Forbes. Archived from the original on 24 May 2015. Retrieved 22 May 2015.

[6]Screenshot from https://orteil.dashnet.org/cookieclicker/

The game is one of the first and most important in the genre of incremental games and has a dedicated fanbase. It has been widely described as addictive, and it has been noted that the game almost does not require a human to play it. The game features geometric growth: the player begins baking handfuls of cookies, but can quickly reach billions of cookies, and eventually attain duodecillions of cookies or beyond. The game has no clear ending. The generator is Cookie Clicker's base production item. Increase cookie revenue per second by purchasing generators and upgrading generators. Each generator has a separate CPS (Cookie Per Second), and with each additional purchase, the payout goes up. Currently, 18 buildings are available for purchase. Each building has an upgrade of 13 stories.

5.3.2 AdVenture Capitalist

AdVenture Capitalist (Figure 5.3) is a free-to-play incremental video game developed and published by Hyper Hippo Productions which was first released in 2014. AdVenture Capitalist allows players to live like a capitalist and invest funds into certain products to generate revenue, by starting out with a single lemonade stand. The more revenue generated, the higher the player's cash is. A player can receive an angel bonus when resetting their progress, which provides a boost in all products' revenue for the next progress timeline.

Figure 5.3: Official promotional poster for the Adventure Capitalist

There are three areas: Earth, Moon, and Mars. The player starts out on Earth with a single investment: a lemonade stand. Tapping it rewards money over time. When enough money is earned, more stands can be bought. Saving up allows the purchase of different investments at the cost of more time until payout. Once managers are hired for investments, they will run automatically, allowing successful idle play, even when offline. Upgrades can be bought to add multipliers to any investment.

5.3.3 Clicker Heroes

Clicker Heroes (Figure 5.4) is an idle game that was developed originally released for browsers by Playsaurus in 2014, then released onto the Steam platform for Microsoft Windows and OS X [133]. In Click Heroes, the player clicks on enemies to make damage, eventually killing them and earning gold, which can be used to upgrade and purchase characters. The amount of automatic damage to enemies and increased damage per second will vary depending on the character purchased. The game runs without requiring the player to do anything [7], but the player must kill 10 enemies in a level in order to reach the next level, and as the level increases, there will be restrictions such as a time limit, the goal of this game is to upgrade the player to obtain Hero Souls.

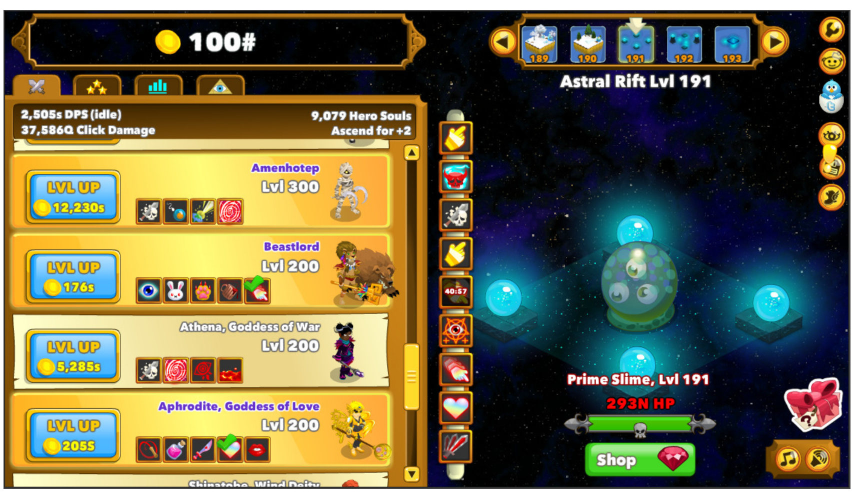


Figure 5.4: Promotional poster for the Clicker Heroes on Steam

The game encourages both active and passive playing experiences, where the player can sit there clicking, or simply let the game killing monsters automatically. These allow

[7]https://www.kotaku.com.au/2015/05/i-left-clicker-heroes-running-all-night-and-heres-what-happened/

players to check in regularly and spend the gold on upgrades. The perfect combination of active and passive play experiences has made the game widely acclaimed. If Clicker Heroes gains traction in the mobile gaming market, it could become the top mobile game of 2015, said Forbes writer Paul Tassi[8].

5.4 Data Collection and Analysis

Mass in mind (m) is the essence of uncertainty in games [1]. Therefore, we set experimental simulation to collect data and get the value of m for the idle game genre. By analyzing the math and value design under the mechanical design of idle games, a fundamental growth pattern beneath the surface of idle gameplay was found. We then simulate the game by their grow equation and check the set of 50,000 seconds for each game, and collect data by their number growth.

5.4.1 The sequential growth rate in idle games

Idle games are known for their remarkable exponential growth in in-game currency. It can grow to huge, incredible numbers. Linear improvements with exponential costs were revealed by Percorella [134] and King [134]. Mathematics behind idle game design is revealed and different idle digital growth models are derived.

For the benchmark, this study applies the simplest model and assumes that the player is always making the optimal choice on the resource allocation in game. Such that the income and cost formula was given as Equation 5.3 and Equation 5.4.

$$cost_{nextgenerator} = cost_{base} \times \text{growth rate}^{\text{number of generators owned}} \tag{5.3}$$

$$income_{total} = income_{base} \times \text{number of generators owned} \tag{5.4}$$

Linear growth, polynomial growth (degree ≥ 2) and exponential growth (as examples in Figure 5.5[9]) are by far the most common used growth rates for incremental game

[8] forbes.com/sites/insertcoin/2014/10/14/why-clicker-heroes-could-be-the-top-mobile-game-of-2015/?sh=4b4aa8c73576

[9] reddit.com/r/incrementalgames/comments/2ztcfk/linearpolynomialexponentialandmoregrowth

designs. The reason is that they bound each other in order (linear $\leq$ polynomial $\leq$ exponential) and can be combined to balance the progress in a game in terms of production and prices.

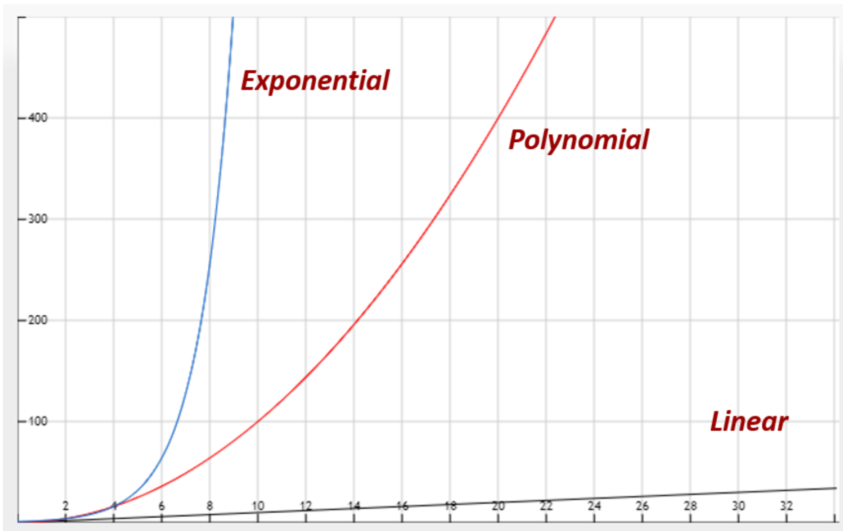


Figure 5.5: Function curve diagram of holy trinity of idle game math design.

