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Doctoral Dissertation

Resolving Uncertain Supply Chain Management Problems through the Development of Innovative Fuzzy Optimization Models

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Abstract

This research develops advanced fuzzy optimization models to enhance resilience and sustainability in Supply Chain Aggregate Production Planning (SCAPP) under uncertain conditions. By employing fuzzy logic, the study quantifies and incorporates uncertainties such as fluctuating operation costs and demand into production planning, enabling decision-makers to manage unpredictable changes in supply chain operations. The proposed approach prioritizes operational efficiency, risk mitigation, reliability, and sustainability, creating a robust framework to adapt to volatility and ensure a steady flow of goods.

The findings demonstrate that the innovative fuzzy optimization model significantly improves adaptability and resilience in supply chains. The model minimizes cost fluctuations, risk mitigation, optimizes resource utilization, and addresses multiple conflicting objectives in uncertain environments. Empirical results validate its practical effectiveness as a valuable tool for modern supply chain strategies, offering companies a reliable means of maintaining stability and sustainability among disruptions.

This research aligns with contemporary trends emphasizing resilient supply chain models to manage uncertainty and variability. By enhancing traditional SCAPP methodologies with advanced fuzzy and risk mitigation techniques, the study addresses critical challenges in managing uncertainty while contributing novel insights to academic literature. Key innovations include a focus on multi-objective optimization, explicit risk mitigation strategies, and the integration of theoretical advancements with practical applications. These contributions establish a new benchmark for improving the adaptability, precision, and applicability of SCAPP models, offering significant value for both researchers and industry practitioners.

Keywords: Supply Chain Management, Uncertainty, Sustainability, Resilience Index, Multi-Criteria Decision-Making, Fuzzy Linear Programming, Monte Carlo Simulation

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Contents

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List of Symbols/Abbreviations

ACO	Ant Colony Optimization
CCP	Chance-Constrained Programming
EV	Expected Value
FLP	Fuzzy Linear Programming
GA	Genetic Algorithms
IFLP	Intuitionistic Fuzzy Linear Programming
IP	Integer Programming
LOM	Largest of Maximum
LP	Linear Programming
MCVaRG	Mean Conditional Value at Risk Gap
MOFLP	Multiple Objective Fuzzy Linear Programming
MOM	Mean of Maximum
NLP	Non-Linear Programming
PDF	Probability Density Function
PET	Polyethylene Terephthalate
PSO	Particle Swarm Optimization
RI	Resilience Index
SA	Simulated Annealing
SCAPP	Supply Chain Aggregate Production Planning
SCM	Supply Chain Management
SSCAP	Sustainable Supply Chain Aggregate Production Planning
TFNs	Triangular Fuzzy Numbers
TrFNs	Trapezoidal Fuzzy Numbers
WA	Weighted Average

Chapter 1
Introduction

The introduction chapter serves as the foundation for this research, outlining the essential components that guide the study's purpose, direction, and contributions. It begins with the research background, providing detailed exploration and significance of the problem being investigated, as well as the theoretical and practical challenges in the domain of supply chain management under uncertainty. The problem statement follows, identifying specific gaps, limitations, or inefficiencies in current methodologies and practices, which this research aims to address. Subsequently, the contributions of the research are highlighted, emphasizing the novel insights, methodologies, and practical solutions that this study brings to the field. Finally, the dissertation overview provides a concise summary of the structure and content of this dissertation, guiding readers through the subsequent chapters and their interconnections. This comprehensive introduction ensures a clear understanding of the study's objectives and its importance in advancing both academic knowledge and real-world applications.

1.1 Research Background

The dynamic and uncertain nature of modern supply chains has underscored the critical need for robust and adaptive planning strategies to navigate an increasingly volatile global landscape. Supply Chain Aggregate Production Planning (SCAPP) serves as a strategic framework that focuses on aligning production capacities, resource allocation, and demand forecasts across interconnected stages of supply chain networks. This alignment is vital for optimizing the flow of goods and ensuring operational efficiency. However, traditional SCAPP models often fall short in effectively addressing the complexities introduced by uncertainty, variability, and conflicting objectives, which are inherent in modern supply chain operations.

In an era characterized by frequent disruptions such as natural disasters, geopolitical tensions, and economic instability, businesses face significant challenges in maintaining continuity and meeting customer demands. Traditional models struggle to account for the unpredictable changes in demand, supplier reliability, and market conditions, often resulting in inefficiencies or sub-optimal decisions. The need for innovative approaches that can integrate and manage these uncertainties is paramount.

Fuzzy optimization models have emerged as a transformation solution to these challenges, offering a way to handle the vagueness and ambiguity inherent in supply chain systems. By employing fuzzy logic, these models go beyond traditional deterministic approaches, quantifying uncertainties and incorporating them into the decision-making process. This capability allows for the development of more reliable and adaptable production plans, empowering supply chain managers to respond proactively to variability and disruptions.

This research addresses these pressing issues by developing advanced fuzzy optimization models tailored to improve sustainability, resilience and operational efficiency in SCAPP under uncertain conditions and conflicting objectives. The proposed models enable decision-makers to simultaneously address cost efficiency, risk mitigation, resource optimization and sustainability consideration while adapting to dynamic market conditions. Furthermore, this research contributes significantly to the academic discourse on utilizing advanced mathematical tools to address real-world supply chain challenges. This research also emphasizes the importance of bridging the gap between theoretical advancements and practical applications, equipping supply chain managers with actionable insights and tools to sustain operations under adverse conditions. Ultimately, this research advances the field's understanding of resilience, adaptability, and sustainability in supply chain management, offering a pathway for organizations to thrive in a rapidly changing environment.

1.2 Problem Statement

The dynamic and uncertain nature of modern supply chains has made effective planning and decision-making increasingly complex. Traditional Supply Chain Aggregate Production Planning (SCAPP) models often fail to address the significant challenges posed by variability, uncertainty, and conflicting objectives. These models are typically designed under the assumption of stable market conditions, which do not reflect the realities of frequent disruptions, fluctuating customer demands, and supply chain volatility. This mismatch results in inefficiencies, such as increased costs and reduced responsiveness, undermining the competitiveness and sustainability of organizations.

Uncertainty in SCAPP, deriving from unpredictable events like supplier delays, economic fluctuations, or global crises, exacerbates the difficulty of aligning production capacities with demand forecasts. Additionally, the presence of conflicting objectives, such as minimizing costs while maintaining customer satisfaction, adds further complexity to decision-making. Traditional methods often lack the capability to provide flexible and adaptive solutions that balance these trade-offs effectively.

To address these challenges, innovative approaches are required that integrate uncertainty and conflicting objectives into the planning process. Existing methodologies often fall short in quantifying and incorporating uncertainties into SCAPP, leaving organizations vulnerable to risks and inefficiencies. Without advanced tools and models capable of addressing these issues, supply chain managers face significant difficulties in achieving operational resilience and sustainability.

This study focuses on addressing these critical gaps by developing innovative fuzzy optimization models for SCAPP. These models aim to enhance decision-making under uncertainty and conflicting objectives, equipping organizations with the tools needed to adapt to dynamic market conditions while maintaining cost efficiency, operational continuity, and sustainability.

1.3 Contributions of Research

1. Development of innovative fuzzy optimization models

This research introduces innovative fuzzy optimization models specifically designed for Supply Chain Aggregate Production Planning (SCAPP). These models enhance the representation and management of uncertainty and conflicting objectives, addressing key gaps in traditional methods and enabling more reliable and adaptive planning processes.

2. Enhancing resilience and sustainability

By focusing on adaptability and robustness, the research contributes to the resilience and sustainability of supply chain operations. The models developed in this study provide practical solutions to sustain operations during disruptions and support long-term performance stability.

3. Bridging theory and practice

This research bridges the gap between theoretical advancements and real-world applications by integrating advanced mathematical tools into practical SCAPP models. This research offers actionable insights for industry professionals, making it a valuable resource for both academic and practice.

4. Advancing knowledge in supply chain optimization

This research adds to the academic discourse on supply chain optimization by introducing innovative methodologies and demonstrating their effectiveness through empirical validation. It sets a foundation for future studies to further refine and expand upon these models.

1.4 Dissertation Overview

Besides this introduction chapter, the remaining chapters of this dissertation are organized as follows:

- Chapter 2: Literature Review

This chapter reviews key literature relevant to Supply Chain Aggregate Production Planning (SCAPP), focusing on fundamental concepts, sustainability, uncertainty, and conflicting objectives. It begins with an overview of supply chain management, providing a foundation for understanding SCAPP and its role in optimizing resource allocation and production planning. The discussion then extends to sustainability, emphasizing the integration of environmental and social considerations in Sustainable Supply Chain Aggregate Production Planning (SSCAP). This chapter further explores uncertainty as a critical challenge in SCAPP, examining its sources and the associated risks that impact decision-making. Finally, conflicting objectives are analyzed, highlighting the trade-offs between cost minimization, efficiency, sustainability, and risk management. By synthesizing these topics, this chapter establishes a strong foundation for understanding the complexities of SCAPP.

- Chapter 3: Fundamentals of Resolving Uncertainty in Supply Chain Management

This chapter explores the fundamental concepts and methodologies used to address uncertainty in supply chain management. It begins with an overview of Fuzzy Set Theory and its application to handle imprecision and vagueness inherent in supply chain systems. The chapter then explores fuzzy numbers and their role in representing uncertain parameters, followed by an introduction to Fuzzy Linear Programming (FLP) as a tool for optimizing supply chain decisions under uncertainty. Further, this chapter examines various techniques for defuzzification, which convert fuzzy outputs into actionable, precise values. Optimization approaches are explored next, offering insights into strategies for effective decision-making in uncertain environments. Additionally, this chapter covers risk measurement methodologies to assess potential negative outcomes and introduces Monte Carlo Simulation as a powerful tool for modeling uncertainty. Lastly, this chapter discusses the Resilience Index (RI), a metric used to evaluate the robustness and adaptability of supply chains in the face of uncertainty and disruptions. Together, these topics provide a comprehensive framework for understanding and managing uncertainty in supply chain operations.

- Chapter 4: A Fuzzy Multi-Criteria Decision-Making for Optimizing Supply Chain Aggregate Production Planning based on Cost Reduction and Risk Mitigation

This chapter introduces an integrated approach for optimizing Supply Chain Aggregate Production Planning (SCAPP) by employing a Fuzzy Multi-Criteria Decision-Making (FMCDM) framework. It is designed to address two critical and often conflicting objectives in supply chain management; cost reduction and risk mitigation, under conditions of uncertainty that are prevalent in real-world applications. By harnessing the capabilities of fuzzy logic, the study aims to assist decision-makers in developing robust and efficient production plans across the supply chain network. This chapter begins with a detailed description of the problem, emphasizing the operational complexities and uncertainties that necessitate a multi-criteria decision-making approach. It also outlines the principal contributions of the study, highlighting the novelty and practical significance of the proposed model. The methodology adopted to tackle the problem is then presented, integrating fuzzy set theory and optimization techniques to effectively manage ambiguity in data and decision preferences. A real-world case study is introduced to demonstrate the practical application and effectiveness of the proposed framework. Subsequently, this chapter presents a comprehensive explanation of the mathematical notations and the formulation of the fuzzy optimization model. The results are discussed in detail, including a comparison with the traditional fuzzy optimization approach and a validation of outcomes to evaluate the model's reliability and performance. This chapter concludes with a discussion of the findings, the advantages of applying proposed approach, and suggestions for future research. Overall, this chapter seeks to bridge the gap between theoretical modeling and practical implementation, offering a structured and adaptable tool for improving supply chain performance under uncertainty.

