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Master’s Thesis

Research on the Computational Complexity of the Crush Ice Game

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Abstract

In this thesis, we investigate the computational complexity of the Crush Ice Game, a physics-based puzzle game played on a hexagonal grid. We present a formal analysis of the game's mechanics, focusing particularly on the interplay between block stability and gravitational effects within its hexagonal grid system. The research transforms the original multiplayer game into a deterministic single-player decision problem while preserving its core mechanical and strategic elements.

Our contribution is twofold. First, we develop a deterministic algorithm for simulating block stability and falling mechanics in a hexagonal grid system, providing a systematic method for evaluating game states. This result implies that the block stability problem of the crush ice game can be solved in polynomial time. Second, we prove that the crush ice puzzle decision problem is NP-complete through a polynomial-time reduction from Planar Monotone 3-SAT. This reduction employs a series of carefully designed gadgets—including variable, clause, and anti-backflow mechanisms—that encode Boolean logic within the game's physical constraints.

The research demonstrates that despite its simple ruleset, the crush ice puzzle exhibits significant computational complexity. Our findings contribute to the broader understanding of complexity theory in physics-based puzzle games and provide insights into the algorithmic challenges of simulating gravity-based mechanics in non-traditional grid systems. Additionally, we identify promising directions for future research.

Keywords: computational complexity, NP-completeness, hexagonal grid systems, physics-based puzzles