The main goal of the game is to unlock as many as possible of the generators to reach a higher and higher currency generating speed. The reward for later generator brings is following the linear growth or polynomial growth, whereas the cost of the next generation is keeping to the exponential growth. Therefore, the difference value between the growth of cost and the growth of income ($\bar{v}_{cost} - \bar{v}_{income}$) is defined as the hardness to win the game as in Equation 5.5.

$$m = \lim_{t \to \infty} \frac{\sum (v_{cost_t} - v_{income_t})}{t} \tag{5.5}$$

5.4.2 Motion in Mind of Idle Games

The sequential growth rate and physics parameters of them are listed in Table 5.1 and plotted as in Figure 5.6. Compared to other game genres, idle games are located at the very left side of the motion in mind functional diagram as shown in Figure 5.6. They have small momentum ($\vec{p_1}$ and $\vec{p_2}$) and small energy (E_p and E_q). Subjective energy (E_p) and objective energy (E_q) are small, thus total energy (E) is also small. Meanwhile they have high velocity either subjective and objective. Such a result suggests that player feels less

Table 5.1: The sequential growth rate and physics parameters of idle games

Game	m	p_1	p_2	Ep	Eq	E
Cookie Clicker	0.15	0.1275	0.08925	0.21675	0.03825	0.255
Adventure Capitalist	0.12	0.1056	0.080256	0.185856	0.025344	0.2112
Clicker Hero	0.07	0.0651	0.055986	0.121086	0.009114	0.1302
Average	0.1133	0.1005	0.0777	0.1782	0.0228	0.2010

attractive and engaging from the perspective of motion in mind compared with other fun games.

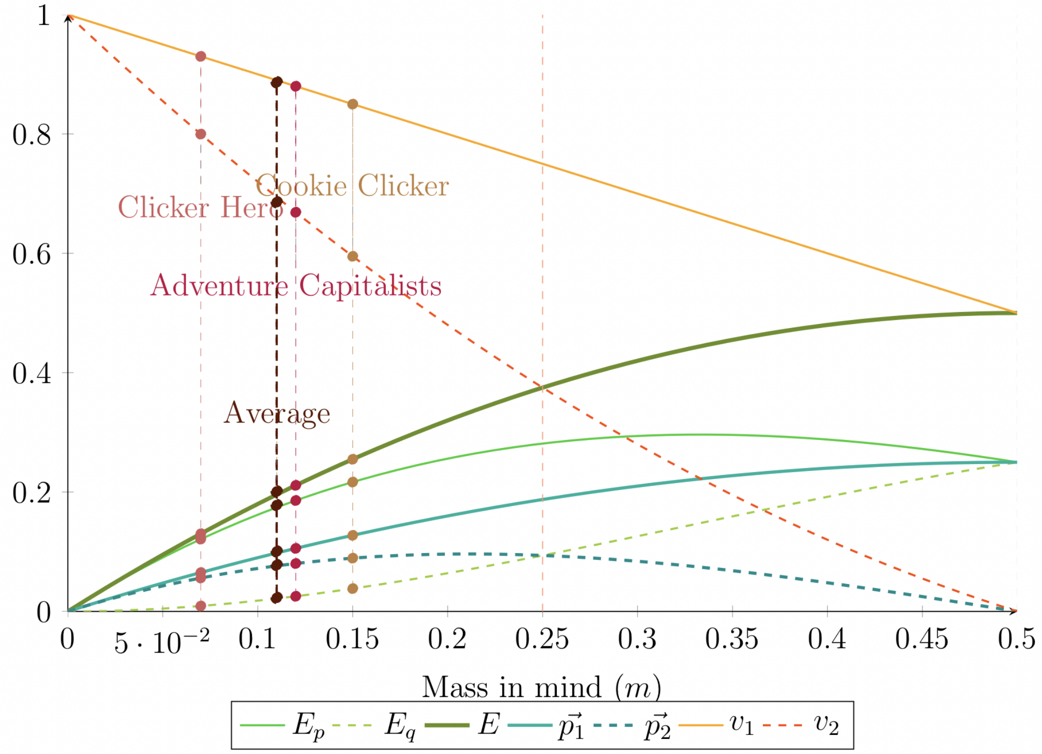


Figure 5.6: Various measures of motion in mind for idle games.

There are two intersections when $m = 0.25$, respectively E_q with $\vec{p_2}$ and E with $\vec{v_2}$. E_q, $\vec{p_2}$ and $\vec{v_2}$ all address the feeling or experience of player expectation, i.e, the motion of mind. And Figure 5.6 shows all of the parameters are positively or negatively correlated with the increase of m (when $m \leq 0.25$).

Conjecture 3 *Game-playing would be affected more by player's interest than information that player perceived due to game's change when $m \leq 0.25$. Therefore, $m \leq 0.25$ is defined as the* ***zone of non-game****.*

Furthermore, each point on the physic parameter function shows different tendency.

For example, E is symmetrical about $m = 0.5$, take two of the same value, say 0.2, we can get the correspondingly two m values. One is near 0.12, the other almost 0.94. The two points have same motion value but absolutely different meaning. (0.12,0.2) is about increasing while (0.95,0.2) is about decreasing.

Even have similar values, since the location of games are different, they have more difference other than the original function reveals. Therefore we derive all the function to observe the trend behind the apparent value.

Figure 5.7 plots the derivatives of each parameter to illustrate more clear based on Equation 5.6, 5.7 5.8, 5.9, 5.10, 5.11. There are three intersections of the first derivatives when $m \leq 0.3$, respectively are $\vec{p_2'}$ with E_q', E_p' with $\vec{p_1'}$ and E_p' with E_q'.

$$\vec{p_1'} = 1 - 2m \tag{5.6}$$

$$\vec{p_2'} = 6m^2 - 6m + 1 \tag{5.7}$$

$$E_p' = 6m^2 - 8m + 2 \tag{5.8}$$

$$E_q' = -6m^2 + 4m \tag{5.9}$$

$$v_1' = -1 \tag{5.10}$$

$$v_2' = 2m^2 - 3m + 1 \tag{5.11}$$

E_q and $\vec{p_2}$ are respectively represents for subjective energy in mind and subjective momentum in mind. Solving $E_q' = \vec{p_2}'$, we get that $m \approx 0.1162$ limited to $m \leq 0.25$. Such value is almost the same of idle game's average m (Table 5.1). The intersection ($\vec{p_2'}$ and E_q') shows their motion trend which for the game-play on the other hand, indicting their desire for continuing the game. This intersection provide a border between the change of E_q and $\vec{p_2}$ as well. E_q's influence is higher at the left side, while $\vec{p_2}$ is dominant more at the right side. Such slight shift in trend can cause a moment of dissonance, but because

