Title	Identifying a crucial role for robustness and spreading in complex network
Author(s)	Liao, Fuxuan; Hayashi, Yukio
Citation	The 8th International Conference on Complex Networks & Their Applications (COMPLEX NETWORKS 2019): 572-574
Issue Date	2019-12-10
Type	Conference Paper
Text version	publisher
URL	http://hdl.handle.net/10119/20008
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Description	The 8th International Conference on Complex Networks & Their Applications (COMPLEX NETWORKS 2019), Lisbon, Portugal, December 10-12, 2019

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COMPLEX NETWORKS 2019

THE 8TH INTERNATIONAL CONFERENCE ON COMPLEX NETWORKS AND THEIR APPLICATIONS

December 10 - 12, 2019
Lisbon, Portugal

BOOK OF ABSTRACT

COMPLEX NETWORKS 2019
The 8th International Conference on Complex Networks & Their Applications
December 10 - 12, 2019 Lisbon, Portugal
Published by the International Conference on Complex Networks & Their Applications.

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COMPLEX NETWORKS 2019
e-mail: hocine.cherifi@u-bourgogne.fr

ISBN: 978-2-9557050-3-2

Identifying a crucial role for robustness and spreading in complex network

Fuxuan Liao and Yukio Hayashi

Japan Advanced Instittue of Science and Technology
1-1 Asahidai, Nomi, Ishikawa 923-1292 Japan

1 Introduction

Although it has been believed for a long time that a high degree node (hub) is the best selection for effective spreading and malicious attack, exciting research of network science has recently unveil that k-core is more important than hub (node) in some sense. The most efficient spreaders are located within the core of network as identified by the k-shell decomposition analysis [1]. For example, if a high degree node in a dangling sub-tree is not necessarily effective for spreading, this node belongs to 1-shell. The spreading from such node in a dangling sub-tree quickly disappears, while the spreading can persist in the core. Thus, in order to find out influence nodes for spreading, a node with high k-shell index is more useful than high degree nodes [1]. Here, k-core is defined as a connected remaining part by recursively removing nodes whose degrees are less than k [2]. The k-shell is the part by deleting (k+1)-core from k-core.

On the other hand, from the asymptotic equivalence of dismantling and decycling problems at infinite graphs in a large class of random networks with light-tailed degree distribution [3], the strong robustness is related to increasing the size of feedback vertex set (FVS) which is necessary to form loops [4]. The existence of many loops may be crucial to maintain the connectivity of network within a finite size. Dismantling (or decycling) problem is to find the minimum set of nodes whose removal yields a graph with the largest connected cluster whose size is at most a constant (or a graph without loops) [3]. Moreover, based on the definition of influencer in Collective Influence (CI), when the minimum set of important nodes for spreading is removed, the propagation is stops; At that time, the network becomes a tree without loops at the critical just before destroying the connectivity [5]. Thus, loops are also related to influencer. From the above viewpoints, it is necessary for improving the robustness to exist many loops in a network, stronger robustness have larger size of FVS. In this paper, we find that not only degree and k-shell are important for network to find out the crucial nodes for spreading and robustness, but also FVS plays the same role. More precisely, our goal is to clarify the relations between FVS and k-shell.

2 Fraction of FVS nodes in k-shells

In order to clarify the relation between FVS and k-shell, we need to get the fraction of FVS nodes in k-shells. The minimum size belongs to NP-hard problem, there is no efficient algorithm to obtain the exact solution. Thus, based on the cavity method in

COMPLEX NETWORKS 2019

The 8^{th} International Conference on Complex Networks and Their Applications. 10 - 12 Dec., 2019, Lisbon, Portugal

statistical physics, we apply efficient belief propagation (BP) algorithm proposed by H-J Zhou [6] to calculate candidate nodes of the FVS. Then, we divide the set of nodes to make the k-shell decomposition into different shells. Finally, the fraction of FVS nodes in each k-shell is calculated.

We investigate the relation between FVS and k-shell for the data of 28 real scale-free networks [7] gathered from different fields. Figure 1 shows typical results in three examples. Here, N, $\mathscr{F}$ and K_s denote the size (the number of nodes included in each subset) of the whole network, FVS, and k-shells, respectively. As shown by red line in Fig. 1, the fraction $\frac{|\mathscr{F} \bigcap K_s|}{|K_s|}$ of FVS in k-shell is increasing as the k-shell index is larger. In other words, the inner core contains a higher fraction of FVS. As shown by blue lines in Fig. 1, the fraction $\frac{|K_s|}{|N|}$ of k-shell in the network size N is decreasing as the k-shell index is larger. Therefore, the inner core is consisted of a few nodes. In blue line, $|K_s|$ is smaller as larger k-shell index, the intersection to FVS also becomes small in the whole FVS, this decreasing is shown by green line. However, in some case, depending on the zig-zag shapes in red and blue lines, there are peaks in green lines.

