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Iterative Model Identification and Tracking with Distributed Sensors

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***Abstract*—In this paper, we propose a new iterative model identification and tracking technique for distributed sensor systems using a factor graph (FG). The idea is initiated from a position identification technique we proposed [4], however, this paper aims to provide an algorithm which is applicable to more generic identification and tracking purposes. With the proposed technique, each sensor performs signal processing for compression of the measurement data and sends the compressed data to the fusion center. The marginal probability of the compressed sensing results are calculated over the FG at the fusion center. At the final stage, a maximum a posteriori probability (MAP) estimate of the model can be obtained during the tracking phase through the FG. The MAP over FG will be used for the prediction at the next state of model identification to further improve the estimation accuracy without requiring unacceptably high computation effort.**

I. Introduction

A general technological trend towards B5G and 6G communications systems is ”distributed”, such as edge computing, distributed sensing network (DSN) [1], millimeter-wave backhaul and access links based networks [2], and various cooperative communications networks [3]. Compared to traditional stand alone radar systems, distributed sensing systems are more flexible to be implemented in practice without requiring large hardware cost. With the development of monitoring techniques in computer science and wireless communications, the DSN-based techniques have been applied to broad areas of applications, for instance, geolocation, autonomous vehicle positioning, and agricultural monitoring, etc [4]–[6].

The primary objective of the distributed sensor-based model identification-and-tracking techniques is to achieve high accuracy of estimation of parameters related to the model, and its robustness against environment change by adequately tracking the model. Particularly, in practice, since measured data sets by the distributed sensors do not straightforwardly represent the system models, mathematical properties of the functions that connects the measurement and the parameters have to be taken into account. In many cases, the connecting functions are non-linear. Furthermore, the computational complexity of the algorithm also need to be considered. Recently, [7] introduced consensus filter-based method for distributed sensor fusion. A vehicle classification method in a wireless DSN is proposed in [8]. Moreover, [9] proposed a framework by using factor graph (FG) for geolocation under Gaussian assumption. The computational complexity can be drastically reduced due to the fact that with the Gaussian assumption, only mean and variance of Gaussian message need to be exchanged and updated through iterations over FG, while the estimation accuracy can be controlled with appropriate approximations. [10] proposed a new unified extended Kalman filter based FG (EKF-FG) for dynamic position tracking. The output from the FG for model identification is used as observation state to refine the prediction state which is obtained from previous state in tracking EKF-FG at each timing step.

This paper investigates distributed sensors-based FG technique for model identification-and-tracking for a system. The system model to be identified is not straightforwardly expressed by the measurements obtained from multiple sources. The main contributions of this paper are summarized as follows:

- A unified model identification and tracking FG algorithm is proposed to accurately estimate the parameters related to the model and to track their states.

- The proposed algorithm can be applied to any kind of non-linear system models, regardless of the relationship between the snapshot and model parameters to be estimated.

This paper is organized as follows. In section II, the schematic model used in this paper is presented. The proposed unified model identification-and-tracking FG and message expression are detailed in section III. The performance of proposed technique is evaluated through computer simulations, and the results are shown in Section IV. Finally, Section V conclude this paper with some concluding remarks. The Cramer-Rao Lower Bound (CRLB) of the model identification is presented in Appendix.

II. System Model

The whole system is composed of the two parts which are: model identification and tracking. We start with model identification system using N distributed sensors, of which self-information such as values set commonly used throughout the iterations, for example, positions in the coordinate, is known to the fusion center. Assume there are I independent sources in the system model, each having M variables in the set $X = \{x_{i,k,1}, x_{i,k,2}, \dots, x_{i,k,M}\}$, which need to be identified. $i = 1, 2, \dots, I$ and $k = 1, 2, \dots, K$ are source and timing indexes, respectively. The source index will be omitted for the simplicity unless required.

The measurements of each distributed sensor at the timing k is given by

$$\hat{y}_{n,k} = h(x_{n,k}) + u_{n,k}, \tag{1}$$

with $n = 1, 2, \dots, N$ the sensor index and $h(x_{n,k})$ a known function representing the relationship between variable set X and distributed sensors self-information, and $u_{n,k} \sim \mathcal{N}(0, \sigma_y^2)$ the measurement error.