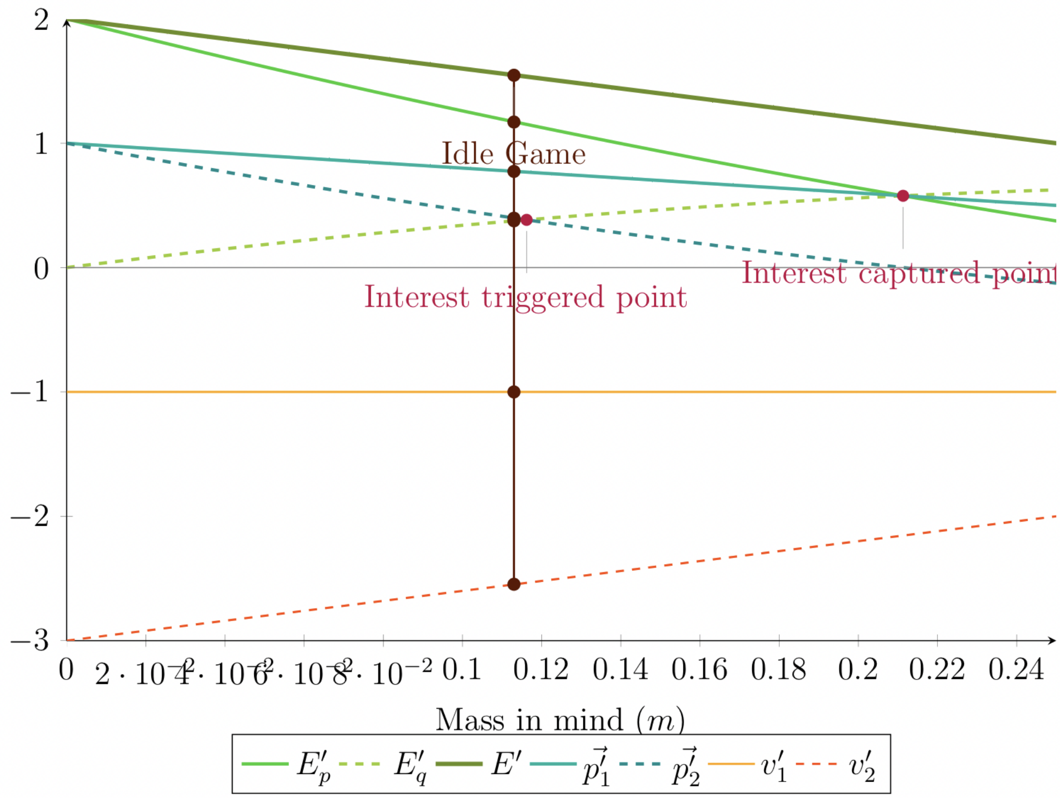


Figure 5.7: Various derivatives of measures of motion in mind for idle games.

of this it also brings a sense of novelty that attracts attention.

Remark 2 *This study defines the intersection of E_q and $\vec{p_2}$ as the* ***interest triggered point****, which contributes to the engagement of idle games, especially compared with those sophisticated games (around m=0.5). Switching between E_q and $\vec{p_2}$ is one of the nature and key attractive factor for idle games.*

Meanwhile, **interest captured point** is also defined by the intersection of E_p, E_q and $\vec{p_1}$. E_p, E_q and $\vec{p_1}$ show the same tendency at this point.

The second derivatives of the idle games are also plotted as shown in Figure 5.8 based on Equation 5.12, 5.13, 5.14, 5.15, 5.16. There is one intersection of total energy E and subjective momentum $\vec{p_2}$, which is also close to one of the target games.

$$\vec{p_1}'' = -2 \tag{5.12}$$

$$\vec{p_2}'' = 12m - 6 \tag{5.13}$$

$$E_p'' = 12m - 8 \tag{5.14}$$

$$E_q'' = -12m + 4 \tag{5.15}$$

$$v_2'' = 4m - 3 \tag{5.16}$$

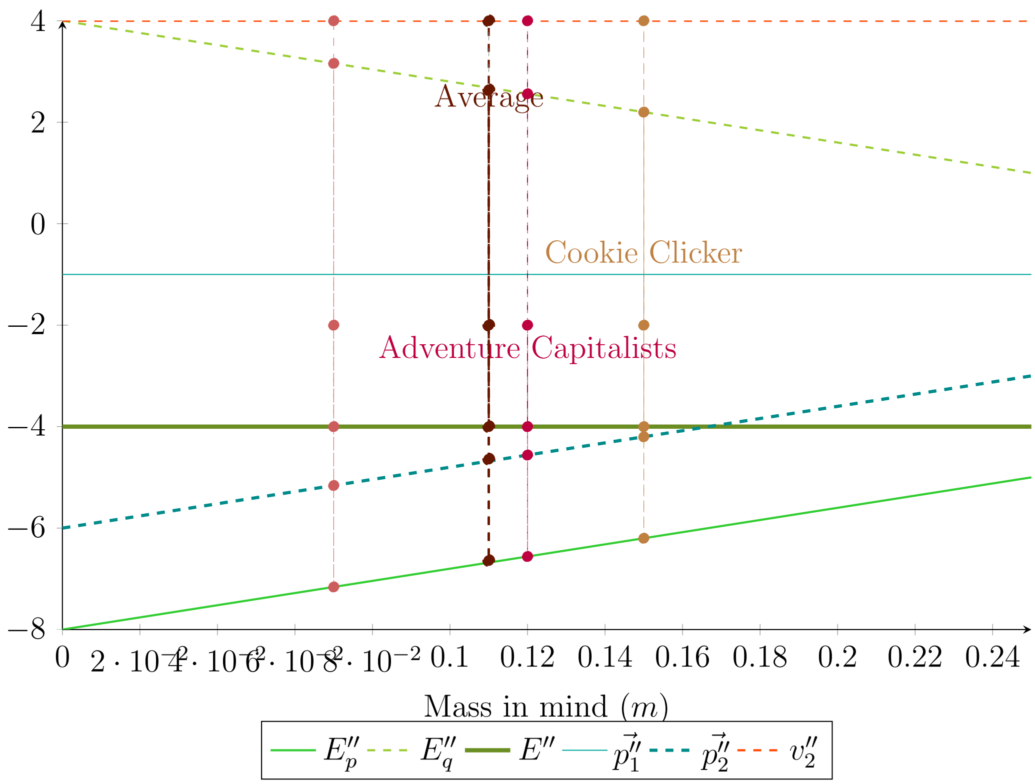


Figure 5.8: Various second derivatives of measures of motion in mind for idle games

There is also one intersection of p_2 and E where $m \approx 0.167$. This point also indicated a very subtle trend change for the information that the player can perceived. The second derivative of subjective momentum (p_2) is increasing, which shows that the resistance for the game-play to attract player is getting smaller, in another word, player's interest is allocated easier though the game-play still hard to attract player's attention. While the second derivative of total energy (E) is constant, which speaks for a stable increasing engagement that the system generates. Therefrom, we denote it as "**insensitive zone**" when $m \leq 0.167$.

