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Reformulating the Dependency Pair Method as Reduction Orders

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Term rewriting is a computational model based on equational theory. This is a model that we regard equations as transformation rules from the left side to the right side, and computation is done by rewriting. We call it rewrite rule and we call the set of rewrite rules term rewriting system (TRS). It is a basis of computer languages and theorem provers. For example, the following is the TRS for addition on Peano numbers.

$$\mathsf{add}(\mathsf{0}, y) \to y \qquad \mathsf{add}(\mathsf{s}(x), y) \to \mathsf{s}(\mathsf{add}(x, y))$$

The term $\mathsf{add}(\mathsf{s}(\mathsf{s}(\mathsf{0})), \mathsf{s}(\mathsf{0}))$ express 2+1. It is rewritten as follows:

$$\mathsf{add}(\mathsf{s}(\mathsf{s}(\mathsf{0})), \mathsf{s}(\mathsf{0})) \to \mathsf{s}(\mathsf{add}(\mathsf{s}(\mathsf{0}), \mathsf{s}(\mathsf{0}))) \to \mathsf{s}(\mathsf{s}(\mathsf{add}(\mathsf{0}, \mathsf{s}(\mathsf{0})))) \to \mathsf{s}(\mathsf{s}(\mathsf{s}(\mathsf{0})))$$

Then we obtain the result: 3, which is expressed by term $\mathsf{s}(\mathsf{s}(\mathsf{s}(\mathsf{0})))$. The above sequence is called a rewrite sequence.

One of the most important property of term rewriting is termination. The definition is that there is no infinite rewrite sequence, so in terminating TRSs one can eventually obtain the result of rewriting. The most basic method for proving termination is using reduction orders. For instance, if there exists a reduction order such that

$$\mathsf{add}(\mathsf{0}, y) > y \qquad \mathsf{add}(\mathsf{s}(x), y) > \mathsf{s}(\mathsf{add}(x, y))$$

in the previous example, we can conclude that the TRS is terminating. In particular, the Knuth-Bendix order (KBO) and the lexicographic path order (LPO) are common and famous reduction orders. Reduction orders are also used in theorem provers based on term rewriting like E and Vampire. In such tools, reduction orders decide the direction of inference, so the success of proof depends on the power (orientability) of the reduction orders. Therefore, if we can define a new strong reduction order, then we can improve the power of these tools.

In this thesis, we introduce a reduction order that simulates the dependency pair method with dependency graphs. The dependency pair method is a powerful method to prove termination. Consider the following TRS, termination of which cannot be shown by KBO and LPO:

$$\mathsf{minus}(x, \mathsf{0}) \to x \qquad \mathsf{minus}(\mathsf{s}(x), \mathsf{s}(y)) \to \mathsf{minus}(x, y)$$
$$\mathsf{quot}(\mathsf{0}, \mathsf{s}(y)) \to \mathsf{0} \qquad \mathsf{quot}(\mathsf{s}(x), \mathsf{s}(y)) \to \mathsf{s}(\mathsf{quot}(\mathsf{minus}(x, y), \mathsf{s}(y)))$$

We obtain the following set of dependency pairs.

$$\mathsf{minus}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) \to \mathsf{minus}^\sharp(x,y) \quad \mathsf{quot}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) \to \mathsf{quot}^\sharp(\mathsf{minus}(x,y),\mathsf{s}(y))$$
$$\mathsf{quot}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) \to \mathsf{minus}^\sharp(x,y)$$

The basic dependency pair method is splits the role of reduction into the pair of two relations with weaker constraints to prove termination. Such a pair is called a reduction pair. In the example, if there exists a reduction pairs $(\gtrsim, >)$ such that

$$\mathsf{minus}(x,\mathsf{0}) \gtrsim x \qquad \mathsf{minus}(\mathsf{s}(x),\mathsf{s}(y)) \gtrsim \mathsf{minus}(x,y)$$
$$\mathsf{quot}(0,\mathsf{s}(y)) \gtrsim \mathsf{0} \qquad \mathsf{quot}(\mathsf{s}(x),\mathsf{s}(y)) \gtrsim \mathsf{s}(\mathsf{quot}(\mathsf{minus}(x,y),\mathsf{s}(y)))$$

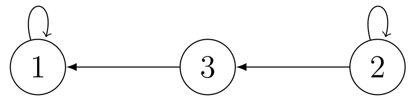
and

$$\mathsf{minus}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) > \mathsf{minus}^\sharp(x,y) \quad \mathsf{quot}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) > \mathsf{quot}^\sharp(\mathsf{minus}(x,y),\mathsf{s}(y))$$
$$\mathsf{quot}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) > \mathsf{minus}^\sharp(x,y)$$

then we can conclude that the TRS is terminating. The power of this method can be improved by using a dependency graph. First, we label the dependency pair such

$$1: \quad \mathsf{minus}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) \to \mathsf{minus}^\sharp(x,y)$$
$$2: \quad \mathsf{quot}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) \to \mathsf{quot}^\sharp(\mathsf{minus}(x,y),\mathsf{s}(y))$$
$$3: \quad \mathsf{quot}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) \to \mathsf{minus}^\sharp(x,y)$$

We obtain the following graph by computing reachability between the rules and the rules.



In this graph, there are two cycle. These cycles tell us it may occur infinite rewrite sequence. So if we have reduction pairs to orient each cycle with the original TRS, then we can conclude termination. Recall the previous example. If there exists a reduction pairs $(\gtrsim_1, >_1)$ and $(\gtrsim_2, >_2)$ such that

$$\mathsf{minus}(x,\mathsf{0}) \gtrsim_1 x \qquad \mathsf{minus}(\mathsf{s}(x),\mathsf{s}(y)) \gtrsim_1 \mathsf{minus}(x,y)$$
$$\mathsf{quot}(0,\mathsf{s}(y)) \gtrsim_1 \mathsf{0} \qquad \mathsf{quot}(\mathsf{s}(x),\mathsf{s}(y)) \gtrsim_1 \mathsf{s}(\mathsf{quot}(\mathsf{minus}(x,y),\mathsf{s}(y)))$$

$$1: \ \mathsf{minus}^\sharp(\mathsf{s}(x),\mathsf{s}(y)) >_1 \mathsf{minus}^\sharp(x,y)$$

and

$$\mathsf{minus}(x, \mathsf{0}) \gtrsim_2 x \qquad \mathsf{minus}(\mathsf{s}(x), \mathsf{s}(y)) \gtrsim_2 \mathsf{minus}(x, y)$$
$$\mathsf{quot}(0, \mathsf{s}(y)) \gtrsim_2 \mathsf{0} \qquad \mathsf{quot}(\mathsf{s}(x), \mathsf{s}(y)) \gtrsim_2 \mathsf{s}(\mathsf{quot}(\mathsf{minus}(x, y), \mathsf{s}(y)))$$

$$2: \ \mathsf{quot}^\sharp(\mathsf{s}(x), \mathsf{s}(y)) >_2 \mathsf{quot}^\sharp(\mathsf{minus}(x, y), \mathsf{s}(y))$$

then we succeed to prove termination.

Dershowitz (2013) shows that the basic one can be simulated by reduction orders called monotonic semantic path orders (MSPOs). However, simulating the dependency pair method with the dependency graph technique is an open problem. In this thesis, we extend his approach by taking multiple reduction pairs to solve this problem.