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LDPC-based multi-relay lossy forwarding for correlated source transmission over orthogonal Rayleigh fading channels

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***Abstract*—In this paper, we design a communication and coding strategy for Lossy Forwarding (LF) systems with multiple relay nodes or helpers based on Low-Density Parity-check Codes (LDPC) with message passing decoding applied on a Tanner-graph that maps all the network. The system performance is investigated under the cases of a fixed number of helpers, and a random multiple shifted-Poisson distributed helpers for orthogonal Rayleigh fading channels. All the practical results will be also compared and validated with respect to theoretical outage probabilities.**

***Index Terms*—LDPC codes, lossy forwarding, joint message passing decoding, cooperative communication, multiple relays.**

I. Introduction

In the recent years, with the development of Internet of communicating things technologies, cooperative communications have been recognized as a promising technology [1]. Cooperative communication allows the usage of the information overheard by neighboring nodes in a network to provide better communication between a source and its destination. In wireless communications, cooperative communication can handle the benefits of multiple antenna systems by using single antenna devices. In fact, the extra communication links provided by the intermediate nodes, called relays act as virtual MIMO systems by providing diversity that helps combating fading.

Many relaying strategies are proposed, and the main methods include detect and forward, amplify and forward, and coded cooperation [2]. Coded cooperation generally performs better than other cooperative methods at moderate to high signal-to-noise ratios (SNR). However, in this strategy relaying is conditioned to error-free reception of data at the relay to prevent error propagation. However, this involves extra resources for error detection, and sub-optimal exploitation of the received information. As an enhancement, a new Lossy Forwarding (LF) strategy was proposed [3]. The latter enables the relay to forward the information even though its not correctly reconstructed and leaves the global recovery function to the destination that performs joint decoding on the received lossy multiple correlated sequences.

The theoretical performance bounds of the three-node LF relaying system can be computed using the theorem of source coding with side information since we only need to recover the original source information [4]. In this case, the relay is acting as a helper. Source coding with multiple helpers is still an open problem in the network information theory. Many practical communication solutions were also proposed in the framework of LF cooperation for different network setups. In [5] authors designed a coding strategy for the three-nodes LF cooperative system, and the proposed solution was generalized for the CEO Problem in [6]. In [7], authors considered power and rate allocations for the orthogonal multiple-access relay channel (MARC) with lossy source-relay links. A detailed survey of the practical systems designed for LF systems including the two-way relay network, and the general multi-source multi-relay networks are provided in [3]. The previously mentioned practical schemes rely on convolutional codes with interleaving and doped accumulator. The same coding method was applied to the case of multiple relays where authors in [7] demonstrated based on theoretical outage probability computation and simulation results that using K helpers enable a diversity order of $(K+1)$. However, in the presented results, the gap between theoretical and practical results is almost of 5 dB, which can be improved substantially.

Motivated by their capacity approaching property, and flexible graph representation, many works tackled the LF cooperation problem using Low-Density Parity-Check (LDPC) codes. The case of three-nodes cooperative network was considered in [9] with application to multiple description coded sources where spatial and inter-descriptions correlations were exploited by joint decoding. A similar system is also proposed in [10] with double LDPC codes used for compression and error protection. The application of LDPC codes to the CEO Problem is also treated in [11] for time-correlated sources where no direct source-destination link is assumed. The same system applies for [13] where authors considered Low-Density Generator-Matrix (LDGM) codes for binary quantization jointly with LDPC codes for syndrome generation.