Our finding might provide a new angle to observe all the games or activities locating in close region. In idle game case, even they all located closely (0.07 - 0.15), Clicker Heroes ($m = 0.07$) is supposed to be the most attracting one because of the change trend behind the motion.

5.4.3 Mechanism of Idle Games

Previous studies on idle games have a general belief that idle game seems boring at first but will be addicted after played for some time[10]. Percorella pointed out that the prestige mechanism attributes mostly to idle games addiction by resetting most elements of the game, especially the generators and multipliers. At meantime, players receive various forms of in-game currency (such as reputation currency) and/or a continuous multiplier. Moreover, the reset game continues at a permanent upper speed. It's like "New Game +". Diagram of the prestige loops is given in Figure 5.9.

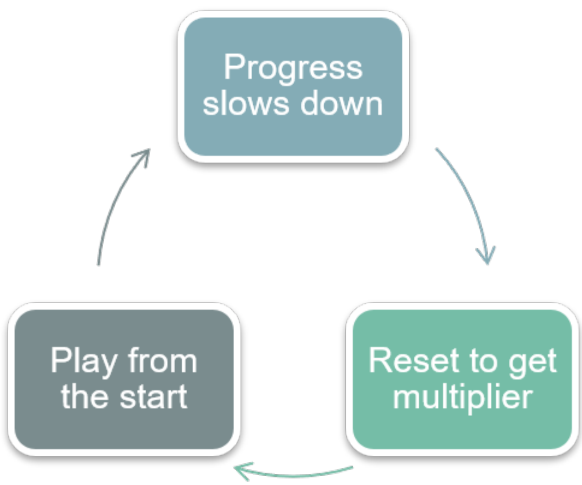


Figure 5.9: Prestige loops in idle games

While the game-play requires strategies on resource allocation, this study regards the player for experiment will always follow the optimal strategy to update generators and multipliers. The value of $\frac{Cost(\alpha)}{nps} + \frac{Cost(\alpha)}{Rate(\alpha)}$ has been proven to be the optimal strategy to solve idle game resource allocation, where the $Cost(\alpha)$ is cost for next generator/-multiplier, $Rate(\alpha)$ is the permanent efficiency increased and nps stands for number incremented per second[11].

[10] https://www.buzzfeed.com/awesome/the-18-stages-of-a-full-blown-cookie-clicker-addiction
[11] gamedevelopment.tutsplus.com/series/numbers-getting-bigger--cms-847

Idle games manage to remove almost all of the redundancy associated with the game and make the game itself so addictive. This study assumes that the player is skillful and always make the optimal resource allocation strategy. When noticing the income growth reward cannot catch up with the cost for next generator, he would choose prestige and reset the game for a permanent attribute bonus.

The prestige study of the idle game by Pecorella[12] was incorporated and further discussed with the perspective of motion in mind. We compared idle game play with prestige and without prestige as presented in Figure 5.10[13]. It is obviously that with prestige player gains more income within same time.

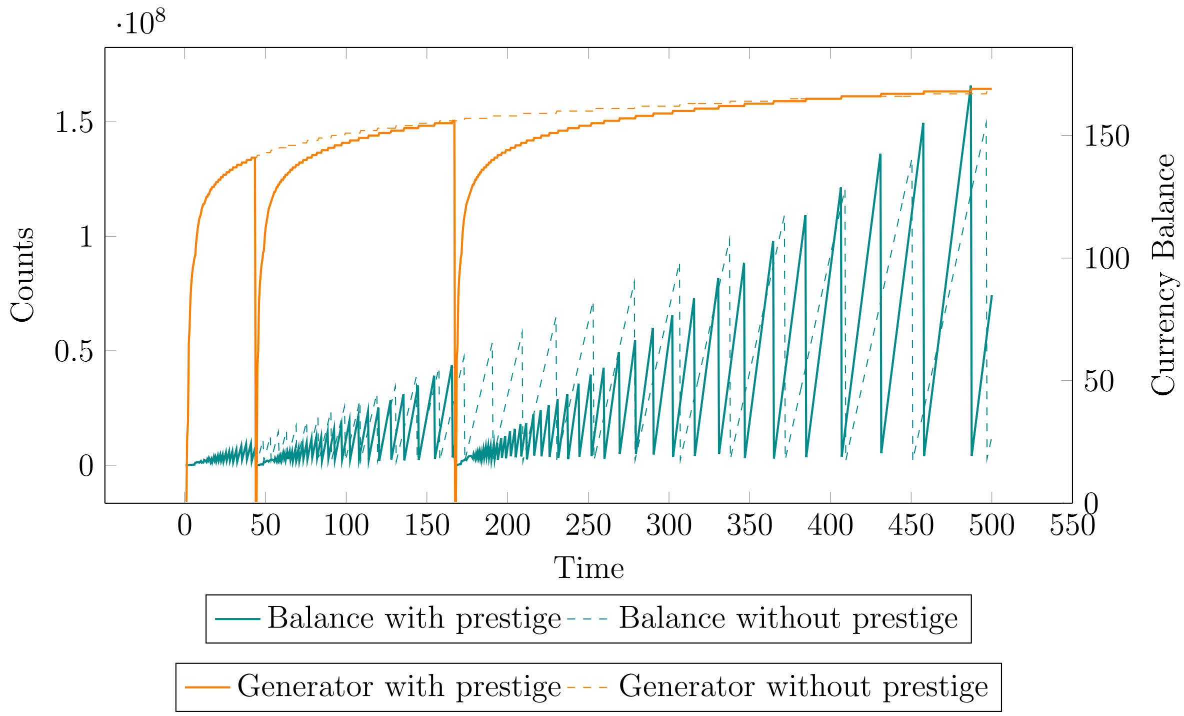
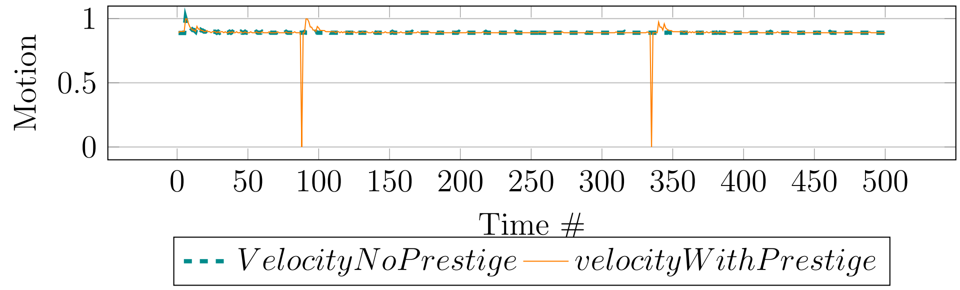