- Chapter 5: A New Integrated Fuzzy Optimization Approach for Sustainable Supply Chain Planning subjected to Sustainability and Uncertain Environments

This chapter presents a novel integrated fuzzy optimization approach designed to enhance sustainable supply chain planning in the face of increasing complexity and uncertainty. The proposed framework simultaneously addresses economic, environmental, and social dimensions of sustainability while incorporating the imprecise and ambiguous nature of real-world supply chain environments. This chapter begins with a comprehensive problem description that highlights the challenges of aligning sustainability goals with supply chain performance under uncertain conditions. The main contributions of the study are then outlined, emphasizing the development of an innovative model that integrates fuzzy logic with sustainability-driven decision-making. The methodology section details the techniques and tools employed to construct the proposed model, incorporating fuzzy set theory and multi-objective optimization to address uncertainty and competing priorities. A real-world case study is introduced to demonstrate the applicability and effectiveness of the proposed approach in a practical context. This is followed by a formal presentation of the mathematical notations and the formulation of the fuzzy optimization model. The results are analyzed in depth, including comparisons with traditional models, validation of the findings, and an exploration of the social and environmental implications of the planning decisions. Finally, this chapter concludes with a discussion of key insights, contributions to the field of sustainable supply chain management, and recommendations for future research. Through this comprehensive approach, this chapter aims to contribute to the development of more resilient, responsible, and adaptable supply chain systems in uncertain and sustainability-sensitive environments.

- Chapter 6: Advanced Fuzzy Mathematical Modeling with Monte Carlo Simulation: A Comprehensive Framework for Analyzing Supply Chain Aggregate Production Planning under Uncertain Environments

This chapter introduces an advanced analytical framework that integrates fuzzy mathematical modeling with Monte Carlo simulation to address the complexities of Supply Chain Aggregate Production Planning (SCAPP) under uncertainty. Recognizing the limitations of conventional deterministic models in capturing the inherent vagueness and variability in supply chain environments, this study proposes a robust approach that combines the strengths of fuzzy set theory and Monte Carlo simulation. This chapter begins with a detailed description of the problem, emphasizing the challenges posed by imprecise data. It further outlines the main contributions of the research, highlighting the innovation in blending fuzzy optimization with probabilistic simulation to enhance decision-making accuracy and reliability. The methodology section presents fuzzy optimization approach and Monte Carlo simulation in detail, explaining how both techniques are used in tandem to model uncertainty and evaluate alternative production planning strategies. A real-world case study is included to demonstrate the practical applicability and effectiveness of the proposed framework. This is followed by a formal presentation of the mathematical notations and the model formulation, which provide the foundation for the subsequent analysis. The results section presents key findings, including a comparative evaluation, to validate the performance and benefits of the model.

Finally, this chapter concludes with a discussion of the implications of the results, the advantages of integrating fuzzy modeling with simulation techniques, and directions for future research. This chapter aims to offer a comprehensive and adaptable solution for improving the resilience and efficiency of SCAPP in uncertain supply chain environments.

- Chapter 7: Fuzzy Optimization with Resilience Metrics for Sustainable Supply Chain Planning under Uncertain and Disruption Environments

This chapter presents a novel fuzzy optimization framework that incorporates resilience metrics to support sustainable supply chain planning in environments characterized by uncertainty and disruption. As modern supply chains face increasing exposure to risks such as natural disasters, geopolitical tensions, and global pandemics, the ability to maintain operational continuity while adhering to sustainability goals has become critically important. This study addresses these challenges by integrating fuzzy logic to handle imprecise information and embedding resilience metrics to evaluate the supply chain's capacity to absorb and recover from disruptions. This chapter begins with a detailed problem description, outlining the complexity of achieving sustainability in the presence of unpredictable and disruptive events. It also highlights the main contributions of the research, particularly the development of a resilience-oriented fuzzy optimization model that offers both theoretical advancement and practical utility. The methodology section elaborates on the approach used to construct the model, combining fuzzy set theory with resilience assessment techniques to support robust decision-making. A case study is presented to demonstrate the applicability of the proposed model in a real-world context, followed by a comprehensive explanation of the mathematical notations and model formulation. The results are then analyzed, including comparative assessments to validate the model's performance and effectiveness. This chapter concludes with a discussion of the findings, emphasizing the benefits of integrating resilience into sustainable supply chain planning, and offers suggestions for future research. Overall, this chapter aims to enhance the strategic capabilities of supply chain systems by promoting resilience and sustainability under uncertain and disruptive conditions.

- Chapter 8: Discussion and Conclusion

This final chapter provides a comprehensive discussion of the findings and conclusions drawn from the preceding chapters. It synthesizes the key insights gained from the research, offering a critical analysis of the proposed models and methodologies in the context of real-world supply chain challenges. This chapter begins by highlighting the managerial implications of the study, focusing on how the proposed fuzzy optimization approaches can be applied to improve decision-making in supply chain planning, particularly under uncertain and disruptive conditions. Then, the limitations of the study are provided. Subsequently, the limitations of the study are discussed in various aspects. This chapter concludes with recommendations for future research, suggesting areas for further exploration and improvement to enhance the applicability of the proposed frameworks in supply chain contexts. By reflecting on both the strengths and limitations of the study, this chapter aims to provide a balanced perspective on the contributions of the research and its potential impact on the field of supply chain management.

Chapter 2
Literature Review

This chapter reviews key literature relevant to Supply Chain Aggregate Production Planning (SCAPP), focusing on fundamental concepts, sustainability, uncertainty, and conflicting objectives. It begins with an overview of supply chain management, providing a foundation for understanding SCAPP and its role in optimizing resource allocation and production planning. The discussion then extends to sustainability, emphasizing the integration of environmental and social considerations in Sustainable Supply Chain Aggregate Production Planning (SSCAP). This chapter further explores uncertainty as a critical challenge in SCAPP, examining its sources and the associated risks that impact decision-making. Finally, conflicting objectives are analyzed, highlighting the trade-offs between cost minimization, efficiency, sustainability, and risk management. By synthesizing these topics, this chapter establishes a strong foundation for understanding the complexities of SCAPP.

2.1 Supply Chain

A supply chain is a dynamic and complex network of interconnected entities, resources, activities, and technologies collaboratively working to create, produce, and deliver products or services from their point of origin to the final customer. It encompasses a wide range of processes, starting from the procurement of raw materials and extending through manufacturing, distribution, and retail, ultimately concluding with the delivery of finished goods or services to end-users [1][2].

According to Figure 2.1, the supply chain includes several key components, such as suppliers who provide the raw materials, manufacturers who convert these materials into finished products, distributors and wholesalers who handle bulk deliveries, retailers who make the products accessible to customers, and logistics providers who ensure seamless transportation and storage of goods throughout the process. Beyond the physical flow of goods, a supply chain also involves efficient management of information and finances to synchronize operations, maintain transparency, and optimize decision-making across all levels [3].

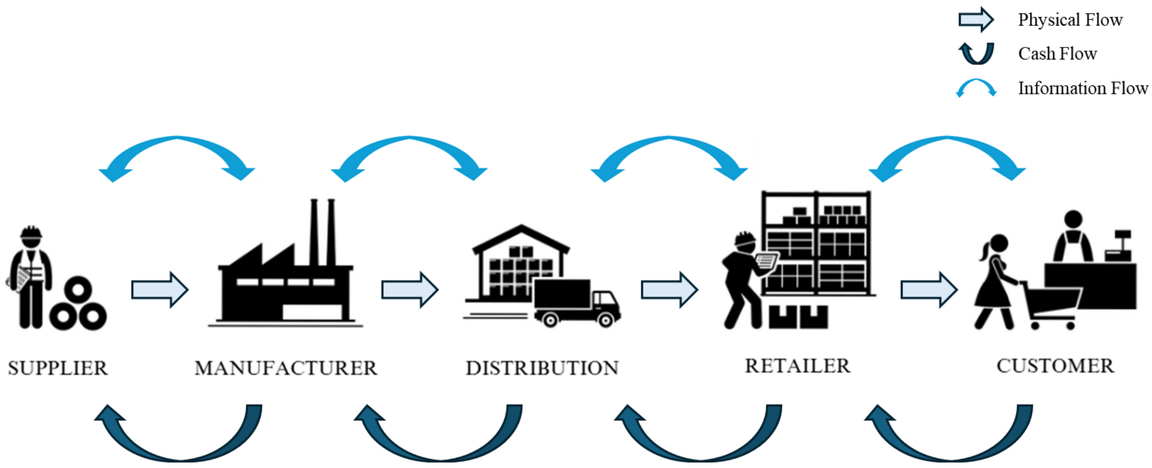

Figure 2.1: The structure of a typical supply chain.

Effective Supply Chain Management (SCM) is critical for achieving operational excellence. It seeks to optimize the interconnected processes to enhance productivity, reduce operational costs, and improve overall customer satisfaction [4]. By streamlining supply chain activities and utilizing advanced technologies, SCM contributes to increasing the responsiveness of the supply chain to fluctuating market demands. Furthermore, a well-managed supply chain fosters competitiveness by enabling organizations to deliver high-quality products or services promptly and cost-effectively, positioning them as leaders in their respective markets.

In today's globalized and digitally interconnected world, supply chain management also integrates strategies for risk mitigation, sustainability, and resilience. This involves addressing uncertainties such as supply disruptions, geopolitical challenges, and environmental considerations while ensuring ethical practices and compliance with international standards. By adopting innovative solutions like predictive analytic, artificial intelligence, and automation, organizations can further enhance the agility and reliability of their supply chains, ensuring long-term success in an increasingly competitive landscape [5][6].

2.2 Supply Chain Aggregate Production Planning

Supply Chain Aggregate Production Planning (SCAPP) is a strategic process that focuses on developing, analyzing, and maintaining a high-level schedule for the operations of an organization. It integrates production and inventory decisions to effectively meet anticipated demand over a specified planning horizon [7]. SCAPP plays a vital role in ensuring that production capacity, resources, and operational goals are harmonized across multiple stages of the supply chain, enabling organizations to deliver products and services efficiently while minimizing costs and maintaining high customer satisfaction [8].

The scope of SCAPP encompasses a wide range of interdependent decisions, including demand forecasting, inventory management, production scheduling, resource allocation, and capacity planning. By aligning these critical areas, SCAPP ensures that resources are utilized effectively, production activities are streamlined, and the supply chain operates at optimal efficiency. The aim is to balance cost minimization with the need for flexibility and reliability in meeting customer demand, creating a robust operational framework that supports long-term competitiveness.

Given the inherent complexity and interconnection of supply chains, SCAPP often involves addressing uncertainties such as fluctuating customer demand, variable lead times, and unpredictable resource availability. To tackle these challenges, sophisticated modeling techniques are employed, including fuzzy optimization, stochastic programming, robust optimization, and other advanced mathematical approaches. These methodologies enable decision-makers to incorporate variability and uncertainty into their planning processes, resulting in more resilient and adaptive production strategies [9][10].

In addition to addressing uncertainty, SCAPP integrates multiple performance objectives, such as maximizing resource utilization, minimizing waste, reducing lead times, and ensuring sustainability. Modern SCAPP approaches leveraging cutting-edge technologies, including artificial intelligence, machine learning, and data analytics, to enhance forecasting accuracy, identify inefficiencies, and adapt quickly to market changes [11][12]. By providing a comprehensive and adaptive framework, SCAPP enables organizations to achieve a competitive advantage while maintaining cost-effective and reliable operations.

The importance of SCAPP has grown significantly in today's globalized and highly competitive marketplace. As supply chains become more complex and customer expectations increase, organizations must develop aggregate production plans that not only address operational goals but also account for external factors such as supply chain disruptions, environmental considerations, and regulatory requirements [13]. By implementing effective SCAPP practices, organizations can ensure the seamless alignment of supply chain activities, optimize performance, and achieve long-term sustainability and success.

2.3 Sustainability

Sustainability refers to the practice of meeting the needs of the present generation without compromising the ability of future generations to meet their own needs. It encompasses a balance among economic, environmental, and social objectives, often referred to as the triple bottom line [14][15]. In the context of businesses and industries, sustainability involves adopting practices that reduce environmental impact, promote social responsibility, and ensure long-term economic viability. Key principles of sustainability include resource conservation, waste reduction, emissions control, ethical labor practices, and fostering community well-being [16]. As global awareness of climate change and resource depletion grows, sustainability has become a critical factor in organizational decision-making, influencing strategies, policies, and practices across various sectors.

Sustainability is built upon three foundational pillars: economic sustainability, environmental sustainability, and social sustainability [17]. These pillars, often referred to as the triple bottom line, represent the interconnected dimensions of sustainable development. Together, they provide a comprehensive framework for balancing growth, resource conservation, and societal well-being as shown in Figure 2.2.