This paper aims at proposing a similar and a less complex communication scheme based on LDPC codes with joint decoding for multi-relay LF systems. A direct-link is assumed between the source and the destination, and many other

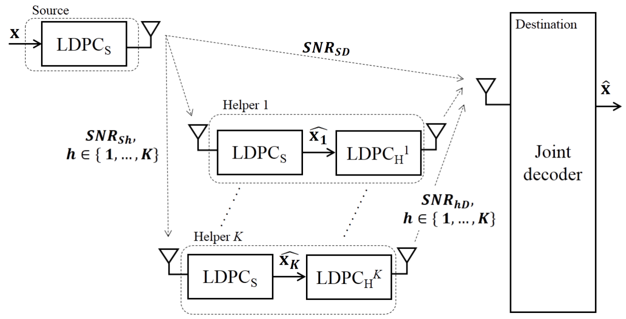

Fig. 1. System model for multi-relay lossy forwarding over orthogonal channels

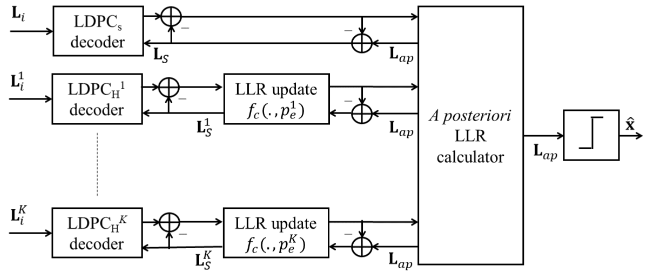

Fig. 2. The proposed JSC decoding strategy exploiting helpers spatial correlation.

alternative links are available based on helpers to provide flexible diversity to the receiver. The joint decoding is based on parallel and iterative extrinsic messages transfer between the LDPC elementary decoders so that to exploit the spatial correlation. The practical results will be investigated with a fixed and random number of helpers to cover the possible application cases in mobile networks. Finally, simulation results are compared to theoretical outage probabilities investigated in [8] for validation and comparison.

The paper is organized as: Section II gives a detailed overview of the system model, notations and practical assumptions. Section III emphasizes the efficiency of the proposed coding scheme by providing simulation results for fixed and random number of helpers. Finally, Section IV gives some concluding remarks.

II. System model

In this paper, we investigate a half duplex relay system where a single source transmits its information denoted $\mathbf{x}$ to a unique destination with the help of K relays or helpers. The aim of the destination is to deliver an estimate $\hat{\mathbf{x}}$ of the source data with low error rates.

A. Communication phases and practical assumptions

As depicted in Fig. 1, the goal of a source node is to deliver an k-bits i.i.d. information vector $\mathbf{x}$ to the destination. To this aim, K helping nodes contribute to the information transfer using LF relaying. As a first phase, the source broadcasts an LDPC encoded version of $\mathbf{x}$. The encoded code-word denoted $\mathbf{b}$ is of length n bits, and the corresponding channel coding rate is defined as $R_c^s = k/n$. As previously stated, according to the lossy forwarding strategy, every helper $h \in \{1, \ldots, K\}$ decodes the received noisy version of $\mathbf{b}$, and generates an estimate denoted $\hat{\mathbf{x}}_h$ that is re-encoded and transmitted even though some errors are found. Every helper h will use a rate R_c^h LDPC code to generate a new code-word denoted $\mathbf{b}_h$. The system involves high correlation levels between the information forwarded to the destination by the source and the helpers, depending on the elementary source-helpers channels state. In fact, the correlation amount depends on the intra-link error probability defined as $p_e^h = P(x_i \neq \hat{x}_i^h)$ for $i = (1, \ldots, k)$. The destination node recovers the source information on the direct link, and the helpers sequences that will be used as side information. The system is equivalent to two phases half-duplex relaying, where orthogonal access is assumed between the source and the helpers transmissions. We also assume a Binary-Phase Shift Keying (BPSK) modulation for all the communicating nodes. All the channels are affected by a block Rayleigh fading and Additive White Gaussian Noise (AWGN) with mean power $\sigma_n^2 = N_0$. For $i = (1, \ldots, n)$, given the modulated version of x_i as y_i, which is transmitted by the source, the received sequence at the destination is given by:

$$r_i = \sqrt{G}.h.y_i + n_i \tag{1}$$

where G denotes the geometric gain of the source-destination link, h the channel gain described as a complex Gaussian variable with $CN \sim (0, 1)$ that changes block by block, and n_i is the AWGN random complex noise sample with $CN \sim (0, 1)$.