Figure 5.10: Schematic diagram of prestige comparison in the idle games, with only one generator. Though it will be slight different with real play, progression is similar. Calculation is based on Equation 5.4 and Equation 5.3. The $income_{base}$ is set as 20, and the $cost_{base}$ is 4, and growth rate is 1.11. And we assume when a player gains 30% of prestige currency, they will gain a new prestige in the game

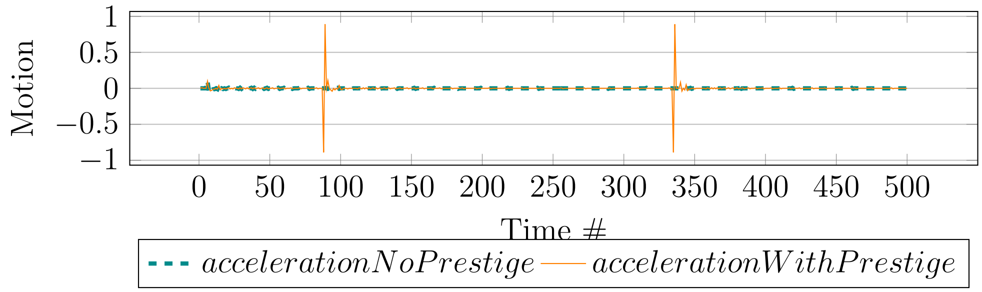
Counting each prestige as a step, we could get the information dynamics paradigm for idle game as shown in Figure 5.11. Jerk is regarded discomfort in game as mentioned in Section 4.3.3. Idle games have been an approach to realize gamification on memorizing

[12]https://blog.kongregate.com/the-math-of-idle-games-part-iii/

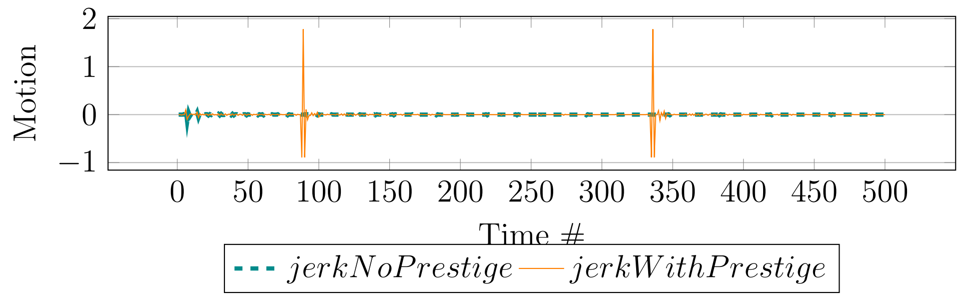
[13]Original data source can be retrieved from: https://docs.google.com/document/d/1BJRYYtByVuF41wjo70zPg96aTcQxfb_6fhd68bhXGpU/edit?usp=sharing



(a) Velocity changes with and without prestige in idle games



(b) Acceleration changes with and without prestige in idle games



(c) Jerk changes with and without prestige in idle games

Figure 5.11: Information dynamics in idle games with respect to jerk in mind when prestige

text. Different from the discussion of in-process performance in Chapter 4, “suprise” or jerk is found to be the main cause of uncertainty for a long term when the game have no end for idle game and idle-like activities. While the arcade game shows the jerk dynamic by gradually decreasing vibration as processing, jerk for long term idle game is irregular and has no obviously pattern.

Every jerk will bring a moment of excitement and freshness, making players get a moment of pleasure beyond expectation. Irregular jerk will attract the attention of players deeply, making players’ expectations for the future become uncertain. Conversely, it’s much harder for the player to give up. Add to that the fact that idle game itself has very little resistance to overcome (i.e., m is very small), and players don’t expect much from the game, and it's much easier for them to get that kind of unexpected surprise.

Compared to other competitive games, the game-play of idle games seems very shallow and unchallenging, with no noticeable improvement in skill. But the long-term uncertainty that comes with prestige makes what might otherwise seem like a boring setting fun (the anticipation of the future that keeps the game interesting), and has a longer life cycle than other kinds of games.

Remark 3 *Little m means little difficulty for players to overcome which will bring the kind of casual entertainment to the players, since the they do not expect much considering the little experience they have invested. Meanwhile, the irregular jerk from prestige to prestige maintains player's expectation for the game process that keeps the player's long-term interest, willingness to play the game as well as the game's activity.*

5.5 Discussion

5.5.1 Prestige as jerk in idle games

Idle games are a fascinating genre, from a psychological point of view, from a design point of view, and from a natural-business point of view. Survey data [135] has shown that idle games have huge amount of dedicated player as well as much higher retention compared with their arcade game counterparts. Games that reward the player for repeated actions are called Skinner Boxes because they enforce a behavior [136], which can quickly escalate into addictive or mindless behavior. These rewards plunge back deep into the human's brain, accumulating desire and loss aversion. Studies show that the pain of losing something hurts twice as much as the pleasure of gaining it [137]. This comes from hunter-gatherer times. Accumulating things like food is the key to life, and things that need to be possessed are thought to be a continuation from then on. Loss aversion is even stronger. Idle games allow the player to click a lot, their reward is a number that goes up, then the reward becomes a purchase, and the purchase accelerates their reward, and so on. Idle games allow players to accumulate, which will stimulate the brain's pleasure. Meanwhile, the pain of loss is almost zero, because each loss (purchase) only allows the player to accumulate things faster. All of these mechanics contribute to idle games as one of the most addictive genres available today.

In addition, idle game seems to have no end, which would give the players a sense of “they will never lose”. However, considering their value design that exponential growth finally will exceed polynomial growth, that is where the idle game meets an **invisible ending**. Such ending presents as that the player’s income velocity would never catch up with the cost. Instead of declare the ending to the player, idle game hides the fact and allow the player continue playing. They will never feel bad about losing the game. When the players sense they are not moving forward, or they are processing at a extremely slow velocity, they would choose to prestige. The prestige point actually happened after the invisible end. Such end is defined **“the trick of idle games”** in this study.

Meanwhile, researcher have noticed that the game design in idle game always sets multiplier increase variable (growth rate in Equation 5.3) value between ”1.07∼1.15”(which correspondingly in this study is $m \in [0.07, 0.15]$) [134]. But they failed to come up with the reason for the setting except designer’s experience. Our result on the other hand, supports the inner motivation for doing so, that is when $m \in [0.07, 0.15]$ game-play has a sense of dissonance and trigger the player’s interest.