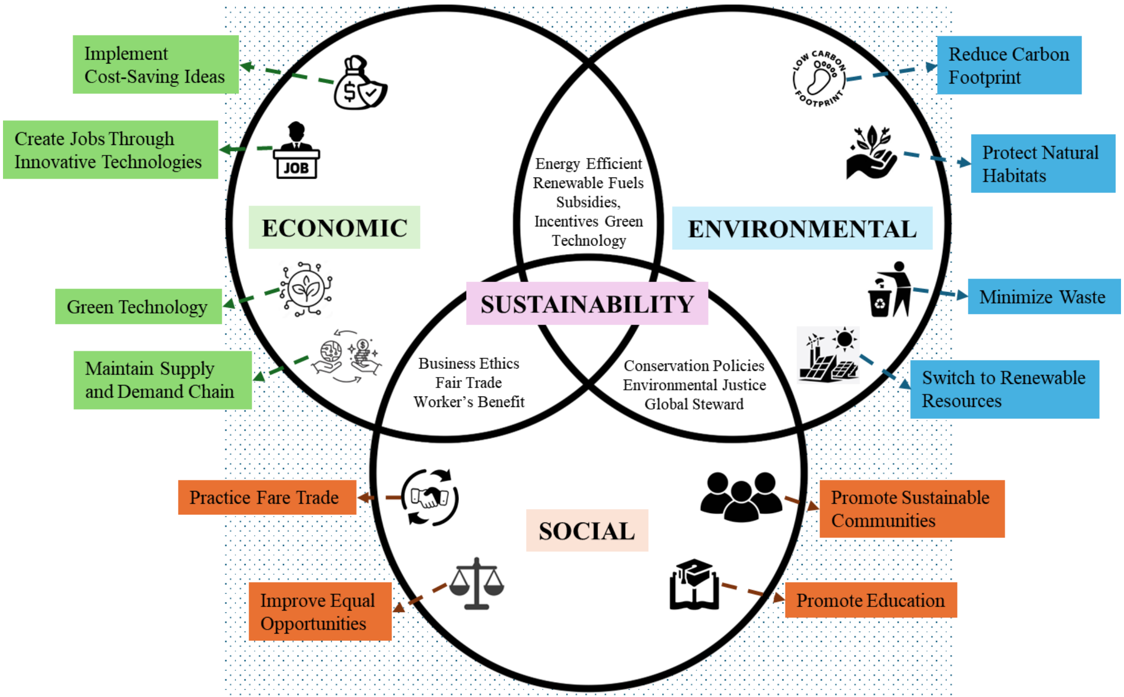

Figure 2.2: The three pillars of sustainability.

Sustainability is built upon three interdependent pillars: economic, environmental, and social sustainability, each essential for achieving long-term global well-being. Economic sustainability emphasizes efficient resource management to ensure profitability and financial stability without compromising future generations, advocating for equitable growth, innovation, and resilience. Environmental sustainability focuses on protecting and restoring natural ecosystems, minimizing waste and pollution, conserving biodiversity, and promoting renewable energy use to safeguard the planet's resources. Social sustainability highlights the importance of equity, inclusion, and community well-being, addressing poverty, access to education and healthcare, and the promotion of human rights and cultural diversity. For sustainability to be truly effective, these pillars must function cohesively, aligning economic goals with environmental stewardship and social equity to foster a balanced and enduring system that supports prosperity, conservation, and social advancement.

Sustainable Supply Chain Aggregate Production Planning (SCAPP) is an advanced production planning approach that incorporates sustainability principles into the strategic alignment of production capacity and resource allocation across the supply chain. In contrast to traditional SCAPP, which focuses primarily on cost efficiency and demand fulfillment, sustainable SCAPP also considers the environmental and social impacts of production and distribution. It emphasizes practices such as reducing carbon emissions, optimizing resource use, minimizing waste, and upholding ethical labor standards [18]. By leveraging innovative techniques and technologies such as green logistics, renewable energy, circular economy principles, and eco-friendly materials, sustainable SCAPP aims to reduce the ecological footprint of supply chain operations [19]. It also involves sophisticated decision-making processes that model trade-offs between conflicting objectives, such as cost reduction and emission control, under uncertain and variable conditions [20].

By implementing sustainable SCAPP, organizations can enhance their resilience, improve stakeholder relationships, and align with global sustainability standards, such as the United Nations' Sustainable Development Goals. Furthermore, it fosters long-term competitiveness by addressing growing consumer demand for environmentally and socially responsible products, reducing regulatory risks, and ensuring supply chain stability in the face of environmental and social challenges. Sustainable SCAPP is not only a necessity for organizations seeking to thrive in a rapidly evolving global landscape but also a key contributor to a sustainable future for the planet.

2.4 Uncertainty

The concept of uncertainty spans across various disciplines, each offering its distinct interpretations and applications, which derives from the unique contexts within which uncertainty arises. Fields such as the physical sciences, engineering, statistics, economics, finance, insurance, philosophy, and psychology each contribute their own understanding of uncertainty. In the physical sciences and engineering, uncertainty typically refers to measurement errors, inconsistencies, and variability in experimental outcomes, which can affect the reliability of data and predictions. In statistics, uncertainty is closely tied to the probabilistic nature of data, inference, and the variability of estimates [21]. Within economics and finance, uncertainty is often discussed in terms of market fluctuations, price volatility, and the risks associated with financial decisions, such as investment strategies [22]. Insurance, on the other hand, approaches uncertainty through the lens of risk assessment, actuarial science, and the estimation of the likelihood of events occurring. Philosophers explore the epistemological aspects of uncertainty, pondering what can be known with certainty and the limits of human knowledge, often raising questions about the very nature of knowledge itself [23]. Meanwhile, psychology examines the emotional and cognitive responses of individuals to uncertainty, particularly how people perceive and manage uncertainty in their decision-making processes, as well as the psychological impact of unpredictable outcomes [24].

The phenomenon of uncertainty arises from a complex interplay of various factors, largely deriving from a lack of understanding, incomplete knowledge, insufficient data, and the inherent variability found within natural processes. These underlying causes create an environment in which precise predictions become difficult, and reliable assessments are often elusive. Uncertainty is further exacerbated by three primary sources, as identified by Lawrence & Lorsch (1967) [25] and Duncan (1972) [26]. Uncertainty arises in several ways, particularly when critical information about environmental factors is inaccessible, hindering a full understanding of external influences. Without knowledge of key market trends, regulatory changes, or competitor strategies, businesses face challenges in navigating dynamic environments and forecasting future developments. Additionally, uncertainty emerges when the expected outcomes of decisions are unclear or ambiguous, especially in complex situations with multiple potential scenarios that carry varying risks and rewards. This lack of clarity undermines effective strategy formation and planning. Lastly, uncertainty intensifies when it becomes impossible to quantify confidence in each scenario. In such cases, the inability to assign probabilities or confidence levels to outcomes destabilizes decision-making, making it reactive rather than proactive, and further exacerbating the instability of the environment.

Within the context of business and supply chain operations, uncertainty arises from two principal sources: environmental uncertainty and system uncertainty [27]. Environmental and system uncertainty are two key factors that influence supply chain operations. Environmental uncertainty refers to external factors, such as supplier performance, market trends, shifts in consumer behavior, and economic conditions, which can cause fluctuations in supply and demand. These unpredictable variables make it challenging for businesses to forecast demand and align their operations accordingly, necessitating flexible and adaptive strategies. System uncertainty arises from internal processes within the organization, such as operational inefficiencies, technological failures, human errors, and poor coordination between departments. These internal challenges can disrupt smooth business functioning, leading to delays, bottlenecks, and increased operational costs. Together, environmental and system uncertainties create a volatile business environment, requiring organizations to implement robust risk management strategies, leverage data-driven insights, and maintain operational flexibility to adapt to both external and internal pressures and ensure long-term success.

Uncertainty, as a broad and multifaceted concept, can be further categorized into four distinct types: data uncertainty, model uncertainty, parameter uncertainty, and scenario uncertainty [28]. Each of these types arises from different sources and presents unique challenges for decision-making and risk management. Understanding these categories is essential for organizations to develop comprehensive strategies to address uncertainty in complex and dynamic environments. Data, model, parameter, and scenario uncertainties are key factors influencing decision-making in dynamic environments. Data uncertainty arises from incomplete, outdated, or inaccurate data, which can lead to flawed decisions, such as stockouts or inefficiencies in resource allocation [29]. Addressing this requires improved data collection and validation techniques. Model uncertainty pertains to the simplifications or assumptions inherent in mathematical models, which may not fully capture real-world complexities, leading to inaccurate predictions [30]. Refining models and conducting sensitivity analyses can help mitigate this uncertainty. Parameter uncertainty arises from imprecise estimates of model parameters, such as demand rates or production costs, which are subject to variability [31]. Techniques like sensitivity analysis or robust optimization can address this uncertainty by quantifying potential outcomes. Scenario uncertainty refers to unpredictable external factors, such as market shifts or regulatory changes, that affect future outcomes [32]. Scenario planning, simulation, and forecasting are used to prepare for various possible futures. Understanding and managing these uncertainties through tailored strategies enables organizations to make informed, resilient decisions in uncertain and complex environments.

The concept of risk, as discussed by Rachev et al. (2011) [33], is closely linked to uncertainty and exposure, forming a crucial aspect of decision-making in unpredictable environments. Risk is viewed as a subjective phenomenon, arising when uncertainty exists, and individuals or organizations are exposed to potential adverse outcomes. Rachev et al. (2011) defines risk as consisting of two components: the inherent uncertainty of future events and the exposure to negative consequences that may result from these uncertain conditions. This understanding underscores the importance of considering uncertainty when managing risk, as it introduces variability and unpredictability into decision-making processes. Uncertainty leads to multiple potential outcomes, each with varying probabilities, generating both favorable and unfavorable risks [34]. The subjective nature of risk means that different individuals or organizations may perceive and respond to the same level of uncertainty in different ways, depending on their experiences and risk tolerance [35]. Effective risk management, therefore, requires a dual approach: quantifying uncertainty and evaluating its potential impact on decision-making. Various risk measurement techniques have been developed to help decision-makers understand and mitigate risks, providing a structured framework for making more informed and reliable strategic choices.

2.5 Conflicting Objective

Multiple objective functions refer to optimization problems where decision-makers are required to simultaneously achieve more than one goal. In these types of problems, there are typically two or more objectives that may either be complementary or, more often, conflicting. As a result, decision-makers must make trade-offs between these objectives, as improving one may lead to the deterioration of another. These objectives are usually formulated as mathematical functions that need to be either maximized or minimized, covering a range of criteria such as profit, cost, efficiency, time, risk, quality, or sustainability.

The challenge in multi-objective optimization arises from the inherent tension between conflicting goals. For example, a business might aim to maximize profit while minimizing costs, but the strategies to achieve these goals might not align perfectly, requiring a decision on how to strike the right balance between them. Solving multi-objective optimization problems often means finding a set of solutions that offer the best trade-offs, where no single objective can be improved without sacrificing the performance of others. This requires advanced methods and tools for analysis, as well as a deep understanding of how different objectives interact with one another. A solution that works well for one objective may be detrimental to another, so the objective is not to optimize each one independently but to find a feasible solution that satisfies all objectives to a satisfactory degree. This approach is critical across many sectors, from manufacturing and logistics to finance and healthcare [36][37].

In the specific context of supply chain aggregate production planning (SCAPP), multiple objective functions are optimization models that aim to achieve multiple, often conflicting, goals related to production and resource allocation within the supply chain. SCAPP involves managing a variety of interconnected tasks such as determining optimal production quantities, scheduling, inventory management, and distribution. Common objectives in SCAPP include minimizing production costs, maximizing customer service levels, optimizing inventory levels, reducing lead times, and minimizing environmental impacts. These goals often conflict with one another, such as minimizing costs while ensuring high levels of service or reducing energy consumption while maintaining productivity.

The inclusion of multiple objectives in SCAPP is crucial because it provides a more holistic view of the decision-making process, acknowledging that supply chain efficiency is not just about cutting costs but also about optimizing other performance metrics. These metrics may include service levels (e.g., ensuring products are available to customers when needed), environmental sustainability (e.g., reducing carbon emissions or energy consumption), or operational efficiency (e.g., reducing idle times or waste). Incorporating these multiple objectives helps managers make more balanced decisions that consider the broader implications for the entire supply chain, rather than focusing solely on financial outcomes [38][39].