For the tracking part, we utilize the multiple variables discrete state-space model (SSM), similar to [10]. The M variables from the I sources can be presented in the vector $\mathbf{x}_k = [x_{i,k,1}, x_{i,k,2}, \dots, x_{i,k,M}]^T$ with $i = 1, 2, \dots, I$ at timing k. The SSM is defined as

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}) + \mathbf{w}_k, \tag{2}$$

where the function $f(\cdot)$ can be linear or non-linear. Hence, regardless of whether the model is linear or non-linear, the algorithm derived in this paper can be commonly used. $\mathbf{w}_k = [w_{i,k,1}, w_{i,k,2}, \dots, w_{i,k,M}]^T$ is the white Gaussian driving force vector. It should be noticed that keeping the messages exchanged over the FG being Gaussian distributed is convenient, because the distribution is expressed only by mean and variance. Therefore, the first order Taylor series (TS) is utilized to approximate the function $f(\cdot)$, given by

$$f(\mathbf{x}_{k-1}) \approx f(\boldsymbol{\alpha}) + f'(\boldsymbol{\alpha})(\mathbf{x}_{k-1} - \boldsymbol{\alpha}) \tag{3}$$

with $\boldsymbol{\alpha}$ being the center point of the TS expansion, $f(\mathbf{x}_{k-1})$ is the current state and $f'(\boldsymbol{\alpha})(\mathbf{x}_{k-1} - \boldsymbol{\alpha})$ is the difference between the current state $\mathbf{x}_{k-1}$ and the next state $\mathbf{x}_k$. Assuming the difference between two states is not intractably large, the derivative can be expressed by state gradient as $\mathbf{v}_{k-1} = f'(\boldsymbol{\alpha})(\mathbf{x}_{k-1} - \boldsymbol{\alpha})$. Then (2) can be further expressed as

$$\mathbf{x}_k \approx \mathbf{x}_{k-1} + \mathbf{v}_{k-1} + \mathbf{w}_k. \tag{4}$$

The changing speed $\mathbf{v}_{k-1}$ can be updated by extended Kalman filter (EKF). The observation state $\mathbf{j}_k$ of EKF is given by

$$\mathbf{j}_k = g(\mathbf{x}_k) + \mathbf{e}_k \tag{5}$$

where $g(\mathbf{x}_k)$ returns the mean of $\mathbf{x}_k$ from model identification FG and $\mathbf{e}_k \sim \mathcal{N}(0, \sigma_e^2)$ is the observation noise. Due to the fact that $\mathbf{e}_k$ is unknown, its variance σ_e^2 is also unknown. Therefore, the smallest σ_e^2 calculated from the CRLB in Appendix, is utilized [11].

III. Proposed algorithm

A. FG for model identification

The proposed distributed sensors based model identification and tracking technique with FG is given in this section. The source and sensor indexes are omitted for the simplicity. It should be emphasized that the first order TS expansion is used to approximate the function $h(x_{n,k})$, in the same way as $f(\mathbf{x}_k)$, centered at a point $\boldsymbol{\beta}$, to keep the message Gaussianity exchanged over the FG, expressed as

$$\begin{aligned} y_k \approx & h(\boldsymbol{\beta}) + \frac{\partial h(x_{n,k})}{\partial x_{k,1}}(x_{k,1} - \beta_1) + \frac{\partial h(x_{n,k})}{\partial x_{k,2}}(x_{k,2} - \beta_2) \\ & + \dots + \frac{\partial h(x_{n,k})}{\partial x_{k,M}}(x_{k,M} - \beta_M) \end{aligned} \tag{6}$$

with $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_m]^T$. The center point $\boldsymbol{\beta}$ is determined by the predicted state $\mathbf{x}_{k|k-1}$ from tracking phase, which will be discussed in the next subsection. Then, (6) can be rewritten as

$$y_k \approx \lambda_{k,1}x_{k,1} + \lambda_{k,2}x_{k,2} + \dots + \lambda_{k,M}x_{k,M} + \gamma_k \tag{7}$$

where $\lambda_{k,m}$, $m = 1, 2, \dots, M$, and γ_k are the constants, given by

$$\lambda_{k,m} = \frac{\partial h(x_{n,k})}{\partial x_{k,m}}, \tag{8}$$

$$\gamma_k = h(\boldsymbol{\beta}) - \sum_{m=1}^{M} \frac{\partial h(x_{n,k})}{\partial x_{k,m}} \beta_m \tag{9}$$