5.5.2 Universal Gravitation in Games

Ermi et al. divided immersion to: sensory immersion, which is the player sees; the challenging-based immersion, which relate to the player’s balance between skill and challenge, i.e, the flow; and imaginative immersion, which is about player perception [138]. According to Brockmyer, engagement in a game is a universal metric [139]. Therefore, the engagement can be developed gradually at levels of immersion, presence, flow and psychological absorption (total engagement). O’Bren et al. consistently defined the term of engagement and pointed out it is a process that comprised of for distinct stages: point of engagement, period of engagement, disengagement and re-engagement [4]. Bouvier et al. also pointed out that engaged behavior in digital game touches on player’s attention and player’s consciousness [43]. Their model reveals that digital game engagement of players contains four states: outside the game environment, inside the game environment, towards the game content and inside the game.

To summarize, engagement contains **engaging** and **re-engaging**. Given the uncertainty posed by the game experience, addictive activities require both immersion and a

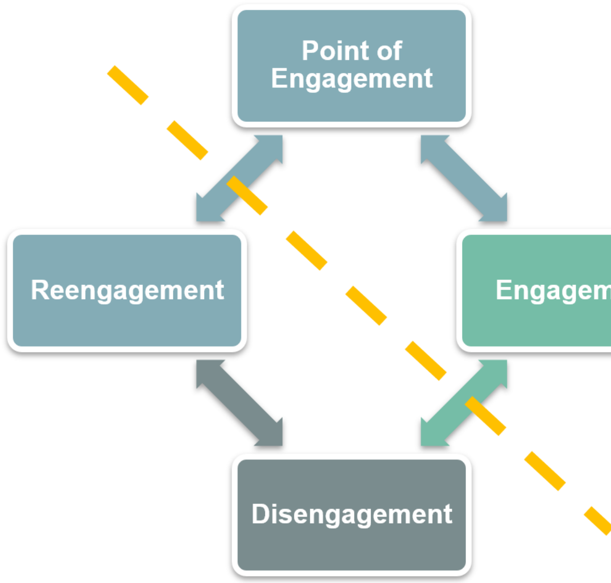


Figure 5.12: Engagement model [4]

strong motivation to continue playing. This may be due to the pleasure illusion of wanting to prolong or continue or repeat the game during or after the event [1].

Newton proposed the idea of **Law of universal gravitation** in 1687 [140], and denoted as Equation 5.17. Any two particles are attracted by forces in the direction of the interlocentric line, the magnitude of which is proportional to the product of their masses and inversely proportional to the square of their distance, independent of the chemical composition of the two objects and the type of medium between them. Newton conjectured that the force of attraction between the earth and the sun might be the same force, following the same laws, as the earth's pull on objects around it.

$$F = G\frac{m_1 m_2}{r^2} \tag{5.17}$$

The motion in mind model is gradually formed and verified by the game results [1,6]. By discussing the derivatives, this study therefore extends discussion about range of the model.

Conjecture 4 *We assume game information as one abstract object while human mind is another. There is an invisible gravitational force between mind and game play. The force attracts mind and game play, thus affects the game engagement.*

To this end, formulas are built around the gravitational analogy, the most prominent of which are the measurements of the gravitational constant, mass and distance based on the game progression model. Table 5.2 shows the analogical connection between the physics model and the game progress model.

Table 5.2: Analogical link between gravitation in physics and mind

Notation	Physics	Mind
F	force	attraction
G	gravitational constant	mind-gravitational constant/ GR
m_1	mass of object 1	hardness to motivate the player
m_2	mass of object 2	hardness to move the game
r	distance between objects	informational progress distinguish

While the game of refinement theory and the earlier study of motion in mind have focused on the process of game information. The law of gravitation in mind ($F_{attraction}$) can offer the explanation for the long term game analytic. There are various game genres in the market and attracting tremendous players. As presented in Equation 5.17, m_1 and m_2 are expected similar since they are interactive with each other, which are proportional to the attraction. Meanwhile, r signifies the game information differences from the player perceived and the player expected.

Researchers studying play habits describe game habit as a tendency to automatically activate a situational cue that occurs simultaneously with a response to a past performance [141]. In the case of an idle game, every time a player checks progress, buys a generator, or gives as much prestige as possible to unlock a new achievement, all the feedback makes the player more likely to continue playing. Each time the player opens the game again, the game gives the player the same or more achievements as "the player didn't leave", making the player feel as if he hadn't left at all. They might check the game during a break from work or before meeting friends and collect in-game currency. This habit is like a kind of never left the distance. Informational progress distinguish (r) is inversely proportional to the $F_{attraction}$. A turn-based game, such as a MOBA or most arcade games, is clearly one that has to be restarted every time, and the player's expectations are likely to be influenced by or even stop at the last round. When the player leaves the game, the information that the game offers clears to zero. This creates a significant gap between the information that the game can provide, i.e, the information that the player perceives, and the information that the player expects. In other words, a higher value of r will affect the $F_{attraction}$. In idle games, even if the player is not in the game, the game is still playing, and whenever the player comes back to the game, the information provided by the game always satisfies the player's expectations. In other words, the value of r will be very small, or even infinitely close to zero, which will cause the $F_{attraction}$ to increase. This also explains why the "anti-game rules" of idle games have such high retention rates [135]. Future study will further verify this assumption.

5.6 Chapter Conclusion

Idle game is a relatively new type of game, which is different from the traditional game design concept and lacks of uncertainty, and is disputed by people as "whether this is a game or not". Existing studies have fully analyzed the historical changes of idle game and the game mechanism design to explain the reasons why it has attracted people's attention and obsession, but all of them are relatively abstract. There is very little literature on the nature of the idle game and the actual player experience. From the perspective of motion in mind, an innovative and profound game experience, we discussed the causes of uncertainty in idle game and the real fun of the game. Uncertainty is the expectation to solve a game – the expectation of closing the gap between income and cost in idle game domain.

This chapter found that not only the equations of motion in mind model would help to analyze the idle games, but also the derivatives of the functions are also able to. And it showed that idle games with $m = 0.11$ are mainly affected by E_q and $\vec{p_2}$. And it has a balance between interest and boredom. Idle game has a long-term jerk which can maintain the freshness to the player. With barely no time lag, players have the feeling of synchronization, i.e., the resonance with the game. It can also be applied to deliberate practice activities.

It provides a good perspective for people to understand the charm of idle game, and also provides some starting points for the development and design of idle game in the future.

Chapter 6

Conclusion

6.1 General Discussion of Uncertainty in Games

Seesaw is regarded as one of the most mature and fun game modes. In this paper, we first discuss the human player-versus-human player rules based card game, which can be competed by multiple players at the same time but can maintain the same competition for each player. It then explores single-player arcade games where human players simply compete against the rules of the game and can significantly observe individual player skill improvement through practice. Finally, we discuss idle games, which are not competitive and even encourage players not to participate in the progress of the game.