Conflicting objectives refer to goals in decision-making and optimization problems that cannot be achieved simultaneously without one or more objectives being compromised. This conflict arises because improving one objective often leads to the deterioration of another, compelling decision-makers to weigh and prioritize different factors. The presence of conflicting objectives is a common challenge in many fields, from engineering and finance to supply chain management and project planning. Each of these fields often involves multiple stakeholders, each with different priorities and constraints, further complicating the decision-making process.

In product design, for example, there is often a conflict between optimizing cost efficiency and ensuring high product performance or aesthetic quality. Reducing the cost of materials or simplifying the design to lower production costs may compromise the product's functionality, durability, or visual appeal. Similarly, in supply chain management, balancing the trade-offs between cost, service level, and lead time is a classic example of conflicting objectives. Lowering transportation costs might result in longer delivery times, which could affect customer satisfaction or order fulfillment rates. These conflicts can also arise in broader project planning scenarios, where project timelines might need to be balanced with resource constraints and cost limitations. For instance, accelerating the timeline of a project by increasing resources may drive up costs, potentially conflicting with budgetary constraints. In these cases, identifying an optimal solution involves understanding how different objectives interact and finding a way to make the most acceptable compromise between them.

In multi-objective optimization, the goal is not necessarily to find a single solution that perfectly satisfies all objectives, but to identify a set of potential solutions that represent the best trade-offs. These solutions, often visualized through the concept of the Pareto front, reflect different combinations of performance across the objectives, where no objective can be improved without making another worse. By presenting decision-makers with these alternatives, they can better understand the impacts of their choices and select the solution that best aligns with their priorities, constraints, and strategic goals. This approach helps stakeholders navigate the complexities of multi-dimensional decision-making, enabling them to make informed and balanced decisions that achieve optimal outcomes in a dynamic, interconnected environment [40][41].

Chapter 3
Fundamentals of Resolving Uncertainty in Supply Chain Management

This chapter explores the fundamental concepts and methodologies used to address uncertainty in supply chain management. It begins with an overview of Fuzzy Set Theory and its application to handle imprecision and vagueness inherent in supply chain systems. The chapter then explores fuzzy numbers and their role in representing uncertain parameters, followed by an introduction to Fuzzy Linear Programming (FLP) as a tool for optimizing supply chain decisions under uncertainty. Further, this chapter examines various techniques for defuzzification, which convert fuzzy outputs into actionable, precise values. Optimization approaches are explored next, offering insights into strategies for effective decision-making in uncertain environments. Additionally, this chapter covers risk measurement methodologies to assess potential negative outcomes and introduces Monte Carlo Simulation as a powerful tool for modeling uncertainty. Lastly, this chapter discusses the Resilience Index (RI), a metric used to evaluate the robustness and adaptability of supply chains in the face of uncertainty and disruptions. Together, these topics provide a comprehensive framework for understanding and managing uncertainty in supply chain operations.

3.1 Fuzzy Set Theory

Fuzzy logic, a pivotal area of modern computational intelligence, has its roots in the groundbreaking work of Lotfi Zadeh in the 1960s [42], when he introduced the concept of fuzzy sets as a way to mathematically represent and manage uncertainty. His work laid the foundation for a revolutionary approach to reasoning that accommodates imprecision and ambiguity, contrasting sharply with traditional binary logic, which relies on clear-cut distinctions between true and false. Since Zadeh's introduction of fuzzy logic, it has gained widespread recognition and adoption across various domains, owing to its ability to bridge the gap between human-like reasoning and computational systems. By embracing the inherent fuzziness of the real world, fuzzy logic allows systems to function in environments where information is incomplete, imprecise, or uncertain, making it an essential tool for developing adaptive, intelligent systems. As technology progresses, the significance of fuzzy logic in shaping artificial intelligence, decision-making, and human-machine interaction is expected to increase, reinforcing its status as a cornerstone of modern computational science.

Fuzzy logic transcends the rigid, deterministic world of traditional binary logic by incorporating the concept of partial truth. Instead of adhering to the dichotomy of true or false, fuzzy logic introduces a spectrum of truth values, allowing for degrees of truth ranging between 0 and 1. This flexibility makes it a powerful framework for modeling and processing imprecise information, closely mirroring the way humans naturally reason and make decisions. In practical terms, fuzzy logic utilizes fuzzy sets, linguistic variables, and fuzzy rules to formalize and quantify qualitative knowledge. These features enable fuzzy systems to accommodate the complexity and uncertainty found in real-world situations, providing a robust mechanism for decision-making and problem-solving across a wide range of applications, from industrial control systems to healthcare and finance.

The origins of fuzzy set theory can be traced back to Zadeh's seminal work in 1965, which revolutionized the way we think about logic and reasoning. Prior to this, traditional logic systems operated on crisp binary values, where propositions could only be true or false. In contrast, fuzzy set theory allowed truth values to be expressed as intermediate values between these extremes, thereby embracing the ambiguous nature of real-world phenomena. This development reflected a shift in how uncertainty and imprecision were viewed, recognizing that many aspects of human cognition and decision-making involve degrees of certainty rather than absolute knowledge. By introducing partial truth, fuzzy set theory provided a powerful tool for addressing complex systems where precise categorization or exact measurements are unattainable, thus creating a more realistic, adaptable computational framework.

One of the key milestones in the integration of fuzzy logic into optimization and decision-making processes was achieved by Zimmermann in 1976, when he applied fuzzy set theory to Linear Programming (LP) problems, marking a significant advancement in optimization techniques under uncertainty [43]. Zimmermann's approach introduced fuzzy goals and constraints into LP models, allowing decision-makers to account for uncertainty and imprecision when formulating optimization problems. This was a major step forward, as traditional optimization models often assume precise values for all variables and parameters, which is unrealistic in many practical scenarios. Building on Zimmermann's work, Benjamin Bellman and Lotfi Zadeh further extended these concepts by developing fuzzy decision models with linear membership functions, demonstrating the equivalence between these fuzzy decision models and traditional LP problems [44]. This breakthrough not only expanded the scope of fuzzy logic but also highlighted its versatility in providing decision-makers with a powerful framework to navigate uncertainty and complexity in real-world systems.

Fuzzy set theory is concretely represented through mathematical constructs such as Triangular Fuzzy Numbers (TFNs) and Trapezoidal Fuzzy Numbers (TrFNs), which are commonly used to model uncertainty in various applications. Triangular Fuzzy Numbers are defined by three parameters: (a, b, c) where $a \leq b \leq c$. These parameters represent the lower bound, the most likely value, and the upper bound of a fuzzy number, respectively [45][46]. The membership function of a TFN rises linearly from to and then falls linearly from to, representing a distribution of possible values centered around the most likely value, with diminishing confidence as values move further away from it. TFNs are particularly useful for modeling uncertain values that are expected to be close to a central value with some degree of variation, making them ideal for scenarios where imprecision is prevalent.

Trapezoidal Fuzzy Numbers, on the other hand, extend the concept of TFNs by allowing for a broader range of possible values and greater flexibility in representing uncertainty [47][48]. TrFNs are defined by four parameters: (a, b, c, d), where a ≤ b ≤ c ≤ d. The membership function for a TrFN remains constant at a value of 1 between the parameters b and c, indicating complete membership, and then linearly decreases from 1 to 0 as values move away from this central interval towards a and d. This structure allows TrFNs to model situations where a certain range of values is more likely or acceptable, with membership gradually diminishing as the value moves outside of this range. Trapezoidal fuzzy numbers are widely used in applications where the uncertainty spans a wider range, such as in risk assessment, production planning, and scenario modeling.

Together, these fuzzy constructs provide a flexible and adaptive approach to modeling uncertainty, making fuzzy logic an indispensable tool for decision-making and optimization in complex, dynamic environments. By incorporating the ability to handle partial truths and uncertain data, fuzzy logic systems can more accurately reflect the imprecision inherent in real-world problems, offering significant advantages over traditional approaches based on crisp, binary logic.

3.2 Fuzzy Number

A fuzzy number is a specialized form of a fuzzy set defined on the real number line, where each element within the set is associated with a degree of membership that lies within the interval [0, 1]. This degree of membership represents the extent to which a particular value belongs to the fuzzy number, reflecting its relative truth or certainty [49]. In contrast to classical numbers, which are crisp and possess precise, well-defined values, fuzzy numbers are inherently flexible and are designed to model uncertainty, vagueness, or imprecision. They are particularly useful in situations where exact data is unavailable or where ambiguity is an inherent aspect of the problem.

Fuzzy numbers are characterized by a membership function, a mathematical representation that assigns a degree of membership to each possible value in the domain. This function typically exhibits certain properties, such as being normalized (where at least one value has a membership of 1, indicating full membership) and possessing a convex shape, which reflects the gradual transition from higher to lower membership degrees. Common types of fuzzy numbers include triangular fuzzy numbers and trapezoidal fuzzy numbers, each defined by their unique membership functions.

These characteristics make fuzzy numbers a powerful tool in modeling and solving real-world problems involving imprecise or subjective information. They find applications across various domains, such as decision-making, optimization, control systems, and artificial intelligence, where they help to capture and process human reasoning and uncertainty. For example, in supply chain planning, fuzzy numbers can represent uncertain demand or fluctuating production costs, enabling more robust and flexible planning solutions [50].

Definition 2.1: A fuzzy number $\widetilde{A}$ is a convex and normalized fuzzy set on the set of real numbers *R* such that:

1. **There exists at least one $x_0 \in R$ with $\mu_{\widetilde{A}}(x_0) = 1$.**
   This condition ensures that the fuzzy number reaches full membership at some value, meaning this value is the most representative or plausible value of the fuzzy number. This point is often referred to as the mode of the fuzzy number and corresponds to the peak of the membership function.

2. **$\mu_{\widetilde{A}}(x)$ is piecewise continuous.**
   This means that the function doesn't jump abruptly. It is either continuous or composed of continuous segments. This property ensures that the fuzzy number changes gradually and smoothly over its range, which is important for modeling imprecise values realistically.

Assume that the membership function of a fuzzy number $\widetilde{A}$ is given as follows:

$$\mu_{\tilde{A}}(x) = \begin{Bmatrix} 1 - \frac{a^m - x}{a^m - a^o}, & a^o \le x \le a^m \\ 1 - \frac{x - a^m}{a^P - a^m}, & a^m \le x \le a^p \\ 0, & otherwise \end{Bmatrix} \quad (3.1)$$

This function defines Triangular Fuzzy Numbers (TFNs), which is a simple effective way to represent uncertainty value.

- $a^L = a^m - a^o$: The left spread, indicating how far the fuzzy number extends to the left of the mode (toward smaller values). It controls the rate of increase in membership value from 0 up to 1.
- a^m: The mode, or the most plausible value. It's the point where the fuzzy number reaches maximum membership (value of 1).
- $a^U = a^m + a^p$: The right spread, indicating how far the fuzzy number extends to the right of the mode (toward larger values). It controls the rate of decrease in membership from 1 down to 0.

Triangular Fuzzy Numbers (TFNs) are a fundamental concept in fuzzy set theory, widely used to represent uncertainty, imprecision, and vagueness in real-world decision-making problems. A TFN is characterized by a triangular-shaped membership function defined by three parameters: the lower bound ($a^L = a^m - a^o$), the mode or peak (a^m), and the upper bound ($a^U = a^m + a^p$). These parameters collectively define the range and shape of the fuzzy number, providing a simple effective means of modeling uncertain or imprecise quantities. The membership function of a TFN is a continuous, piecewise-linear curve. It increases linearly from zero at the lower bound to one at the mode, then decreases linearly back to zero at the upper bound as shown in Figure 3.1. This gradual progression reflects the inherent fuzziness of human perception and reasoning, where truth values are not binary but vary smoothly. The intuitive shape of the triangular function allows for easy interpretation and straightforward implementation, making TFNs especially suitable for applications requiring simplicity without sacrificing the ability to model uncertainty.