In Fig. 2, in order to make the results more intuitive, we visualize the k-shell decomposition. Colored nodes except gray ones in each circle represent a node in FVS. The red lines show the connections of FVS nodes in a same shell, yellow lines show the connections between nodes in FVS included in different shells. As shown small rings in the Fig. 2, the large k-shell indexes have the high fractions of FVS, even if each $|\mathscr{F} \bigcap K_s|$ is small.

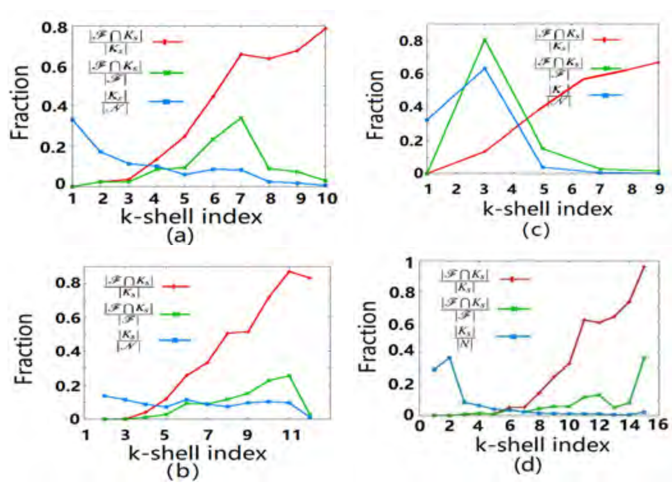

Fig. 1. Results for real networks in four different domains. (a) biological network of yeasts with N = 2224, # of FVS = 363, maximum k-shell index k = 10 (b) technological US power grid with N = 4941, # of FVS = 516, maximum k-shell index k = 5 (c) social email networks with N = 1133, # of FVS = 370, maximum k-shell index k = 12 (d) Japanese language networks with N = 2698, # of FVS = 136, maximum k-shell index k = 15.

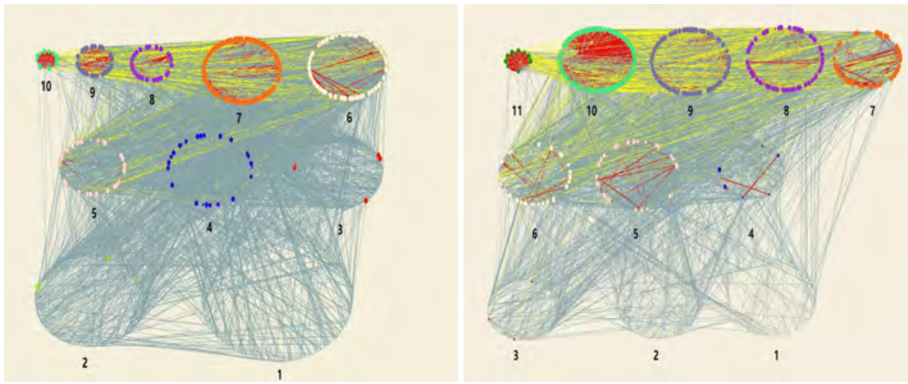

Fig. 2. Visualization by Pajek as typical result. From left to right, biological network of yeasts with N = 2224, # of FVS = 363, maximum k-shell index k = 10, technological US power grid with N = 4941, # of FVS = 516, maximum k-shell index k = 12. The numbers represent k-shell indexes.

Summary. Our research is an extension to investigate the relation between degree and k-shell [1]. We suggest that FVS and k-shell play a crucial role for robustness and spreading. We will elucidate that the nodes in FVS become important not only for robustness but also for efficient spreading. In addition, to find FVS is a NP-hard problem, while the k-shell decomposition is a P-problem; It is expected that the gap in computational effort leads to develop a new direction by solving the P-problem to estimate the FVS.

Acknowledgements

This research is supported in part by ISPS KAKENHI Great Number JP.17H01729.

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