A general interpretation of the growth rate showed that all of the target arcade games followed the distinct trend of reaching new high scores, albeit at a different rate of change. On the surface, these target arcade games were seen to encourage winning the game as fast as possible or play as long as possible. However, such conditions showed that these games showed signs of sudden popularity and mental fatigue. As such, different technique was adopted to counter such setbacks.

Multi-player card games are evaluated by derivation of branching factors and game length. By quantifying the milestones (Figure 4.8), overall performance of single-player arcade games tends to be the master of playing [2], which is on a count of forward game information process with acceleration. From further inspection of the target arcade games, it was found that each game has its dynamics of challenge, which makes their attractiveness vary from one instance to another. Some games provide excitement in the middle or late milestones of the game, meaning that reaching a higher or the highest score is satisfying. Meanwhile, others focus on early to middle or throughout many game milestones by changing the level's difficulty. It is possible with the analysis of the average informational acceleration and the average jerk.

However, the primary mechanism that makes an ideal infinite arcade game lies in analyzing the in-game process of all the target games. It was found that FB and PN are significantly different compared to BC and TR when measured through the velocity of the in-game process. Insights from the analysis revolve around manipulation and mechanistic usage of information along the process of game-playing, where sufficient thrills (from accelerated velocity) and some hint of surprise (from the sudden change of accelerated velocity) provide a prominently better game-playing experience.

Comparing BC with TR, BC remains less popular and the least attractive, possibly due to the insufficient vibration (or jerk dynamic) to make the game-playing experience much more enduring.

For a two-person game, when both players have the same possibility of winning or losing, the game would be most exciting since the challenge is equivalent to the player's skill level. As they practice more, so do their opponents, and players always stay in the state of challenge equivalent to skill; Hence, corresponding to the concept of *flow* in a psychological perspective. However, a single-agent game had no apparent opponent, and the challenge would decrease as the player's skill increases. Therefore, to maintain the player's interest and retain in the flow state, the game's difficulty should consistently increase at an appropriate proportion as the player's skill improves. TR's enduring popularity and game mechanics show that jerk dynamics is crucial for designing ideal infinite arcade games where sufficient "applicable vibration" would retain the flow state's play experience.

To sum up, velocity is taken as the main measure of the state of motion in mind, but the main reason cannot be well understood only by the uncertainty necessary to be solved and time. The acceleration, however, can describe the motion driven by the force of the mind. Acceleration in mind is a feeling during play, with positive cues for pleasure and negative cues for tension. The change in acceleration, jerk, highlights a sudden, unexpected change, and such that the player feels more engaged. Basically, relative velocity and resultant force is crucial to understand the game without specific length. And jerk dynamic also plays a important role to make this kind of game attractive.

From chapter to chapter, the model of motion in mind is also established, optimized its range, and evolved into a relative sophisticated shape. The nature of entertainment in different areas of the motion in mind model is meaningful to establish entertainment science.

All of the target games are listed in Table 6.1 and plotted as Figure 6.1.

Table 6.1: Motion in mind for all the events mentioned in this thesis

game	m	v_1	v_2	v_R	$\vec{p_1}$	$\vec{p_2}$	E_p	E_q	E	F	F_R
DouDiZhu (3)	0.795	0.205	-0.121	0.326	0.1628	-0.0961	0.0667	0.2590	0.3257	0.1810	-0.6143
DouDiZhu (4)	0.661	0.339	-0.109	0.448	0.2240	-0.0722	0.1518	0.2962	0.4481	-0.3555	-1.0166
Tien Len (4)	0.802	0.198	-0.120	0.317	0.1586	-0.0959	0.0627	0.2545	0.3172	0.2091	-0.5931
Killer (4)	0.803	0.197	-0.119	0.317	0.1583	-0.0959	0.0624	0.2542	0.3166	0.2113	-0.5915
Winner (3)	0.563	0.437	-0.055	0.492	0.2461	-0.0308	0.2153	0.2769	0.4922	-0.7498	-1.3124
Winner (4)	0.680	0.320	-0.115	0.435	0.2174	-0.0785	0.1389	0.2959	0.4348	-0.2780	-0.9585
Big two (4)	0.665	0.335	-0.111	0.445	0.2227	-0.0736	0.1490	0.2963	0.4453	-0.3387	-1.0041
Daifugo (4)	0.851	0.149	-0.105	0.254	0.1269	-0.0891	0.0379	0.2160	0.2538	0.4033	-0.4475
Chinese Chess	0.776	0.224	-0.124	0.347	0.1736	-0.0960	0.0776	0.2695	0.3471	0.1059	-0.6706
Go	0.399	0.601	0.121	0.480	0.2398	0.0484	0.2882	0.1914	0.4796	-1.4038	-1.8029
Mahjong	0.895	0.105	-0.083	0.188	0.0939	-0.0742	0.0197	0.1681	0.1879	0.5802	-0.3148
Flappy Bird	0.149	0.851	0.597	0.254	0.1268	0.0890	0.2158	0.0378	0.2536	-2.4040	-2.5530
BrickCarRacing	0.450	0.550	0.055	0.495	0.2475	0.0248	0.2723	0.2228	0.4950	-1.2000	-1.6500
Pong	0.680	0.320	-0.115	0.435	0.2176	-0.0783	0.1393	0.2959	0.4352	-0.2800	-0.9600
Tetris	0.46	0.540	0.043	0.497	0.2484	0.0199	0.2683	0.2285	0.4968	-1.1600	-1.6200
Coockie Clicker	0.15	0.850	0.595	0.255	0.1275	0.0893	0.2168	0.0383	0.2550	-2.4000	-2.5500
AdVenture Captalist	0.12	0.880	0.669	0.211	0.1056	0.0803	0.1859	0.0253	0.2112	-2.5200	-2.6400
Clicker Heroes	0.07	0.930	0.800	0.130	0.0651	0.0560	0.1211	0.0091	0.1302	-2.7200	-2.7900