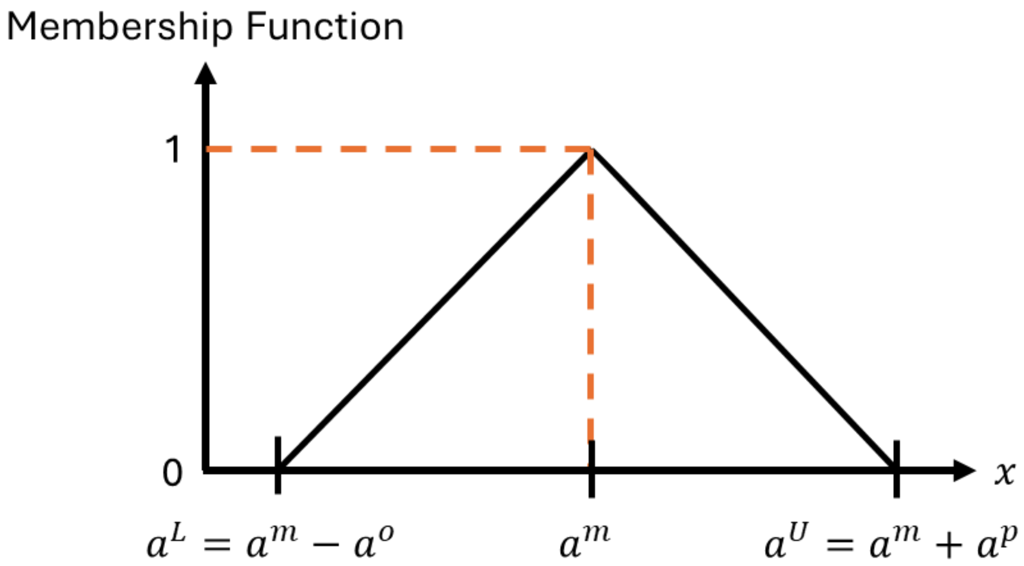

Figure 3.1. Triangular distribution.

Dubois et al. (2004) described the triangular distribution as an upper envelope that encompasses all probability distributions derived from symmetric Probability Density Functions (PDF) sharing the same support [51]. This interpretation emphasizes the triangular distribution's flexibility and robustness in modeling uncertainty over a bounded range. As a result, triangular distribution is widely recognized as a practical and intuitive tool in probability theory, statistics, and decision sciences. It is particularly valuable when data is limited or imprecise, providing a straightforward way to represent uncertainty.

The triangular distribution is a continuous probability distribution defined over the interval $X \in [a^o, a^p]$, where a^o and a^m denote the minimum and maximum values, respectively, and a^m represents the mode, or the value at which the probability density function reaches its maximum. The PDF of the triangular distribution is piecewise linear, increasing linearly from a^o to a^m, and then decreasing linearly from a^m to a^p. This structure forms a triangular shape and reflects a natural, gradual change in likelihood across the defined interval. The distribution is thus well-suited for capturing uncertainty in situations where only approximate estimates are available.

$$P(X) = \begin{cases} \frac{2(x-a^o)}{(a^m-a^o)(a^p-a^o)}, & a^o < x < a^p \\ \frac{2(a^m-x)}{(a^m-a^o)(a^m-a^p)}, & a^p < x < a^m \end{cases} \quad (3.2)$$

Definition 2.2: A fuzzy number $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) | x \in R\}$ is said to be non-negative if and only if its membership function satisfies $\mu_{\widetilde{A}}(x) = 0$ for all $x < 0$.

For a triangular fuzzy number $\widetilde{A} = (a^o, a^m, a^p)$, this condition holds if and only if $a^m - a^o \geq 0$. This ensures that the entire support of the fuzzy number lies within the non-negative real number, i.e., $[a^m - a^o, a^m + a^p] \subseteq [0, \infty)$.

Assume $\widetilde{A} = (-2, 4, 10)$,

- Left endpoint: $a^m - a^o = 4 - (-2) = 6$
- Right endpoint: $a^m + a^p = 4 + 10 = 14$

Therefore, support = [6, 14], which is entirely non-negative because there is no part of the fuzzy number below zero.

Definition 2.3: Let $\widetilde{A} = (a_A^o, a_A^m, a_A^p)$ and $\widetilde{B} = (a_B^o, a_B^m, a_B^p)$ be two triangular fuzzy numbers, where each tuple represents left spread, mode, and right spread, respectively. Therefore, $\widetilde{A}$ and $\widetilde{B}$ are considered equal if and only if the corresponding parameters are equal, that is:

$$a_A^o = a_B^o,\ a_A^m = a_B^m, \text{ and } a_A^p = a_B^p.$$

Because of a triangular fuzzy number, which is uniquely defined by these three parameters, equality of parameters implies equality of the fuzzy numbers.

The arithmetic of two triangular fuzzy numbers is defined as [52]:

1. **Addition**
   $\tilde{A} \oplus \tilde{B} = (a_A^o + a_B^o, a_A^m + a_B^m, a_A^p + a_B^p)$
2. **Subtraction**
   $\tilde{A} - \tilde{B} = (a_A^o - a_B^o, a_A^m - a_B^m, a_A^p - a_B^p,)$
   This is acceptable if both fuzzy numbers are strictly positive and for simplified analysis**.**
3. **Multiplication**
   $\tilde{A} \otimes \tilde{B} = (a_A^o \times a_B^o, a_A^m \times a_B^m, a_A^p \times a_B^p)$
   This assumes that all values are positive to preserve the triangular shape.
4. **Scalar Multiplication**
   For any scalar λ, $\lambda\tilde{A} = \lambda(a_A^o, a_A^m, a_A^p) = \begin{Bmatrix} (\lambda a_A^o, \lambda a_A^m, \lambda a_A^p), & if\ \lambda \geq 0 \\ (\lambda a_A^p, \lambda a_A^m, \lambda a_A^o), & if\ \lambda < 0 \end{Bmatrix}$
5. **Division**
   $\tilde{A} \div \tilde{B} = (a_A^o \div a_B^o, a_A^m \div a_B^m, a_A^p \div a_B^p,)$ where all elements of $\tilde{B}$ are positive and non-zero.

Definition 2.4: A triangular fuzzy number $\widetilde{A} = (a^o, a^m, a^p)$ is said to be symmetric if the difference between the mode and the lower bound equals the difference between the upper bound and the mode.

$$a^m - a^o = a^p - a^m$$

Definition 2.5: A matrix A is said to be non-negative and is denoted by $A \geq 0$ if all elements of A are non-negative numbers.

Definition 2.6: A fuzzy vector $\tilde{b} = (\widetilde{b_i})_{m\times 1}$ is said to be non-negative and is denoted by $\tilde{b} \geq 0$, if each element of $\widetilde{b_i}$ is a non-negative fuzzy number, that is $\widetilde{b_i} \geq 0$ for all i.

The unsymmetrical triangular distribution is a statistical model commonly used to represent skewed data, meaning data that are not symmetrically distributed around a central value (mode), as illustrated in Figure 3.2. In contrast to the symmetrical triangular distribution, which assumes that values on either side of the mode are equally probable (resulting in equal slopes on both sides), the unsymmetrical triangular distribution allows for asymmetrical slopes, meaning that the likelihood of values can differ on the left and right sides of the mode [53].

The ability to adjust the position of the mode between the minimum and maximum values provides this flexibility to model a wide range of real-world phenomena where outcomes are not evenly distributed. This makes the unsymmetrical triangular distribution particularly useful in fields such as risk analysis, cost estimation, and decision-making under uncertainty.

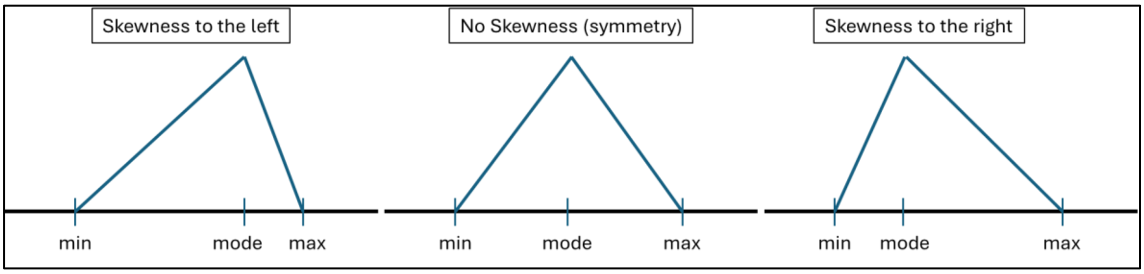

Figure 3.2. Skewness forms in triangular fuzzy numbers.

Figure 3.2 demonstrates the three possible skewness configurations of triangular fuzzy numbers: Left-skewed, Symmetrical (no skewness), and Right-skewed.

From a cost and risk perspective:

1. Left skewness implies a higher probability of achieving lower costs and reduced uncertainty.
2. Symmetry suggests an equal likelihood of encountering either lower or higher costs and corresponding uncertainty.
3. Right skewness indicates a greater probability of experiencing higher costs and increased uncertainty.

This interpretation is valuable for modeling uncertain parameters in optimization problems, particularly when analyzing downside risk or planning under imprecise conditions.

3.3 Fuzzy Linear Programming

Mathematical programming is a robust and versatile computational technique used to optimize problems expressed through linear or non-linear algebraic equations. This method offers a structured, systematic approach to finding optimal solutions by mathematically formulating the problem and then seeking the best possible outcome within defined constraints [54][55]. One of its significant advantages is its ability to provide definitive, precise solutions under certain conditions, which makes it particularly valuable in situations where exact, deterministic results are both desired and achievable. Whether the objective is to maximize profit, minimize costs, or optimize efficiency, mathematical programming offers a powerful tool for decision-making.

A key strength of mathematical programming is its capacity to tackle complex optimization problems through rigorous mathematical formulations. It allows analysts and decision-makers to address a broad range of challenges across various disciplines, including operations research, engineering, finance, logistics, and many other fields. By leveraging this technique, organizations can make data-driven decisions that are aligned with their strategic goals and constraints, ultimately optimizing processes and enhancing performance. The ability to model both linear and non-linear problems enables mathematical programming to handle diverse optimization tasks, ranging from resource allocation to production scheduling and beyond.

Linear Programming (LP), a fundamental method in mathematical programming, was introduced by George Dantzig in the 1940s [56] and became widely recognized in 1963. It is known for its effectiveness in solving optimization challenges and serves as the foundation for many real-world applications. At the heart of LP is the simplex method, a powerful algorithm that operates in two main phases. The first phase involves identifying an initial extreme point, or vertex, within the feasible region of the problem. If no feasible solution exists due to inconsistent constraints, the algorithm will flag this inconsistency. In the second phase, the algorithm evaluates whether the identified extreme point is optimal. If it is not, the method iterates to the next feasible extreme point, continuing until the optimal solution is found. This iterative nature of LP ensures that the process is both systematic and efficient, guaranteeing that the best possible solution is attained in a well-defined manner [57].

Linear programming focuses on optimizing the allocation of limited resources to achieve a desired outcome. Its versatility lies in its ability to model a wide array of decision-making scenarios, accommodating a variety of objective functions. These objectives can be either singular or multifaceted. LP provides a comprehensive framework for making decisions regarding resource distribution, helping organizations streamline their operations, improve productivity, and allocate resources in the most efficient way. By employing LP models, businesses and governments can make well-informed decisions that are strategically aligned with their overarching goals and constraints [58].

Fuzzy Linear Programming (FLP) is an advanced extension of traditional linear programming that incorporates fuzzy logic to handle uncertainty and imprecision in problem parameters. While classical LP assumes that all data is precise and known, FLP acknowledges that, in many real-world scenarios, the data is inherently uncertain or vague [59]. This uncertainty might derive from the difficulty of obtaining exact values for parameters such as costs, demand, or resource availability, which are often subject to fluctuations or imprecision. FLP thus provides a more flexible and realistic modeling approach that allows decision-makers to handle this ambiguity.

In FLP, the objective functions, constraints, and decision variables can be expressed using fuzzy sets, enabling the incorporation of fuzzy numbers such as triangular or trapezoidal fuzzy numbers. These fuzzy numbers represent uncertain coefficients in the objective function and constraints. Solutions to FLP problems are typically derived by transforming the fuzzy model into an equivalent crisp optimization problem. This transformation is done through methods called the defuzzification process, which help convert fuzzy parameters into a precise form that can then be solved using conventional linear programming techniques.