Every helper h forwards $y_i^h, i = (1, \ldots, n)$: a BPSK modulated version of its reconstructed version of x_i previously called $\hat{x}_i^h$. Hence, the corresponding received sequence at the destination is:

$$r_i^h = \sqrt{G^h}.h^h.y_i^h + n_i^h \tag{2}$$

where G^h represents the helper h to destination link geometric gain, h^h and n_i^h are respectively the channel gain and the complex noise sample distributed according to $CN \sim (0, 1)$.

We notice that under the block fading channel assumption, the intra-link error probability p_e^h changes for every packet transmission but remains constant for all the bits in one packet. In this work, we propose to use the intra-link error probability model presented in [8] computed based on the instantaneous link received SNR Γ^h as:

$$p_e^h = \begin{cases} H^{-1}(1 - \Phi(\Gamma^h)), & \text{if } \Phi^{-1}(0) \leq \Gamma^h \leq \Phi^{-1}(1) \\ 0, & \text{for } \Gamma^h \geq \Phi^{-1}(1). \end{cases} \tag{3}$$

where $\Phi(\Gamma^h) = \frac{1}{R_c^s} \log_2(1 + \Gamma^h)$, and $\Phi^{-1}(.)$ is its inverse function. $H^{-1}(.)$ is the inverse function of the binary entropy function $H(x) = -x \log_2(x) - (1 - x) \log_2(1 - x)$.

B. LDPC joint decoding with helpers correlation exploitation

The system model considered in this paper supposes no source compression, and no distributed processing. Thus, the design of the joint decoder that can best exploit the system correlation properties plays a very important role.

In fact, in the proposed decoder the helpers spatial correlations are exploited through global iterations connecting the different elementary LDPC decoders. As depicted in the block diagram of Fig. 2, during local iterations, all the LDPC decoders are working independently. For every node elementary decoder, the systematic variable nodes of the LDPC Tanner graph of node number m receives the channel observations LLRs $\mathbf{L}_i^h$ as intrinsic information, and an extrinsic information $\mathbf{L}_S^{(h)}$ fusing all the other helpers data and provided by the last global iteration. The latter is equal to zero before the first global iteration. The LLRs are exchanged between the variable and check nodes of the elementary LDPC decoders until a maximum number of local iterations is reached. Then, a global iteration is performed to update the spatial correlation information using an LLR updating function taking into account the observation error probability p^h (equal to 0 for the source, which is equivalent to no updating). During a global iteration, the *a posteriori* LLR calculator fuses all the information generated by the elementary local decoders to deliver a full *a posteriori* information $\mathbf{L}_{ap}$. Each local decoder extracts its corresponding extrinsic information $\mathbf{L}_S^h$ that will be appended in the next local iteration to improve the system performance by exploiting the spatial correlation.

Let's focus on detailed information transfer between the different decoders for a network component (source or helper) h, at the local iteration number $l \in \{1, \ldots, L\}$, and a global iteration $g \in \{1, \ldots, G\}$. We denote $w_{v,c}^{(l,h)}$ and $w_{c,v}^{(l,h)}$ respectively the LLR messages passed from the v-th variable node to the c-th check node and inversely at the local iteration l for sensor h. For the initialization, the LLRs to be used for the global iteration are first set to zero for all the decoders. The check-to-variable node messages are also initialized to zero for the first local iteration. The process for the LLR message updating for a sensor h follow the Sum-Product (SP) decoding with additional LLRs from the other network elements decoders for every global iteration.

The LLR messages of the h-th elementary decoder to be forwarded from the systematic variable nodes to the corresponding check nodes exploit the spatial correlation by inducing the updated LLR delivered by the other network elements $L_{S,v}^{(g-1,h)}$ at the previous global iteration. For $v \in \{1, \ldots, k\}$, we have:

$$w_{v,c}^{(l,h)} = L_v^h + \sum_{c' \neq c} w_{c',v}^{(l,h)} + L_{S,v}^{(g-1,h)} \tag{4}$$