The model parameters to be identified can then be derived as:

$$x_{k,m} = \frac{y_k - \gamma_k - \sum\limits_{m'=1, m' \neq m}^{M} \lambda_{k,m'} x_{k,m'}}{\lambda_{k,m}} \tag{10}$$

The FG for the model identification is illustrated in the upper part of Fig.1. Noted that the Fig.1 is used for estimating one source parameters related to the model. It should be emphasized that the algorithm for multiple sources model identification can be derived in the same way. The measured

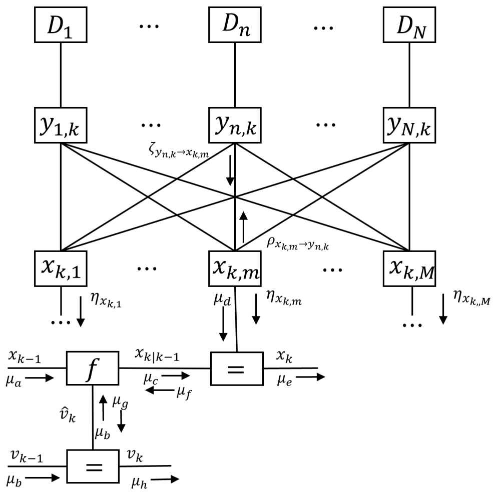

Fig. 1. Proposed FG structure for model identification and tracking

messages from the distributed sensors are the input to the measurement function node (FN) D_n for calculating mean and variance, i.e., $(m_{y_{n,k}}, \sigma^2_{y_{n,k}})$. The calculated mean and variance from Measurement FN is passed through the iteration FN. In this node, messages from multiple distributed sensors are exchanged and updated. Let $\zeta_{y_{n,k}\to x_{k,m}}$ and $\rho_{x_{k,m}\to y_{n,k}}$ denote the downward and upward messages between Iteration FN and Estimation FN, respectively. The iteration process is then described as follows, according to (8)-(10):

- Update of downward message

$$m_{\zeta_{y_{n,k}\to x_{k,m}}} = \frac{1}{\lambda_{k,m}} m_{y_{n,k}} - \frac{\sum\limits_{m'=1,m'\neq m}^{M} \lambda_{k,m'} m_{\rho_{x_{k,m'}\to y_{n,k}}}}{\lambda_{k,m}} - \frac{\gamma_k}{\lambda_{k,m}} \quad (11)$$

$$\sigma^2_{\zeta_{y_{n,k}\to x_{k,m}}} = \frac{1}{\lambda^2_{k,m}} \sigma^2_{y_{n,k}} + \frac{\sum\limits_{m'=1,m'\neq m}^{M} \lambda^2_{k,m'} \sigma^2_{\rho_{x_{k,m'}\to y_{n,k}}}}{\lambda^2_{k,m}} \quad (12)$$

- Update of upward message

$$\frac{1}{\sigma^2_{\rho_{x_{k,m}\to y_{n,k}}}} = \sum_{n'=1,n'\neq n}^{N} \frac{1}{\sigma^2_{\zeta_{y_{n'},k}\to x_{k,m}}} \quad (13)$$

$$m_{\rho_{x_{k,m}\to y_{n,k}}} = \sigma^2_{\rho_{x_{k,m}\to y_{n,k}}} \cdot \sum_{n'=1,n'\neq n}^{N} \frac{m_{\zeta_{y_{n'},k}\to x_{k,m}}}{\sigma^2_{\zeta_{y_{n'},k}\to x_{k,m}}} \quad (14)$$

The iteration is stopped when estimates are converged or pre-defined maximum iteration number due to the complexity or latency restriction is reached. At last, the estimates corresponding to the identified model are achieved by $(m_{x_{k,m}}, \sigma^2_{x_{k,m}})$, given by

$$\frac{1}{\sigma^2_{x_{k,m}}} = \sum_{n=1}^{N} \frac{1}{\sigma^2_{y_{n,k}\to x_{k,m}}} \quad (15)$$

and

$$m_{x_{k,m}} = \sigma^2_{x_{k,m}} \cdot \sum_{n=1}^{N} \frac{m_{y_{n,k}\to x_{k,m}}}{\sigma^2_{y_{n,k}\to x_{k,m}}}. \quad (16)$$

It should be noted that the result $m_{x_{k,m}}$ of the FG iteration can be seen as observation state of the EKF tracking phase. From (5), the variance of the observation noise is not equal to the estimation variance $\sigma^2_{x_{k,m}}$. Since the observation noise variance is unknown, we use the smallest value calculated from CRLB to replace with it, as shown in Appendix.