FLP's value lies in its ability to provide decision-makers with a structured framework for addressing ambiguity and vagueness [60]. By doing so, it allows for more reliable and practical optimization results. FLP is particularly useful in applications where uncertainty is a significant factor, such as in supply chain management, resource allocation, financial planning, and project scheduling. For instance, when companies are faced with uncertain demand forecasts or variable production costs, FLP enables them to make more informed and adaptable decisions, improving the robustness of their optimization models.

Ultimately, fuzzy linear programming extends the capability of traditional optimization techniques, making it more applicable to real-world problems where precision is not always feasible. It bridges the gap between the idealized world of deterministic models and the uncertain, often unpredictable nature of practical decision-making, offering a powerful tool for navigating complex optimization challenges in an increasingly uncertain world. By integrating fuzzy logic with linear programming, FLP empowers decision-makers to make more flexible and reliable choices, enhancing the effectiveness of their strategic initiatives across a variety of industries.

3.4 Defuzzification Approach

Defuzzification approaches are techniques designed to convert fuzzy sets or outputs into precise, crisp numerical values suitable for decision-making or further computational analysis. In fuzzy logic systems, data is often represented using linguistic variables such as "high," "medium," or "low," with associated membership functions that quantify the degree of truth or relevance for each category. This representation is particularly advantageous for modeling and addressing imprecision, vagueness, and uncertainty commonly encountered in real-world problems. However, practical applications typically require actionable and well-defined outputs to inform decisions, control processes, or perform numerical evaluations [61]. The transition from fuzzy, imprecise outputs to clear, definitive numerical values is the fundamental purpose of defuzzification.

Defuzzification serves as a critical step in fuzzy logic systems, bridging the gap between the abstract, inference-driven outcomes generated by fuzzy reasoning and the concrete, actionable requirements of real-world applications. Without defuzzification, the outputs of fuzzy models would remain abstract and potentially ambiguous, significantly limiting their usability in practical domains such as control systems, optimization, decision-making, and predictive modeling [62]. By transforming fuzzy outputs into crisp values, defuzzification ensures that the insights derived from fuzzy systems are not only interpretable but also directly applicable. This makes it an indispensable component in diverse fields, including automated systems, resource allocation, predictive analytic, and supply chain optimization.

The necessity of defuzzification lies in its ability to make fuzzy logic systems functional and relevant to real-world contexts. Fuzzy systems often operate in environments characterized by uncertainty, variability, and incomplete information. While fuzzy representations excel at accommodating these complexities, real-world systems require precise outcomes for tasks such as controlling machinery, allocating resources, or making strategic decisions. For instance, an automated climate control system might use fuzzy inputs like "slightly warm" or "very humid" but ultimately output a specific temperature or humidity level to maintain optimal conditions. Similarly, in optimization problems, fuzzy constraints and objectives must be translated into specific decisions or solutions.

A variety of defuzzification approaches have been developed, each catering to specific application needs and performance criteria. These methods differ in their computational complexity, interpretability, and suitability for particular types of fuzzy systems. Common defuzzification techniques include:

1. Centroid method

The centroid method calculates the center of gravity of the fuzzy set's membership function. This approach is widely used due to its simplicity and ability to produce results that balance all possible outcomes proportionally [63].

2. Mean of Maximum (MOM)

The Mean of Maximum method identifies the average of all values where the fuzzy set reaches its maximum membership degree. It is computationally efficient and often used in systems requiring quick responses [63].

3. Largest of Maximum (LOM) and Smallest of Maximum (SOM)

These methods select the largest or smallest value with the highest membership degree, respectively. They are particularly useful when extreme values are preferred in decision-making scenarios.

4. Weighted Average (WA) method

This method computes a weighted average of all values within the fuzzy set, considering the membership degree as the weight. It offers a balance between precision and computational efficiency.

Choosing an appropriate defuzzification approach depends on several factors, including the complexity of the problem, the desired level of computational efficiency, and the degree of precision required for the application. For instance, in real-time control systems, methods like the mean of maximum or max-membership principle may be preferred due to their simplicity and speed. Conversely, applications requiring a more balanced and representative output, such as financial modeling or resource allocation, may benefit from methods like the centroid or weighted average. In this study, the appropriate defuzzification approach will be selected based on the positions of fuzzy data within model. These can be divided into two types: defuzzification approach for objective function and defuzzification approach for constraints, that will be explained later.

In summary, defuzzification plays a pivotal role in making fuzzy logic systems practical and actionable. By translating fuzzy outputs into crisp numerical values, it ensures that the insights and predictions derived from fuzzy models can be effectively applied in diverse domains. The choice of defuzzification method is critical to the success of fuzzy logic systems, as it directly impacts the accuracy, efficiency, and relevance of the resulting decisions or actions. As fuzzy logic continues to evolve, the development of more advanced and adaptive defuzzification techniques remains an important area of research, further enhancing the applicability of fuzzy systems in solving complex, real-world problems.

3.5 Optimization Approach

Optimization refers to the systematic process of defining problems and employing techniques to determine the most effective solutions. It involves maximizing or minimizing objective functions while adhering to a set of predefined constraints. At its core, optimization theory seeks to identify the most favorable outcomes or decisions within the limitations imposed by resources, time, or other constraints [64]. This discipline is fundamental in various fields, including engineering, economics, logistics, and artificial intelligence, where efficient resource allocation, cost reduction, and performance improvement are critical.

Optimization involves two key aspects: problem formulation and the application of solution methodologies. Problem formulation translates real-world scenarios into structured mathematical or computational models. These models often include objective functions, which represent the goals (such as minimizing costs or maximizing efficiency), and constraints, which define the boundaries or limitations of the problem [65]. To solve optimization problems effectively, researchers and practitioners rely on optimization algorithms, which can be broadly categorized into two main types: mathematical programming and meta-heuristic programming. These categories provide distinct methods for addressing a wide range of optimization challenges.

Mathematical programming encompasses techniques that utilize mathematical models and precise calculations to find optimal solutions. These methods are particularly suitable for structured problems with clearly defined relationships among variables. Key methodologies within this category include:

1. Linear Programming (LP)

It is used for optimizing a linear objective function subject to linear constraints. It is widely applied in industries for tasks like production planning, transportation, and resource allocation.

2. Integer Programming (IP)

It is similar to LP but requires some or all variables to be integers. This method is ideal for problems involving discrete decisions, such as scheduling and facility location.

3. Nonlinear Programming (NLP)

It extends optimization to problems where the objective function or constraints are nonlinear. It is used in complex scenarios such as energy optimization, financial modeling, and engineering design.

Mathematical programming techniques are known for their accuracy and reliability. However, they often require significant computational resources, particularly for large-scale problems with high dimensional or intricate constraints.

Meta-heuristic programming involves heuristic or approximate algorithms inspired by natural phenomena or innovative problem-solving strategies. These methods excel in tackling complex, unstructured, or large-scale optimization problems where traditional mathematical approaches may be infeasible or inefficient. Notable meta-heuristic techniques include:

1. Genetic Algorithms (GA)

It is modeled after the process of natural selection, GAs use operations like mutation, crossover, and selection to evolve solutions over generations [66].

2. Simulated Annealing (SA)

It is inspired by the annealing process in metallurgy, this method explores the solution space by mimicking the cooling of metals to escape local optima.

3. Particle Swarm Optimization (PSO)

It simulates the collective behavior of swarms, such as birds or fish, to identify optimal solutions.

4. Ant Colony Optimization (ACO)

It mimics the behavior of ants searching for food, providing effective solutions for combinatorial problems like routing and scheduling [67].

These methods are flexible and adaptive, making them well-suited for dynamic or uncertain environments. While they may not guarantee a precise global optimum, meta-heuristic algorithms often provide high-quality approximations in a reasonable time frame.

The selection of an appropriate optimization technique depends on several factors:

1. **Nature of the problem**
The structured problems with well-defined constraints and relationships are best suited for mathematical programming, while unstructured or dynamic problems benefit from meta-heuristic approaches.

2. **Scale and complexity**
The large-scale or highly complex problems may require the flexibility of meta-heuristic methods, especially when exact solutions are computationally prohibitive.

3. **Computational resources**
Mathematical programming can be computationally intensive, particularly for nonlinear or integer problems, whereas meta-heuristic methods often balance efficiency and solution quality.

4. **Precision requirements**
Problems demanding exact solutions may favor mathematical programming, while approximate solutions are often sufficient for real-world applications using meta-heuristics.

3.6 Risk Measurement

The concept of risk, as discussed by Rachev et al. (2011) [68], is intrinsically linked to the presence of uncertainty and exposure. Risk is often perceived as a subjective notion that arises when uncertainty is present, and individuals or organizations are exposed to potential adverse outcomes. According to Rachev et al., (2011) risk encompasses both the element of uncertainty where future events or conditions are unpredictable and the exposure to potential negative consequences that may result from these uncertain factors. This perspective highlights that risk cannot be fully understood or managed without considering the inherent uncertainty in any given situation.

Uncertainty refers to the lack of complete knowledge about future outcomes or conditions, which introduces variability and unpredictability into decision-making processes. When uncertainty exists, it implies that there are multiple possible outcomes, each with varying probabilities. Risk, therefore, emerges because of this uncertainty, as it represents the potential for adverse outcomes resulting from the unknown. For instance, in financial investments, uncertainty about market movements creates the risk of financial loss. Similarly, in project management, uncertainty about resource availability or project timelines introduces the risk of delays and cost overruns.

The subjective nature of risk underscores that different individuals or organizations may perceive and respond to risk differently based on their own experiences, knowledge, and risk tolerance. This subjectivity means that the same level of uncertainty can be viewed as risky depending on the context and perspective of the decision-makers. For example, a high-risk investment may be perceived as attractive to a risk-tolerant investor but as too risky for a more conservative investor.

Effective risk management, therefore, involves not only understanding and quantifying uncertainty but also addressing how this uncertainty impacts exposure to potential negative outcomes. Therefore, several risk measurements are proposed to assist decision makers to better understand the nature of the risks they face and develop strategies to mitigate or manage them.

Risk measurement is a critical process in risk management that involves quantifying the potential impact of uncertainties on an organization or investment. This process aims to provide a clear and objective assessment of risk exposure by evaluating the likelihood and severity of adverse outcomes. Risk measurement helps decision-makers understand the extent of potential losses or damage and aids in developing strategies to mitigate these risks effectively [69].

One fundamental approach to risk measurement involves calculating the probability and impact of different risk events. Probability estimates the likelihood of a risk occurring, while impact assesses the potential severity or consequences if the risk does occur. Techniques such as risk assessments, simulations, and statistical analyses are commonly used to evaluate these factors. For instance, in financial risk management, value-at-risk is a widely used measure that quantifies the maximum potential loss over a specified period, given a certain confidence level. This provides a numerical estimate of risk exposure and helps with setting appropriate risk limits [70].

Another key aspect of risk measurement is the use of metrics and indicators to monitor and manage risk over time. These metrics can include measures such as standard deviation, which gauges the volatility of returns, or the beta coefficient, which assesses the sensitivity of an asset's returns to market movements. By tracking these indicators, organizations can identify changes in risk levels and adjust their risk management strategies accordingly. Additionally, scenario analysis and stress testing are valuable tools for examining how different scenarios or extreme conditions might impact risk, allowing organizations to prepare for and mitigate potential adverse outcomes.

In summary, risk measurement is a crucial component of risk management that involves quantifying the probability and impact of potential adverse events. By employing various techniques and metrics, organizations can assess their risk exposure, monitor changes over time, and develop strategies to mitigate potential losses. Integrating quantitative data with qualitative insights provides a more holistic view of risk, enhancing decision-making and helping to safeguard against uncertainties.

3.7 Monte Carlo Simulation

Monte Carlo Simulation is a powerful quantitative technique used to analyze and predict the behavior of systems or processes under conditions of uncertainty. By employing random sampling and statistical modeling, it provides a way to assess the potential outcomes of a model and their associated probabilities. Unlike deterministic models, which rely on fixed input values, Monte Carlo Simulation assigns probability distributions to uncertain variables, reflecting their possible ranges and likelihoods. This approach allows for a comprehensive exploration of variability and risk, making it an indispensable tool for decision-making under uncertainty [71][72][73].

