Title	直観主義論理上での再帰理論の導入と諸概念の分離 [課題研究報告書]
Author(s)	森山, 巧
Citation	
Issue Date	2022-03
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/17662
Rights	
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Introduction to the recursion theory in the intuitionistic logic and separation of concepts

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In this report, we discuss the recursion theory based on the constructive mathematics.

The problem occuring when we discuss recursion theory (or other territories of mathematics, too) in inutitionistic logic is that it is not trivial how to define concepts used in recursion theory. In the constructive sence, even it is not trivial how should we define characteristic functions, therefore computable sets either.

For example, if we define characteristic function f of $A \subset \omega$ as

$$\begin{aligned} x \in A &\to f(x) = 0 \\ x \in A^c &\to f(x) = 1, \end{aligned}$$

we might have some degree of freedom for f, since in the intuitionistic logic, $A \cup A^c$ is not (always) ω.

We introduce four definitions of characteristic functions; all of them are classically equivalent and characterize the concept of characteristic functions, but in the constructive sence those are not equivalent. Accordingly, we obtain four definitions of computability of sets. We call those definitions of characterisitic functions 1-characteristic functions, 2-characteristic functions, 3-characteristic functions, and 4-characteristic functions each other, and similary for definitions of computability. 1-characteristic functions and 1-computability are the strongest concept among them, and 4-characteristic functions and 4-computability are the weakest. 2-characteristic functions and 3-characteristic functions are logically imcomparable concepts, and 2-computability and 3-computability are so.

Of course, it changes that some proposition holds or doesn't hold depending on which of above four definitions we adopt. For example, if both $A \subset \omega$ and A^c are recursively enumerable (r.e.), we can conclude A is computable in classical mathematics. But it is not true in the constructive mathematics, using definitions in the existing literature. But we found above therem is provable in the intuitionistic logic for 3-computability. In fact, 3-computability is equivalent to "Both $A \subset \omega$ and A^c are r.e."

In addition, we point out those definitions of computability can also be characterized by some separability conditions of A and A^c.

Furthermore, we made the variations of definitions of many-to-one and one-to-one reducibilities, which are corresponding to ones of computability.

One of the definitions of reduciblity is useful to prove constructive analog of Rice's theorem; we don't know if we can prove $K \leq_1 A$ for inhabited index sets A such that

$$A \cap \{e : \forall n.[\varphi_e(n) \uparrow]\} = \emptyset$$

in the strongest definition of $\leq_1$ at this time, but we can prove that in a weaker definition, and it is suffice to prove A is not computable in the any sence of four definitions.

We also indicate the famous characterization of the r.e. sets that "be the empty set or the range of some primitive recursive function.", which is probably the epimology of "recursively enumerable", is not true in constructive mathematics, and in fact that this statement characterizes r.e. sets is equivalent to $(\mathrm{LPO})_{\mathbf{Prim.Rec.}}$, one of the non-constructive principles.

This report consists of Chapter 1 for introduction, Chapter 2 for preparing the recursion theory based on the classical logic, Chapter 3 for the intuitonic logic and arithmetic formal systems on it, Chapter 4 for the recursion theory on the intuitionistic logic, Chapter 5 for conclusion and the References.

Our main chapter is Chapter 4. Section 4.1 is for charateristic functions and computabilies, Section 4.2 is for r.e. sets, Section 4.3 is for redefinition of computabilities by separability conditions, Section 4.4 is for many-to-one and one-to-one reducibilities, and Section 4.5 is for analysis for classical theory of recursion theory.