B. FG for model tracking

Due to the space limitation, a tracking algorithm for only one parameter $x_{k,m}$ with the model is presented as EKF-FG. The rest of parameters can be updated in the same way. The completeness of the derivation of EKF-FG is described in [12]. The variable index is omitted for the simplicity. Let μ and η denote the flow message and observation over the tracking phase. The whole tracking process can be divided into three steps:

- State prediction
  The prediction state is obtained from current state x_{k-1} and v_{k-1}, given by

$$\mu(x_{k|k-1}) = \sum_{x_{k-1}} \sum_{v_{k-1}} f(x_{k|k-1}|x_{k-1}, v_{k-1}) \mu_a(x_{k-1}) \mu_b(v_{k-1}), \quad (17)$$

- State update
  The observation state η_k of model identification FG is used to refine the prediction to obtain the next state x_k, given by

$$\mu_e(x_k) = \mu_c(x_{k|k-1}) \mu_d(\eta_k) \quad (18)$$

- Gradient Update
  The state gradient vector v_k is updated by current state v_{k-1} and the changing rate $\hat{v}_k$ between two adjacent states divided by unit time, given by

$$\mu_h(v_k) = \mu_b(v_{k-1}) \sum_{x_{k-1}} \sum_{x_k} f(\hat{v}_k|x_{k-1}, x_k) \mu_a(x_{k-1}) \mu_f(x_k) \quad (19)$$

IV. Simulation results

In this section, results of the simulations conducted to evaluate the performance of the proposed FG based model identification and tracking algorithm using distributed sensors are presented. In the simulations, because of the space limitation, we only focus on single source scenario, however, the proposed algorithm derived in the previous sections is also applicable to multi-source cases. However, it should be noticed that for the multi-source case, measurement data association, such as clustering is needed, as in [4]. Let the set (A_n, B_n),

$n = 1, 2, 3$, denote three sensors self-information, which is unchanged during the iterations. $(A_1, B_1) = (-30, 50)$, $(A_2, B_2) = (10, 5)$ and $(A_3, B_3) = (60, 20)$, respectively. The $x_{1,k}$ and $x_{2,k}$ defined in (6) and (7) are assumed to be given as

$$x_{1,k} = 0.05k^2 + 0.03k + 6 + \omega_{1,k} \tag{20}$$

$$x_{2,k} = -0.02k^2 + 0.07k + \omega_{2,k} \tag{21}$$

in the simulation, where the timing $k = \{1, 2, \ldots, 50\}$ and $\omega_{1,k},\ \omega_{2,k} \sim \mathcal{N}(0, \sigma_\omega^2)$ with $\sigma_\omega^2 = 0.5$. The initial value of source $(x_{1,0}, x_{2,0})$ is set at $(2, -2)$. The non-linear function $h(x_{n,k})$ is defined by

$$h(x_{n,k}) = (A_n - x_1)^2 + (B_n - x_2)^2 \tag{22}$$

According to (6), the function $h(x_{n,k})$ is first of all, approximated at the central point $\beta = (\beta_1, \beta_2) = (x_{1,k|k-1}, x_{2,k|k-1})$ by the first order Taylor expansion, which is given by

$$\begin{aligned} y_k &\approx h(\beta) + \frac{\partial h(x_{n,k})}{\partial x_{k,1}}(x_{k,1} - \beta_1) + \frac{\partial h(x_{n,k})}{\partial x_{k,2}}(x_{k,2} - \beta_2) \\ &= \lambda_1 x_1 + \lambda_2 x_2 + \gamma_k \end{aligned} \tag{23}$$

with

$$\lambda_1 = -2(A_n - x_{1,k|k-1}), \tag{24}$$

$$\lambda_2 = -2(B_n - x_{2,k|k-1}), \tag{25}$$

$$\gamma_k = A_n^2 - x_{1,k|k-1}^2 + B_n^2 - x_{2,k|k-1}^2 \tag{26}$$

It should be noted that the function $h(x_{n,k})$ can be arbitrary function, regardless of linear or non-linear and the number of parameters can also be chosen arbitrarily, depending on the system or models.