For parity variable nodes, we have no extra information to exploit, and the check messages update follows the standard SP decoder with, for $v \in \{k+1, \ldots, n\}$:

$$w_{v,c}^{(l,h)} = L_v^h + \sum_{c' \neq c} w_{c',v}^{(l,h)} \tag{5}$$

After L local iterations, the overall *a posteriori* LLR can be evaluated for every variable node to prepare the next global iteration, and for every local decoder h we have

$$L_{v,app}^{(g,h)} = L_v^h + \sum_{c'} w_{c',v}^{(l,h)} + L_{S,v}^{(g-1,h)} \tag{6}$$

This information can be used for spatial correlation update in the next global iteration. In fact, after performing LDPC SP decoding several iterations, the *a posteriori* LLRs output from the $(K+1)$ elementary decoders are combined. As described previously, the sequences processed at the different forwarding nodes are initially generated by the same originator, and corrupted by random errors with probabilities p_e^h for $1 \leq h \leq K$. Since we suppose that the observation error probabilities are known, we can evaluate the contribution of node h to the LLR of the original sequence $\mathbf{x}$. For every node h, we can obtain a relation between the probabilities of the v-th bits, x_v of $\mathbf{x}$ and $\hat{x}_v^h$ of $\hat{\mathbf{x}}^h$ as:

$$\Pr(x_v = 1) = (1-p^h)\Pr(\hat{x}_v^h = 1) + p^h \Pr(\hat{x}_v^h = 0)$$
$$\Pr(x_v = 0) = (1-p^h)\Pr(\hat{x}_v^h = 0) + p^h \Pr(\hat{x}_v^h = 1)$$

Based on these relations, we can demonstrate the expression relating the LLRs of $\mathbf{x}$ and $\hat{\mathbf{x}}^h$ components as:

$$\begin{aligned} L(x_v) &= \log\left(\frac{\Pr(x_v = 1)}{\Pr(x_v = 0)}\right) \\ &= \frac{(1-p^h)\exp(L(\hat{x}_v^h)) + p^h}{p^h \exp(L(\hat{x}_v^h)) + (1-p^h)}, \end{aligned} \tag{7}$$

which is equivalent to the LLR updating function $f_c(L(\hat{x}_v^h), p^h)$ applied for $L(\hat{x}_v^h)$ given the error probability p^h.

In our system, this function will be applied to extrinsic LLRs excluding the spatial correlation information of the previous global iteration. This extrinsic information will be delivered by every network element as

$$L_{v,ext}^{(g,h)} = L_{v,app}^{(g,h)} - L_{S,v}^{(g-1,h)} \tag{8}$$

Then, we perform LLR update using $f_c(., p^h)$, and sum up over all the network elements including the source, to have:

$$L_{v,app}^{(g)} = \sum_{h=1}^{K} f_c(L_{v,ext}^{(g,h)}, p^h) + L_{v,ext}^{(g)} \tag{9}$$

For every variable node v, and for every network element h, we evaluate the *a priori* information coming from the global iteration as:

$$L_{S,v}^{(g,h)} = f_c(L_{v,app}^{(g)} - L_{v,ext}^{(g,h)}, p^h) \tag{10}$$

After that, novel local iterations are performed until the maximum number of global iterations G is reached.

Finally, the estimated message is obtained by taking the hard decision on the *a posteriori* LLR of all the variable nodes which is calculated as

$$L_v = \sum_{h=1}^{K} f_c(L_{v,app}^{(G,h)}, p^h) + L_{v,app}^{(G)}. \tag{11}$$

C. Relay number distribution

The aim of this paper is to investigate the efficiency of the LF technique with multi relays. The number of the forwarding nodes K is generally supposed to be constant in the main references dealing with such a study. However, with emerging applications like smart cities, industry 4.0, and V2V communications, the number of the helping nodes can change randomly. The Poisson distribution with average

