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Mathematical Characterizations and Computational Complexity of Anti-slide Puzzles

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For a given set $\mathcal{P}$ of pieces and a frame F, an anti-slide puzzle asks us to arrange the pieces so that none of the pieces can slide (an anti-slide arrangement)in the frame F. This "putting in" operation is called packing. Packing puzzles are similar to anti-slide puzzles, but these two puzzles have different reasons for hardness.

Packing puzzles (packing problems) have been well investigated in computer science. Packing puzzles are often designed so that the area inside the frame F and the total area of the pieces in the set $\mathcal{P}$ are almost equal. Thus, "the hardness of packing puzzles lies in the packing". On the other hand, anti-slide puzzles are based on the following condition "packing is trivial to solve". Thus, anti-slide puzzles are required to have the following properties "easy to pack, but hard to make an anti-slide arrangement".

Therefore, when proving the hardness of an anti-slide puzzle, we have to design gadgets that explicitly express the properties that "it is easy to check (in polynomial time) that packing can be done".

The puzzles that make a relative anti-slide arrangement with only a set of pieces $\mathcal{P}$ without a frame F is called interlock puzzles. Interlock puzzles are a special form of anti-slide puzzles.

In this paper, We first propose three models that mathematically characterize the anti-slide states and compare them with the models proposed in previous research of anti-slide and interlock puzzles. In this research of anti-slide puzzles, we consider three possible combinations of piece shapes and piece arrangements: "orthogonal arrangements of polyominoes", "general arrangements of polyominoes", and "general arrangements of general polygones". Here, a polyomino refers to the silhouette obtained from multiple unit squares by arranging them so that two adjacent squares totally share a unit edge. A polygon P is orthogonal when all edges of P are parallel to the x- or y-axes.

There are three types of anti-sliding properties defined in this paper: "weak anti-slide", "strong anti-slide", and "anti-slide for polyominoes". Previous work has dealt only with "weak anti-slide properties for orthogonal arrangements of polyominoes" in terms of our paper. They reported analyse of anti-slide arrangements with the minimum number of pieces, using the tools IP solver and OBDD, under this model. However, this model does not take into account some information (e.g. gravity) necessary for real anti-slide puzzles. Therefore, the solution output by the computer does not always match the realistic solution. In this paper, we proposed the "strong anti-slide" as a model that is closer to the realistic anti-slide property than the models in previous research, taking into account physical information such as gravity.

"Weak anti-slide" and "strong anti-slide" are defined for all of "orthogonal arrangements of polyominoes", "general arrangements of polyominoes" and "general arrangements of polygons". On the other hand, the third anti-slide property defined in this paper, "anti-slide for polyominoes", is defined only for "orthogonal arrangements of polyominoes".

In this paper, we investigate the computational complexities of the anti-slide puzzles on the model "anti-slide for polyominoes". Based on the following condition the orthogonal arrangements of polyominoes, which is the simplest, we can see that "the problem is hard even in the restricted case with bounded conditions".

First, we consider the problem of determining whether a given piece arrangement $\mathcal{A}$ is anti-slide or not. To solve this problem, we represent a given arrangement $\mathcal{A}$ of pieces in an anti-slide puzzle as a directed graph. Next, we prove that "the problem of determining whether a given piece arrangement $\mathcal{A}$ is anti-slide or not" and "the problem of determining whether the directed graph representation of the piece arrangement is strongly connected or not" are equivalent. The problem of determining the strong connectivity of a given directed graph can be solved in linear time. Thus, we can prove that "the problem of determining whether a given piece arrangement is anti-slide or not can be solved in polynomial time".

Next, for a given set $\mathcal{P}$ of pieces and a frame F, we consider the problem of determining whether there exists an arrangement that achieves anti-slide or not. We prove the NP-completeness of anti-slide puzzles using the following procedure. Since it is possible to determine in polynomial time whether a given piece arrangement $\mathcal{A}$ is anti-slide or not, it can be shown that the hardness of anti-slide puzzles falls into the computational complexity class NP. Next, we show the NP-hardness reduction from the 3-Partition problem, which is an NP-complete problem. We complete the proof that anti-slide puzzles is an NP-complete problem.

Finally, for a given set $\mathcal{P}$ of pieces, we clarify the boundary conditions for interlocking. That is, we prove that there exists an interlocking arrangement even if all pieces are x-monotone, but that it is impossible to interlock for any piece arrangements when all pieces are convex polygons.