Title	多面体と多角形の間の折りと展開に関する研究
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Abstract

This paper aims to clarify the folding/unfolding relation between polygons and polyhedra. A polyhedron Q is called unfoldable into a polygon P if we obtain P by cutting a certain set of line segments (not limited to edges) on the surface of Q. Inversely, a polygon P is called foldable into a polyhedron Q if Q is unfoldable into P.

The first part of the thesis is a folding problem that inquires whether a polygon P is foldable into a polyhedron Q for given P and Q. An efficient algorithm for this problem when Q is a box was recently developed. We extend this idea to a class of convex polyhedra. We develop two algorithms for the problem. The first algorithm solves the folding problem for a certain class of convex polyhedra, with a unit length and a unit angle, except for tetramonohedra. The second algorithm handles the exceptional case for the class of tetramonohedra. Combining these algorithms, we can conclude that the folding problem can be solved in pseudo-polynomial time when Q is a polyhedron in a certain class of convex polyhedra, which includes Platonic solids.

The second part of this thesis is a reconfiguration problem on refolding. We show that any pair of polyhedra in several classes of polyhedra is joined by a sequence of $O(1)$ refolding steps, where each refolding step unfolds the current polyhedron into a polygon that is foldable into the next polyhedron. In other words, a polyhedron is refoldable into another polyhedron if they share a common unfolding. Specifically, we prove that (1) any two tetramonohedra are refoldable into each other, (2) any doubly covered triangle is refoldable into a tetramonohedron, (3) any tetrahedron has a 3-step refolding sequence to a tetramonohedron, (4) any (augmented) regular prismatoid and doubly covered regular polygon are refoldable into tetramonohedra, and (5) the regular dodecahedron has a 4-step refolding sequence to a tetramonohedron. In particular, we obtain a ≤ 6-step refolding sequence between any pair of Platonic solids, applying (5) for the dodecahedron and (1) and/or (2) for all other Platonic solids.

The third part of this thesis is about the nonexistence of common unfoldings. We show that the existence of common unfoldings can be reduced to the existence of standard-form common unfoldings under a certain condition. We also develop an algorithm that checks the existence of standard-form common unfoldings, and we implement it on some specific polyhedral class. We obtain the fact that there is no common unfolding with k vertices within $k < 300$ between any strongly-independent and algebraic doubly covered triangles.

Keywords: Computational geometry, Computational origami, Unfolding of polyhedra, Common unfolding, Refolding, Reconfiguration problem.