Monte Carlo Simulation is particularly advantageous in analyzing complex systems where uncertainty plays a significant role. By performing thousands or even millions of iterations, it generates a robust data set of potential outcomes. This process helps decision-makers:

1. **Identify risks**: Quantify the likelihood and impact of adverse scenarios.
2. **Explore variability**: Understand the range of possible results and their probability.
3. **Enhance decision-making**: Provide a probabilistic foundation for evaluating strategies and making informed choices.

The Monte Carlo Simulation process can be broken into several steps, each of which contributes to capturing the complexity and uncertainty of real-world systems:

1. Define the model

A mathematical or computational model is created to represent the system or process being studied. This model includes the relationships between variables, constraints, and objectives that define the system's behavior.

2. Assign input distributions

Uncertain variables in the model are represented using probability distributions that capture their variability. Common distributions include:

- **Normal distribution**: Suitable for variables with a bell-shaped variability pattern.
- **Uniform distribution**: Used when all outcomes within a range are equally likely.
- **Triangular distribution**: Ideal for variables with known minimum, maximum, and most likely values.

3. Generate random samples

The simulation generates random values for each input variable based on their assigned distributions. This randomness reflects the inherent uncertainty and variability of the real-world system.

4. Simulate iteratively

The model is run repeatedly, often thousands or even millions of times, with each iteration using a new set of random inputs. These iterations simulate different scenarios, capturing a wide range of possible outcomes.

5. Analyze results

The outcomes from all iterations are aggregated to form a distribution of possible results. This data is then analyzed to provide insights into:

- **Probabilities**: The likelihood of specific outcomes or scenarios.
- **Risks**: The impact and frequency of adverse events.
- **Variability**: The range and spread of results, highlighting best-case, worst-case, and most likely scenarios.

Monte Carlo Simulation offers several benefits that make it a preferred method for uncertainty analysis:

1. **Flexibility**: Accommodates complex models and diverse types of input data.
2. **Comprehensive insight**: Provides a complete distribution of possible outcomes rather than a single deterministic result.
3. **Risk assessment**: Quantifies uncertainty and helps identify potential risks and their probabilities.
4. **Scenario analysis**: Enables the evaluation of different strategies and their potential impacts.

3.8 Resilience Index

The resilience index in supply chain aggregate production planning is a quantitative metric designed to assess and enhance the robustness and adaptability of a supply chain in response to disruptions, uncertainties, and fluctuations in demand or supply. Resilience, in this context, refers to the ability of a supply chain to maintain its operational performance, recover swiftly from disruptions, and adapt to changing circumstances while minimizing negative impacts on production, costs, and customer satisfaction [74]. The resilience index serves as a comprehensive indicator that integrates multiple dimensions of supply chain performance, providing a systematic way to evaluate and improve the supply chain's capacity to withstand and recover from adverse events.

At its core, the resilience index encapsulates key attributes such as flexibility, redundancy, adaptability, and risk management. Flexibility ensures that supply chain components can adjust quickly to variations in production volumes or product mix. Redundancy refers to maintaining additional capacity or resources to handle unforeseen events. Adaptability highlights the ability to modify strategies and processes in the face of evolving market dynamics. Effective risk management encompasses proactive identification, assessment, and mitigation of potential vulnerabilities within the supply chain. Together, these attributes form the basis for quantifying resilience, enabling decision-makers to prioritize strategies and investments that enhance the overall stability and responsiveness of the supply chain.

In the context of SCAPP, the resilience index acts as a bridge between traditional optimization objectives, such as cost minimization and efficiency maximization, and the imperative to address uncertainty and risk. By incorporating resilience into planning models, businesses can better align production capacities, inventory levels, and resource allocation with the demands of a volatile and dynamic environment [75]. This ensures not only the continuity of operations during disruptions but also the ability to capitalize on opportunities in competitive markets.

The resilience index is often calculated using a multi-criteria approach, integrating quantitative data (e.g., inventory turnover rates, lead times, and production capacities) with qualitative assessments (e.g., supplier reliability, market responsiveness, and workforce adaptability). These inputs are aggregated to provide a holistic view of the supply chain's performance under varying scenarios. Advanced methodologies, including fuzzy logic, stochastic modeling, and multi-objective optimization, are frequently employed to refine the resilience index, making it a practical and actionable tool for real-world applications.

The importance of the resilience index in SCAPP has grown significantly due to the increasing complexity of global supply chains and the frequency of disruptions caused by factors such as natural disasters, geopolitical tensions, pandemics, and technological failures [76]. By integrating the resilience index into decision-making processes, organizations can proactively:

1. **Mitigate risks**: Identify and address vulnerabilities within the supply chain before they escalate into critical issues.
2. **Enhance recovery capabilities**: Ensure a faster return to normal operations following disruptions.
3. **Improve resource utilization**: Balance efficiency with flexibility to optimize costs without compromising stability.
4. **Strengthen competitive advantage**: Build trust with customers and stakeholders by demonstrating a reliable and adaptive supply chain.

The resilience index is a valuable tool for strategic decision-making in SCAPP. It can be used to:

1. **Evaluate supply chain configurations**: Identify weak links and prioritize investments in resilience-enhancing measures, such as supplier diversification or inventory buffers.
2. **Optimize trade-offs**: Balance cost-efficiency with resilience by incorporating the index into multi-objective optimization models.
3. **Monitor performance**: Track changes in resilience over time to ensure continuous improvement and alignment with organizational goals.

Chapter 4
A Fuzzy Multi-Criteria Decision-Making for Optimizing Supply Chain Aggregate Production Planning based on Cost Reduction and Risk Mitigation

This chapter introduces an integrated approach for optimizing Supply Chain Aggregate Production Planning (SCAPP) by employing a Fuzzy Multi-Criteria Decision-Making (FMCDM) framework. It is designed to address two critical and often conflicting objectives in supply chain management; cost reduction and risk mitigation, under conditions of uncertainty that are prevalent in real-world applications. By harnessing the capabilities of fuzzy logic, the study aims to assist decision-makers in developing robust and efficient production plans across the supply chain network. This chapter begins with a detailed description of the problem, emphasizing the operational complexities and uncertainties that necessitate a multi-criteria decision-making approach. It also outlines the principal contributions of the study, highlighting the novelty and practical significance of the proposed model. The methodology adopted to tackle the problem is then presented, integrating fuzzy set theory and optimization techniques to effectively manage ambiguity in data and decision preferences. A real-world case study is introduced to demonstrate the practical application and effectiveness of the proposed framework. Subsequently, this chapter presents a comprehensive explanation of the mathematical notations and the formulation of the fuzzy optimization model. The results are discussed in detail, including a comparison with the traditional fuzzy optimization approach and a validation of outcomes to evaluate the model's reliability and performance. This chapter concludes with a discussion of the findings, the advantages of applying proposed approach, and suggestions for future research. Overall, this chapter seeks to bridge the gap between theoretical modeling and practical implementation, offering a structured and adaptable tool for improving supply chain performance under uncertainty.

4.1 Problem Statement and Contributions

In today's rapidly evolving and increasingly uncertain business landscape, supply chain decision-makers must navigate a multitude of challenges deriving from interdependent operations, fluctuating demand, and volatile market conditions. The complexity of managing these interconnected elements is further exacerbated by unforeseen events such as economic instability, geopolitical tensions, and natural disasters. Traditional decision-making approaches, which often rely on deterministic or static models, struggle to adequately capture the inherent uncertainties and risks associated with supply chain operations. As a result, organizations may experience inefficiencies, increased costs, sub-optimal resource utilization, and diminished overall performance.

To address these challenges, this study presents a novel Multi-Objective Fuzzy Linear Programming (MOFLP) model designed to enhance supply chain decision-making by optimizing production planning, material flows, and resource allocation. The model integrates cost efficiency and downside risk minimization as its primary objectives, ensuring that businesses can achieve a balanced trade-off between profitability and operational resilience. Unlike conventional models, which may inadequately handle uncertainty, this approach incorporates unsymmetrical triangular fuzzy numbers to represent uncertain parameters more accurately. By doing so, the model specifically addresses downside risk, aiming to mitigate negative financial impacts from market fluctuations and unforeseen events.

This framework aims to mitigate the negative effects of financial impact and uncertainties by providing decision-makers with a more robust and flexible planning tool. The incorporation of downside risk analysis ensures that worst-case scenarios are systematically considered, enabling proactive risk management and contingency planning. As a result, businesses can improve cost-effectiveness, enhance supply chain agility, and strengthen resilience against unpredictable situations. Through a comprehensive case study, this research demonstrates the practical application and advantages of the proposed model, highlighting its potential to significantly improve strategic decision-making in modern supply chain management.

This study makes significant contributions to the field of Supply Chain Aggregate Production Planning (SCAPP) through the development of a multi-objective fuzzy linear programming model that integrates cost efficiency optimization and downside risk mitigation. Unlike traditional optimization models that focus primarily on cost minimization, this approach incorporates the Mean-Conditional Value at Risk Gap (MCVaRG) as a downside risk measure, offering a more comprehensive assessment of uncertainty. By including MCVaRG, the model provides decision-makers with a robust tool for managing financial exposure to adverse deviations in production and market conditions, thereby enhancing supply chain resilience and strategic planning under uncertainty. Additionally, this study addresses a critical limitation in traditional fuzzy optimization by incorporating unsymmetrical triangular fuzzy numbers. This advancement allows for a more accurate representation of real-world uncertainties, such as fluctuating demand, variable lead times, and uncertain production capacities. By capturing the inherent skewness of uncertain data, the model significantly improves decision-making precision and reliability, offering an important enhancement to conventional fuzzy optimization methods in supply chain management.

4.2 Methodology

This section presents a comprehensive optimization methodology designed to enhance the efficiency, reliability, and robustness of the supply chain aggregate production planning framework. Given the complexity and uncertainty inherent in modern supply chains, the proposed methodology employs a multi-objective fuzzy linear programming approach to balance cost minimization with downside risk mitigation. By integrating advanced fuzzy optimization techniques, the framework ensures that decision-makers can make more informed and resilient strategic choices under uncertain conditions.

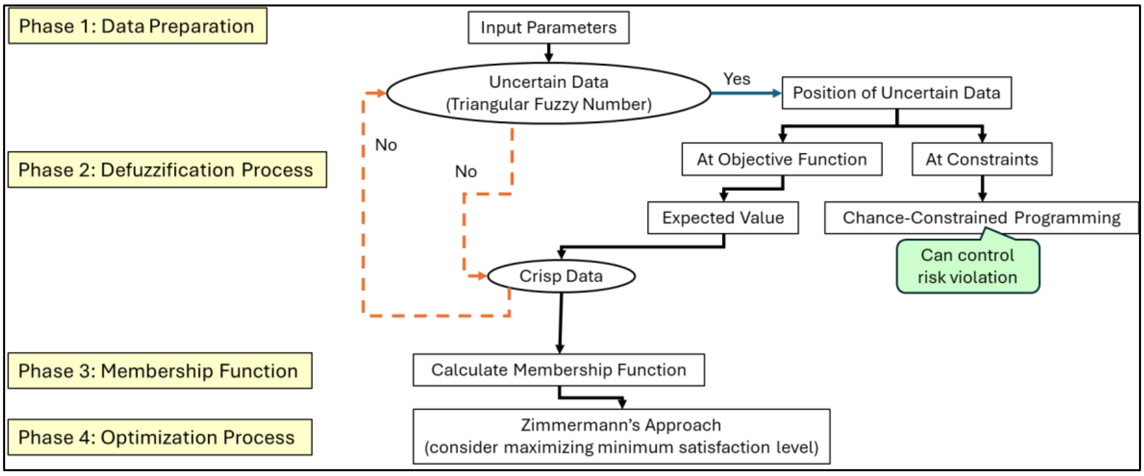

Figure 4.1. The framework of proposed optimization methodology.

The methodology is systematically structured into four critical phases as shown in Figure 4.1, each addressing key aspects of uncertainty modeling and optimization:

- **Phase 1: Data Preparation**

This phase involves systematic collection, classification, and preprocessing of relevant supply chain parameters. All parameters will be categorized into crisp and uncertain. Parameters that are precisely known will be grouped into the crisp category, while parameters that are ambiguous and challenging to ascertain will be grouped into the uncertain class. A particular focus is placed on incorporating uncertain parameters using unsymmetrical triangular fuzzy numbers, which provide a more realistic representation of asymmetric uncertainties in transportation costs, inventory cost, and penalty cost of lost sale. The accuracy and quality of data at this stage are essential to the reliability of the entire optimization process.