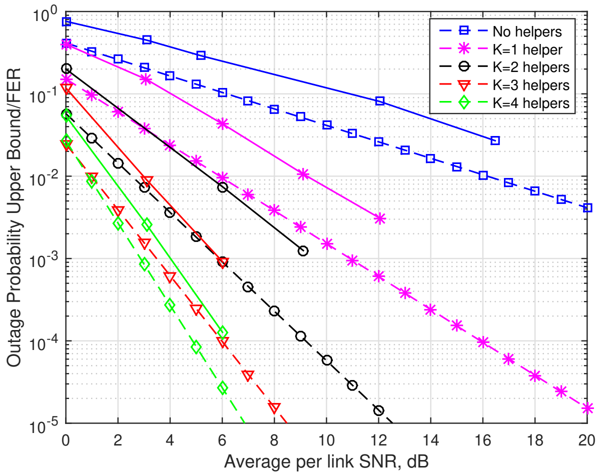

Fig. 3. The proposed joint decoding strategy performance for a fixed number of helpers $K = 0$–4.

number λ can model such a varying scenario where the probability to get u helpers is expressed as:

$$\Pr(u) = e^{-\lambda} \frac{\lambda^u}{u!} \tag{12}$$

Under this assumption, it is possible, depending on λ, to have some cases where no helpers are available ($u = 0$) which corresponds to the point to point communication scheme. In some densely deployed architectures, it is possible to deploy the network elements such that we guarantee a minimum number of K helpers available for cooperation. In this case, the random number of helpers u which follow a shifted-Poisson distribution, expressed as:

$$\Pr(u) = e^{-\lambda} \frac{\lambda^{(u-K)}}{(u-K)!} \tag{13}$$

These two scenarios will be discussed later and the system performance will be investigated according.

III. Simulation Results

In this section, we carry out investigations on the proposed system performance for fixed and random number of helpers. The presented Frame Error Rate (FER) results are based on Monte Carlo simulations. All these simulation results will be compared with respect to the outage probability upper bounds provided in [8].

A. *Fixed number of helpers*

In this paragraph, we investigate the Frame error-rate (FER) system performance in the case where K helpers contribute to the source data forwarding. The objective is to show the gains induced by the global iterations (information exchange between the different LDPC SP decoders) and the effect of the number of the helping nodes on the global system performance. The source-sequence length is equal to $k = 2048$ bits and a rate $R_c^s = R_c^h = 0.5$ irregular LDPC code based on optimized degree distribution [14] is used for all the network elements. The corresponding variable node degree distribution is given by $\lambda(x) = 0.25105x + 0.30938x^2 + 0.00104x^3 + 0.43853x^9$. We also notice that the LDPC codes construction is random, thus we don't consider interleavers. The simulations were made for the case of $K = 0, 1, 2, 3,$ and 4 helpers with $G = 10$ global iterations and $L = 50$ local iterations. Also, we assume that all the nodes have the same average SNRs with respect to the destination ($G = G^h = 1$). The average source-helpers SNR will provide the intra-link helpers error probability that is changing block-by-block and helper-by-helper as specified by equation (3).

Figure 3 presents the outage probability upper bounds (dashed lines), and the simulated FER (solid lines) for a fixed number of helpers K, with respect to the average SNR. The improvements provided by the LF strategy are clear when compared to the point to point case. These improvements are getting higher for increasing number of helpers, and this is justified by higher correlation between the different forwarding nodes. It is demonstrated that $(K + 1)$ diversity gain can be achieved with the proposed LDPC-based system, which is consistent with the theoretical outage probability results. A gap of 1 to 3 dB depending on the helper's number is observed with respect to theoretical outage, and this can be recovered by designing a suitable LDPC codes. We also notice that the increase of the diversity gain with K involves that the error rates decrease remarkably with large K for a given SNR.

B. *Random number of helpers*

As a first experiment, we assume a random number of helpers that follows a Poisson distribution with mean λ. Simulation results are investigated and compared to theoretical outage probability upper bounds. The latter is derived from the fixed helpers u outages $P_{outage}(u)$, and the probability to have u helpers $P(u)$ as:

$$P_{outage} = \sum_{u=0}^{+\infty} P_{outage}(u) P(u) \tag{14}$$

Based on the remark of the last section stating that the error probability decreases fast with increasing value of the number of helpers u, the sum will be dominated by the error rates of the small values of u. Hence, the total error probability can be approximated to the first 4 values of u.