At every measurement timing, each sensor receives 60 samples with standard deviation $\sigma_y = 5$. We set maximum iteration number being 10 and the initial guess for iteration set at $(0, 0)$.

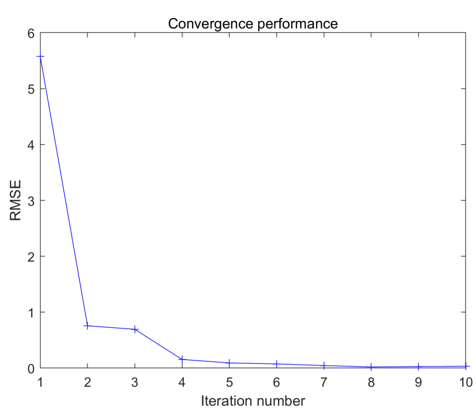

Fig. 2. Performance of convergence

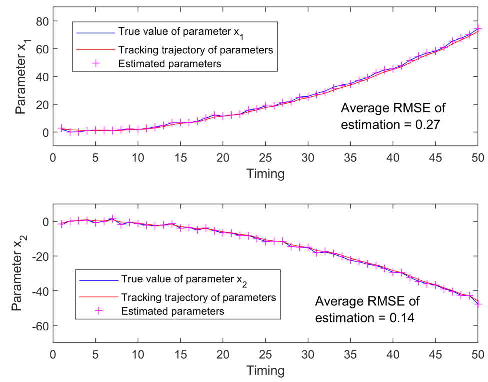

Fig. 3. Performance of proposed tracking algorithm with $\sigma_y = 5$

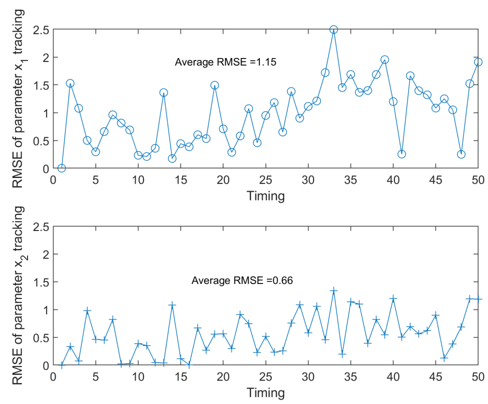

Fig. 4. RMSE evaluation of the timing index

The convergence performance represented by the root mean square error (RMSE) is shown in Fig. 2. It can be found that with 4 or 5 iterations, the estimation result can acquire the actual behavior of the model parameters, yielding very small RMSE value with proposed FG based method. The performance of model tracking is shown in Fig. 3. It is found that the estimations of parameters related to the model by FG are very close to the actual values. In addition, the gaps

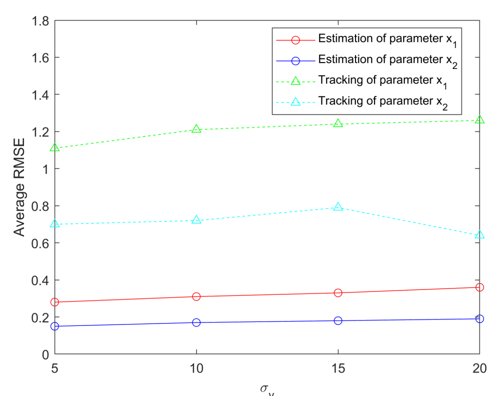

Fig. 5. Average RMSE versus standard deviation with proposed FG based model identification-and-tracking

between the result of FG-EKF and actual trajectories of the two parameters are very narrow. RMSE of model tracking is calculated at each timing, of which results are shown in Fig. 4. The average RMSE is also very small, which demonstrates excellent performance of proposed model identification and tracking algorithm. Finally, the accuracy of proposed technique is evaluated by changing σ_y, of which result is shown in Fig. 5. Obviously, with the increased standard deviation σ_y, also increases RMSE, however, surprisingly the accuracy of model identification and tracking is not very sensitive to the σ_y value change.

