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# Abstract

In this paper, we investigate the computational complexity of two types of pencil puzzles: cryptarithms and arithmetical restorations.

A cryptarithm is a puzzle in which the digits of an arithmetic expression are replaced by letters, and the objective is to assign distinct digits to each letter in a way that the expression becomes valid.

In contrast, arithmetical restorations involve an arithmetic expression with certain digits replaced by blanks, where the goal is to assign appropriate digits to these blanks to complete the expression.

Previous research has examined the complexity of both puzzles under several conditions. Eppstein demonstrated the NP-completeness of cryptarithms for the addition of two numbers, while Matsui proved NP-completeness for arithmetical restorations involving multiplication of two numbers.

In this work, we extend these results by showing that cryptarithms are also NP-complete for multiplication, subtraction, and division of two numbers.

Furthermore, we develop a linear-time algorithm for the number of digits such as counting up the total number of solutions using dynamic programming for solving arithmetical restorations in cases of addition and subtraction.

Extending this result, we develop a polynomial-time algorithm for $k$ and the number of digits of $k$-number addition arithmetical restorations. We also developed a linear-time algorithm for the number of digits of recently proposed double arithmetical restorations. In addition, we prove NP-completeness for arithmetical restorations involving division.

These results provide a comprehensive analysis of the computational complexity of all four basic arithmetic operations for both cryptarithms and arithmetical restorations.