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On the completeness of discrete time logic LinDisc

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It is usually said that, systematic research of modal logic goes back to Aristotle. A contextual various factors at the situation and time that the inference is done in many cases etc. are included in the inference that appears in a daily idea. It cannot explain such an inference enough by the propositional logic. Therefore, the operator that shows the modal concept of "necessarily" and "possible" to treat a daily inference is introduced. And logic that increases the representational power the propositional logic is called the modal logic.

A similar logical system to the modal logic by giving various interpretations to the operator of the modal logic can be obtained. There are that the dynamic logic used for specification or description of procedural program language and verification of validity of program, the logic of knowledge and belief that interprets "Inevitability" as "Know", and the deontic logicused for a logical analysis of the sentence related to the law, etc. The temporal logic interprets "necessary" as "always" and "possible" as "at some point", respectively. The temporal logic is widely applied about linguistics, the verification of the validity of the program, and artificial intelligence, etc.

The temporal logics have the following logics. The linear temporal logic thought that flow of time is number line. And the branching temporal logic thought the branching point exists in flow of time. In addition, each logic includes the discrete time logic, the rational time logic, and the real time logic, etc. The completeness of logic LinDisc of the discrete time to the linear time given by Goldblatt logic is handled in this research.

Logic LinDisc of the discrete time thought about by Goldblatt is logic that added four axioms of Z_F and Z_P that discrete in the future and the past, and D_F, D_P that series in the future in the past to the linear temporal logic. It is known that this logic is determined by integer frame $(\mathbf{Z}, <)$

As for the technique of the proof the completeness where Goldblatt descrived the ouline, it thinks about frame "Dumbbell" in which the special sequence of a cluster whose a first and last cluster(equivalence turned on accessibility relation) consists of non-degenerate cluster and other clusters consists of the degenerate clusters. And the following result is shown.

Theorem 1
If a frame D is dumbbell, then D is a p-morphism image of $(\mathbf{Z}, <)$.

By theorem1 and lemma of p-morphism, if $(\mathbf{Z}, <) \models A$, then A is valid in all the dumbbell.

Thus to prove that the LinDisc is complete with respect to $(\mathbf{Z}, <)$, it suffices to show that it is complete with respect to the class of danbbell, i.e. that any non-theorem of LinDisc is falsified by a model on some danbbell.

The above is the outline of proof of completeness with respect to the discrete time by Goldblatt. However, we pointed out that Theorem1 don't hold in this research, and neither axiom Z_F nor Z_P hold in addition in dumbbell D.

To correct these two gaps, by using the cluster assingment $t = \langle t_1 \text{ and } t_2 \rangle$ innovated by F.Wolter We show that the completeness of the discrete time logic can proved.

t is map from sets of clusters in frame F to $\{m, j\} \times \{m, j\}$ Here, m means maximal and j means joker. If cluster C is a degeneration cluster, it is shown $tC = (m, m)$

The frame used as follows is assumed to be $F = (S, R, t)$with cluster-assignment. Here, for all formulas B and any x and $y \in S$

$$max_R(V(B)) = \{x \in F | \forall y \in F(y \in V(B) \Rightarrow yRx)\}$$
$$min_R(V(B)) = \{x \in F | \forall y \in F(y \in V(B) \Rightarrow xRy)\}.$$

When valuation V consist the following condition for all clusters C, V calls A-good.

1. for all $B \in Sub(A)$ s.t. $B = \langle F \rangle B'$ for some B'

   $$\text{if } t_1C = j \text{ then } C \cap max_R(V(B)) \neq \emptyset$$

2. for all $B \in Sub(A)$ s.t. $B = \langle P \rangle B'$ for some B'

   $$\text{if } t_1C = j \text{ then } C \cap min_R(V(B)) \neq \emptyset$$

Next, we thinks about frame good-dumbbell$(C_1 \cdots C_n)$ with cluster-assignment.

1. the first cluster C_1 and the last cluster C_n are degenerate cluster i.e. $t_1C_1 = m$、$t_2C_n = m$.
2. all other clusters are degenerate i.e. $tC_2 = \cdots = C_{n-1} = (m, m)$.
3. if $n \neq 1$ i.e. $C_1 \neq C_n$ then $t_1C_1 = j$ and $t_2C_n = j$.

Under valuation V is A-good, the following relation exists between $(\mathbf{Z}, <)$ and good-dumbbell D'.

Lemma 1
For any formula A, if A holds in $(\mathbf{Z}, <)$, then it holds in all good-dumbbells with all A-good valuations.

By Lemma 1, to show the completeness of the discrete time logic, it only has to show that it is more complete than with respect to the class of good-dumbbell. In fact, for any non-theorem A of LinDisc A is falsified by a model on some good-dumbbell.

As a result, for some $s \in S$,

$$D' \not\models_s A.$$

By Lemma1, for some $s \in \mathbf{Z}$,

$$(\mathbf{Z}, <) \not\models_s A.$$

Therefore, the completeness of the discrete time logic is shown. In addition, we have found a falsifying model on a finite frame "good-Dumbbell", we also get the strong finite property for the logic Lin Disc. That is, the following result is obtained.

Theorem 2
Lin Disc is characterized by the frame $(\mathbf{Z}, <)$. Moreover it has the finite model property, and hence is decidable.