V. Conclusion

This paper has proposed a distributed sensors-based iterative technique for model identification and tracking. A generic mathematical model is, first of all, introduced for multiple sources multiple variables which are aimed to be identified and tracked. The proposed algorithm is then implemented using a unified factor graphs (FG). Identification accuracy and tracking performances have been evaluated through a series of computer simulations for the case of a single source with sensors. It has been demonstrated that besides its simplicity, fast convergence can be achieved even if the initial guess is very different from the actual values. The proposed technique enables accurate model identification combined with dynamic trajectory tracking. Furthermore, it should be stated that since the tracker can predict the state at the future timings of which state probability can be calculated from the previous state probability, the technique can be used in, for example, collision prediction in autonomous driving system. Such practical applications are left as future study.

VI. Appendix

A. CRLB deviation for observation noise

The observation noise $\mathbf{e}_k \sim \mathcal{N}\ (0, \sigma_e^2)$ will be calculated from CRLB in this sub-section. As in [11], the CRLB can be defined as

$$\mathrm{CRLB} = \frac{1}{M}\mathrm{trace}[F^{-1}(\mathbf{x})] \tag{27}$$

with F being the Fisher information matrix (FIM), of which can be expressed as

$$F(\mathbf{x}) = E\left[\left(\frac{\partial}{\partial \mathbf{x}}\ln p(\widehat{y})\right)^2\right], \tag{28}$$

by giving the measurement $\hat{y}$ with L samples. The PDF of $p(\widehat{y})$ is defined as

$$p(\widehat{y}) = \prod_{l=1}^{L}\frac{1}{\sqrt{2\pi\sigma_y^2}}\exp\left[-\frac{1}{2\sigma_y^2}(\widehat{y}_l - y)^2\right]. \tag{29}$$

Since

$$E\left[\left(\frac{\partial}{\partial y}\ln p(\widehat{y})\right)^2\right] = -E\left[\frac{\partial^2}{\partial y^2}\ln p(\widehat{y})\right], \tag{30}$$

and according to [12], (28) can further be expressed as

$$\begin{aligned} F(\mathbf{x}) &= \frac{\partial y}{\partial \mathbf{x}}^T E\left[\left(\frac{\partial}{\partial y}\ln p(\widehat{y})\right)^T\left(\frac{\partial}{\partial y}\ln p(\widehat{y})\right)\right]\frac{\partial y}{\partial \mathbf{x}} \\ &= \frac{\partial y}{\partial \mathbf{x}}^T E\left[\left(\frac{\partial}{\partial y}\ln p(\widehat{y})\right)^2\right]\frac{\partial y}{\partial \mathbf{x}} \\ &= \frac{\partial y}{\partial \mathbf{x}}^T\left[\frac{L}{\sigma_y^2}\right]\frac{\partial y}{\partial \mathbf{x}}, \end{aligned} \tag{31}$$

where $\frac{\partial y}{\partial \mathbf{x}}$ is the Jacobin matrix, which can be given by

$$\mathbf{J}_{\mathrm{k|k-1}} = \frac{\partial \mathrm{y}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_{1,k|k-1}} & \frac{\partial y_1}{\partial x_{2,k|k-1}} & \cdots \\ \vdots & \vdots & \vdots \\ \frac{\partial y_N}{\partial x_{1,k|k-1}} & \frac{\partial y_N}{\partial x_{2,k|k-1}} & \cdots \end{bmatrix}. \tag{32}$$

In this paper, we replace unknown true value (x_1, x_2) with prediction state $(x_{1,k|k-1}, x_{2,k|k-1})$. Thereby, the CRLB is given by

$$\mathrm{CRLB} = \frac{1}{M}\{\mathrm{trace}[(\mathbf{J}_{k|k-1}^T\mathbf{\Sigma}_y^{-1}\mathbf{J}_{k|k-1})L]\}^{-1}, \tag{33}$$

where $\mathbf{\Sigma}_y = \sigma_y^2\mathbf{I}_M$ denotes Gaussian covariance vector and $\mathbf{I}_M$ is identity vector.

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