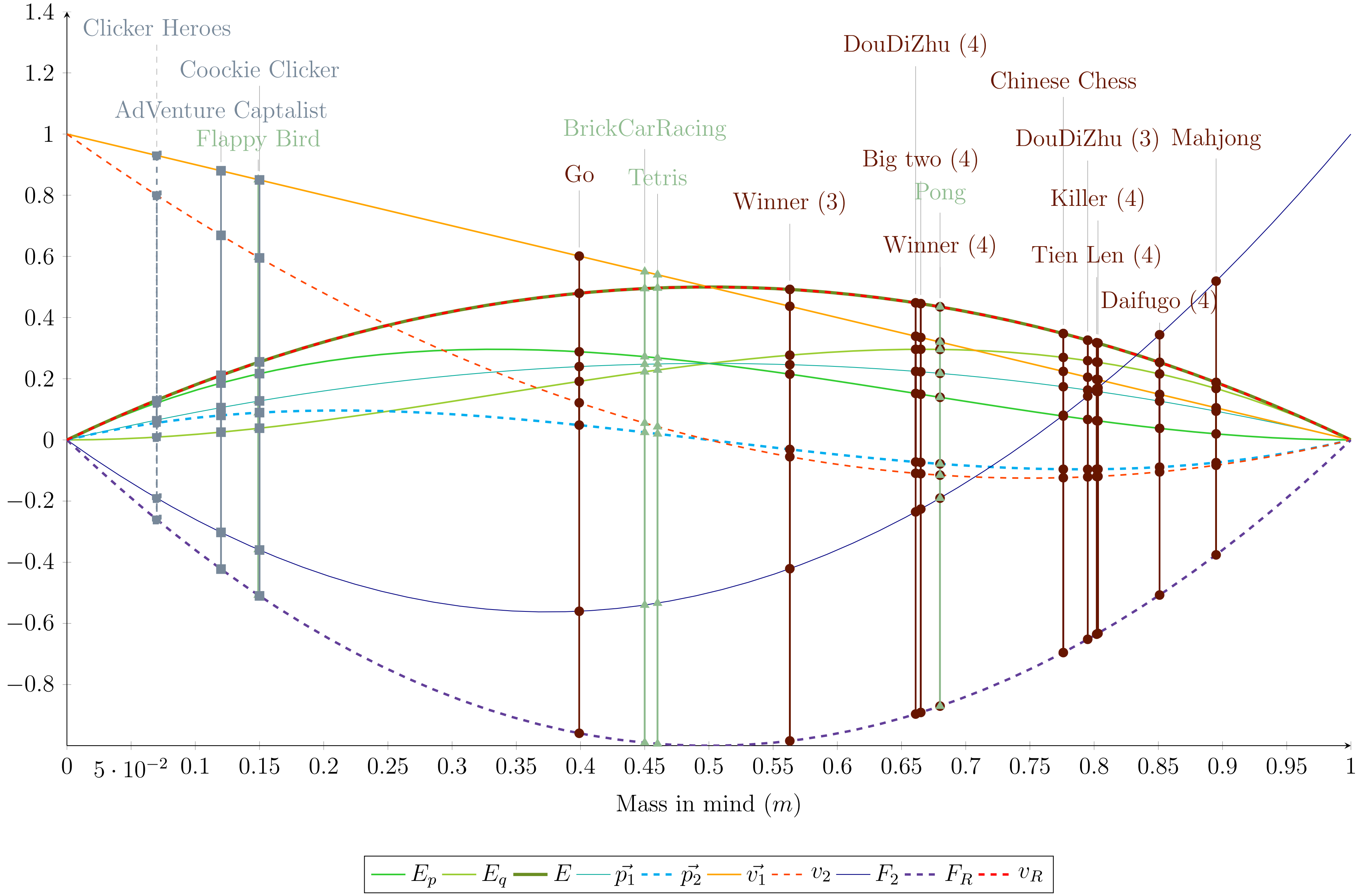


Figure 6.1: Motion in mind of all the targets

6.2 Answers to Research Questions

Research objectives were proposed at the beginning as the guidance of the whole dissertation: (1) to conduct research on competitive games to find out how game setting would affect the game popularity and engagement; (2) to further discuss and improve the model of motion in mind by defining the measurement of single player games without specific length; (3) to reveal the internal laws behind engaged human activities by applying the theory of motion in mind to the entertainment field; (4) to extend the range and further construct the model of motion in mind and to establish entertainment science by incorporating the nature of entertainment in different areas of the model.

For each research problem, this thesis has achieved the following results.

- The acceleration model in mind was first applied to a new study object. A competitive game, DouDiZhu, firstly to be proved as a sophisticated game which can maintain the fairness, uncertainty and entertainment. It testifies the approach of game refinement theory and reveals that settings affect the competitiveness and uncertainty of game outcome thereby affect the engagement and popularity.

- The motion in mind model was further constructed incorporating new concept of relative velocity and resultant force, which better explain the motion in mind when playing single player game. Four single-agent arcade games were tested by potential growth rate. It found that jerk dynamics is essential to deliver the perception of excitement and entertainment to players.

- Derivatives of the model of motion in mind were explored and tested to be useful for analyzing idle games and game-like noncompetitive activities. Long term jerk is crucial to maintain player's or user's freshness and loyalty. By synchronizing the game with people's lives, it breaks the sense of interruption and creates a sense of synchronization that keeps the users' engagement and stickiness.

- The process within the game-play system is first explored and constructed (acceleration in mind, motion in mind with relative velocity and resultant force, jerk in mind), and then the element outside the game-play system is also discussed (gravitation in mind). The science of entertainment using motion in mind models as a method to study games is preliminarily established.

6.3 Concluding Remark

As Ilya Romanovich Prigogine claims, certainty will end [142]. We are facing a probabilistic world, and at the same time we are discovering new possibilities in the world. This new understanding of natural law is expressed by the irreducible probability statement, which is associated with instability and deals with the probability of events at both micro and macro levels, but does not reduce these events to inferable and predictable outcomes.

Players in the game, through the interaction with the game world, produce psychological and physiological satisfaction, get a sense of pleasure and stimulation, so as to produce an immersive experience. This study discussed that the immersion is not only the experience in the game, before the game, after the game, immersion has its profound meaning. We try to summarize some universal methods, for more game design to provide reference, and can bring some inspiration for the design of other non-game products. The design of game's purpose is to provide players with an immersive game world. Thus, a lot of research focused on the player's subjective experience, such as game addiction, immersive. This study shows an approach focusing on the objective results of the game and the game players to study the relation between the subjective experience and objective results. The methodology and measurement of this study adopted and formalized would offer a new standard to quantify the focus of different games.

Taking different games and recreational activities as objects and using the construction of physical models in the brain as media, this study explores the effects of dynamic changes of uncertainty on the entertainment and attraction of games and recreational activities. Through the physical modeling of the thinking movement, it innovatively constructs the operation mode of the human thinking world, and provides a brand-new understanding and angle for the study of either human beings themselves or the nature of entertainment.

The systems and processes at work in physics might different from the psychological processes we are attempting to describe. By adhering to these physical formulae, layers of complexity intrinsic in psychological phenomena might not be considered. Concerning the gravitational potential, the game itself is attractive to players, and the potential energy of this attraction is the energy of the game. This definition contains multiple constructs: an objective reality construct (information required), a psychological expectancy construct, a psychological motivation construct (anticipation), and a psychological motivation/value construct (game's attraction). These constructs are all lumped into a single term in the equations despite their being at least somewhat distinct from each other. To compound this issue, these constructs and others in the formulae provided have overlap, they are not fully distinct as is the case when operating with, say mass and velocity in physics. As a result, formalization would be optimized in future. Furthermore, some definitions, such as the growth rate (v) for arcade game, need further justification.

Overall, the future work would be focused on the specific goal, optimize the physics analogy in favor of a general formalization, and dramatically simplify the scope of the formalization to focus on distinct concepts as a start with future work directed toward proper formalization processes that fully consider the complexities inherent in psychological modeling (e.g., proper treatment of overlapping constructs).

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