Figure 4 depicts the outage probability upper bound and the FER versus the per link SNR for the case of Poisson distributed number of helpers. We consider average number of helpers equal to $\lambda = 0, 1, 2, 3,$ and 4. It is clear that simulation results match the theoretical outages. The results show that even a little enhancement at very low SNR, the error probability approaches the first order diversity for medium to large SNRs. This is justified by the fact that the event where no helpers are available happens with non zero probability, and causes a high level of error rates that will dominate the total average error probability. This case is equivalent to a point to point source destination link achieving a first order diversity. To cope with this problem, a densification of the network can allow us to guarantee a minimum number K of the available helpers.

When the number of helpers obeys to a K shifted Poisson distribution with mean λ, the total error rate can also be computed using equation (14) where $P(u)$ is computed based on equation (13). Results are shown in figure 5 for different number of shifts and a fixed mean $\lambda = 2$. It is clear that

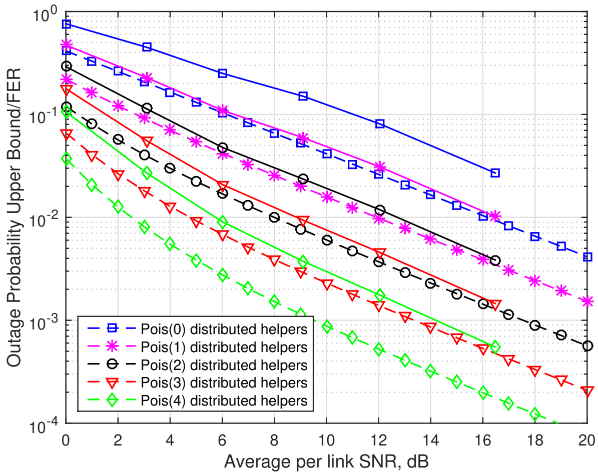

Fig. 4. The proposed joint decoding strategy performance for Poisson distributed random helpers number.

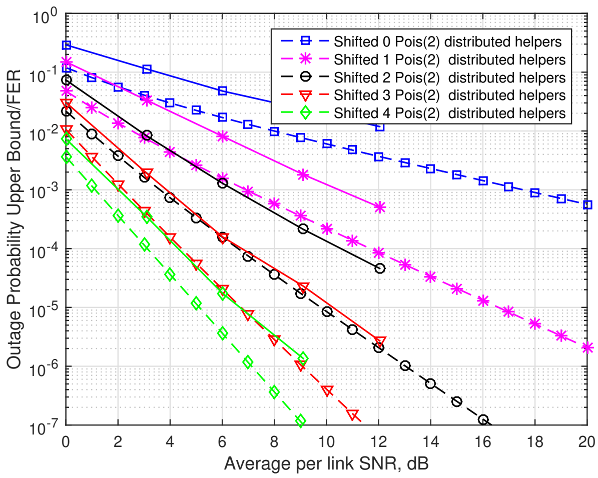

Fig. 5. The proposed joint decoding strategy performance for Shifted Poisson distributed random helpers number.

the simulation results are consistent with theoretical outage probabilities with a gap varying from 2 to 4 dB. We can see that when guaranteeing a minimum number of K helpers, we can reach again the $(K+1)$ diversity order. Hence, for variable environments, the usage of the proposed strategy is motivating when a minimum number of helpers is available, and guaranteed.

IV. Conclusions

In this work, we investigated a new LDPC based coding scheme for multiple helpers lossy forwarding cooperation in the case of orthogonal Rayleigh fading channels. We presented a communication and coding system, with a joint decoder that allows the exploitation of the correlation available with multiple helpers. We showed that for a given number of helpers K we can reach a $(K+1)$ diversity order using the iterative global iterative joint decoder. However, when the number of helpers is varying, the proposed system is dominated by the worst case with small numbers of helpers. Hence, it is recommended for densely deployed architectures where we can have a fixed minimum number of available helpers to cooperate.

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