- **Phase 2: Defuzzification Process**

Since fuzzy data lacks precise numerical values, defuzzification is necessary to translate fuzzy variables into actionable decision parameters. This study employs an advanced defuzzification approach that preserves the integrity of uncertainty while enabling computational feasibility. It ensures that uncertain parameters are transformed into representative crisp values, balancing realism and computational efficiency. By refining the defuzzification process, the defuzzification approach can be classified into two main types; defuzzification approach for objective function and defuzzification approach for constraint, based on the positions of fuzzy parameters in the model.

1. Defuzzification approach for objective function

The Expected Value (EV) approach is a widely used conventional defuzzification method applied to objective functions in fuzzy mathematical programming. It provides a systematic way to transform fuzzy objectives into crisp values, facilitating optimization and decision-making under uncertainty. The core principle of the EV approach is to compute the average overall performance of the objective function as shown below.

$$EV(\tilde{Z}) = \frac{\frac{Z^o+Z^m}{2}+\frac{Z^m+Z^p}{2}}{2} = \frac{Z^o+2Z^m+Z^p}{4} \tag{4.1}$$

where Z^o, Z^m, and Z^p are values of objective functions in optimistic, most likely, and pessimistic situations, respectively.

2. Defuzzification approach for constraint

The Chance-Constrained Programming (CCP) approach is a powerful defuzzification method applied to the constraints in fuzzy optimization problems. CCP integrates uncertainty by ensuring that constraints are satisfied with a specified probability or credibility level (γ). The CCP approach allows constraints to be formulated in a probabilistic manner, ensuring that they hold with a predefined confidence level. Instead of requiring strict feasibility under all possible scenarios, it accommodates uncertainty by defining constraints in terms of acceptable risk levels. This makes it especially useful for supply chain environments where decision-makers must balance feasibility with risk exposure, ensuring that constraints remain valid under realistic conditions.

$$Cr\{\textstyle\sum_{j=1}^{n} a_{ij}x_j \le \tilde{b}_i\} \ge \gamma$$

$$\text{when } (0 \le \gamma \le 0.5): ax \le (2\gamma)b^m + (1-2\gamma)b^p \tag{4.2}$$

$$\text{when } (0.5 < \gamma \le 1): ax \le (2\gamma-1)b^o + (2-2\gamma)b^m \tag{4.3}$$

where b^o, b^m, and b^p are values of available resource in optimistic, most likely, and pessimistic situations, respectively.

- **Phase 3: Membership Function**

The process involves normalizing the different units of multiple objective functions to a standardized scale, typically ranging from 0.0 to 1.0, which represents the level of satisfaction, as illustrated below.

1. Membership function for minimization of the objective function

$$\mu_{Z_i} = \begin{cases} 1, & Z_i \le Z_i^{PIS} \\ \frac{Z_i^{NIS}-Z_i}{Z_i^{NIS}-Z_i^{PIS}}, & Z_i^{PIS} \le Z_i \le Z_i^{NIS} \\ 0, & Z_i \ge Z_i^{NIS} \end{cases} \tag{4.4}$$

2. Membership function for maximization of the objective function

$$\mu_{Z_i} = \begin{cases} 1, & Z_i \ge Z_i^{PIS} \\ \frac{Z_i-Z_i^{NIS}}{Z_i^{PIS}-Z_i^{NIS}}, & Z_i^{NIS} \le Z_i \le Z_i^{PIS} \\ 0, & Z_i \le Z_i^{NIS} \end{cases} \tag{4.5}$$

where Z_i^{NIS} is the maximum bound for minimizing objective of the minimum bound for maximizing objective and Z_i^{PIS} is the maximum bound for maximizing objective or the minimum bound for minimizing objective.

- **Phase 4: Optimization Process**

This process is used to identify the optimal compromise solution for multiple objective fuzzy linear programming problems. Since 1978, Zimmermann's method has been a traditional, weightless fuzzy linear programming approach in MOFLP. It seeks to maximize the minimum satisfaction level across multiple objective functions, treating each function with equal importance, as shown below.

$$\begin{aligned}&\text{Maximize } \mu_Z\\&\text{Subjected to: } x \in F(x)\\&\qquad \mu_Z \leq \mu_{Z_i}, \quad i = 1, 2, \ldots, I\end{aligned} \tag{4.6}$$

where μ_Z is a minimum value of the satisfaction levels from the multiple objective functions and μ_{Z_i} is the satisfaction level of each objective function.

4.3 Case Study

A case study of a small-sized Thai plastic bottle industry is presented to assess and demonstrate the effectiveness of the proposed fuzzy multiple-criteria decision-making model. The supply chain of this Thai plastic bottle industry involves four qualified suppliers who provide PET resin. The suppliers offer the resin at varying prices, which are influenced by both the quality of the resin and the suppliers' ability to manage and control their pricing strategies. The production plant manufactures standardized plastic bottles but operates under a limited manufacturing capacity, which restricts its output. Additionally, the supply chain includes six retailers, each located in different regions and experiencing varying demand levels, as illustrated in Figure 4.2. The planning horizon for the supply chain aggregate production planning within this supply chain is set to span over 6 months. This case study serves to evaluate how the fuzzy MCDM model can optimize the production planning, resource allocation, and supply chain management in the context of uncertainty and competing objectives.

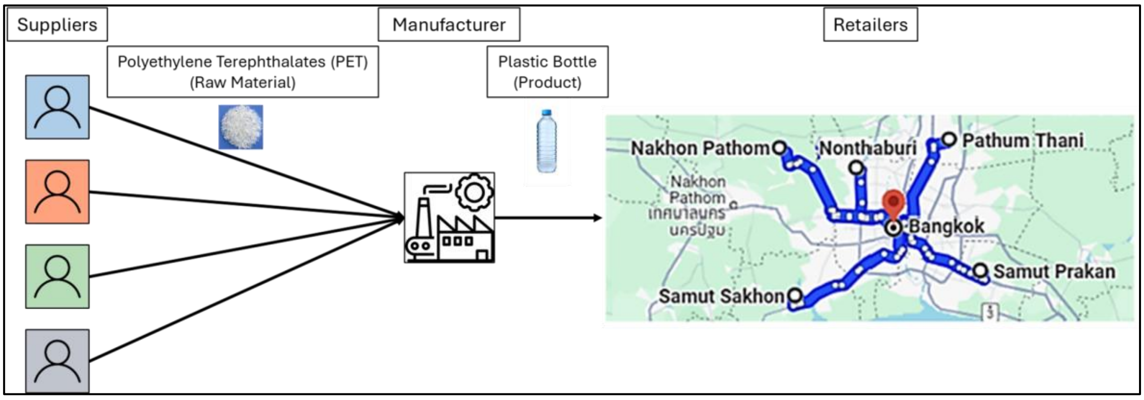

Figure 4.2. Supply chain structure and locations of retailers.

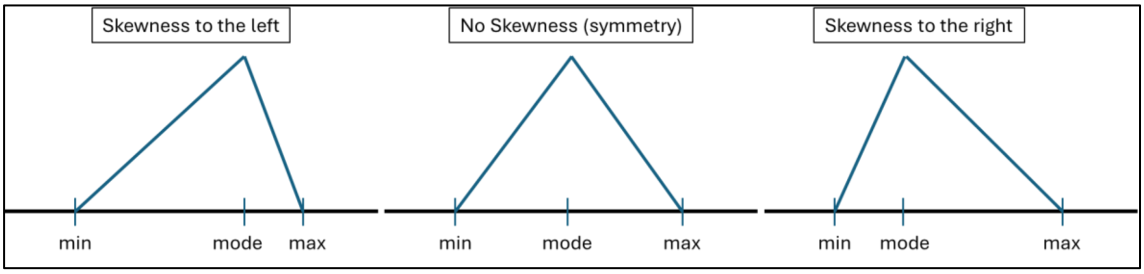

Figure 4.3. Skewness forms in triangular fuzzy numbers.

Figure 4.3 illustrates the unsymmetrical skewness of risks, highlighting three distinct forms of skewness in triangular fuzzy numbers: left-skewed, symmetric, and right-skewed. These forms of skewness represent varying levels of uncertainty and risk in supply chain and production planning scenarios, with significant implications for decision-making in uncertain environments. Each type of skewness reflects the distribution of possible outcomes and their associated costs and risks, which can greatly influence the overall planning and optimization process.

1. Left-skewed

In a left-skewed distribution, the tail of the fuzzy number is longer on the left side, indicating that lower values (such as lower costs and reduced risks) are more probable. This suggests that the system is likely to experience lower-than-expected costs and reduce uncertainty. Decision-makers in this scenario might be more confident in the outcomes, as the model predicts a higher probability of achieving favorable conditions. In supply chain planning, this could imply greater efficiency, where processes are likely to yield cost savings and lower risks with higher certainty.

2. Symmetric

A symmetric distribution, as the name suggests, reflects an equal likelihood of either obtaining lower costs with lower risks or higher costs with higher risks. This type of distribution suggests balanced uncertainty, where outcomes are spread equally around the most likely value. In a symmetric risk scenario, decision-makers face a neutral risk situation, where there is no significant bias toward either favorable or unfavorable outcomes. In supply chain planning, this type of skewness might represent scenarios where fluctuations in demand or resource availability could result in either beneficial or adverse effects on cost and risk.

3. Right-skewed

In a right-skewed distribution, the tail of the fuzzy number is longer on the right side, indicating that higher values (such as higher costs and greater risks) are more probable. This suggests a higher likelihood of encountering adverse outcomes, where the system is more prone to experiencing increased costs and greater risk of uncertainty. In supply chain and production planning, a right-skewed risk profile may indicate scenarios where there are higher chances of unexpected challenges, such as supply chain disruptions or demand surges, leading to higher operational costs and increased vulnerability to risk. Decision-makers in this context must account for worst-case scenarios and incorporate strategies to mitigate potential risks.

There are four qualified suppliers that provide PET resin at different uncertain prices, which are represented by symmetrical Triangular Fuzzy Numbers (TFNs), as detailed in Table 4.1. The pricing of the raw materials supplied by each vendor is influenced by both the quality of the materials they provide and their reliability in maintaining price stability over time. For instance, Supplier 1 offers lower-quality raw materials and exhibits poor reliability, leading to the lowest price among the suppliers. This price is subject to significant fluctuations, with a variation of ±45% from its most likely value. In contrast, Supplier 4 provides high-quality raw materials with a strong reputation for price stability, resulting in the highest price among the suppliers. However, the price fluctuations for Supplier 4 are limited to ±15% of its most likely value, indicating greater price predictability. Despite these price differences, all suppliers can meet an equal maximum supply capacity (denoted as $\mathrm{MaxSupCap}_{st}$) of 25,000 units per period. This uniform supply capacity provides a consistent foundation for supply chain planning, ensuring that each supplier can meet the demand within the constraints of their respective price and quality profiles.

Table 4.1 Fuzzy prices of raw material.

Triangular Fuzzy Number	Fuzzy price of a raw material supplied from supplier *s* in period *d* ($/unit)			
	Supplier 1	Supplier 2	Supplier 3	Supplier 4
Optimistic Value	5.23	6.50	8.44	10.84
Most Likely Value	9.50	10.00	11.25	12.75
Pessimistic Value	13.78	13.50	14.06	14.66
Skewness Type	Symmetry (±45%)	Symmetry (±35%)	Symmetry (±25%)	Symmetry (±15%)

The fuzzy parameters associated with the production plant are represented using symmetrical Triangular Fuzzy Numbers (TFNs), with a variability of ±20% from their most likely values, as detailed in Table 4.2. The production plant is required to meet at least 80% of each retailer's demand. However, this requirement may result in some lost sales at certain retailers, which would incur penalty costs due to the shortfall in meeting the demand. The delivery lead time is considered negligible, meaning that any delays in delivery are not factored into the model. Additionally, there are no subcontracting arrangements included in the supply chain setup. In addition to the fuzzy parameters, Table 4.3 provides a summary of other deterministic parameters used in this case study.

Table 4.2 Fuzzy parameters related to the production plant.

	Most Likely Values
Fuzzy cost of regular time production per unit of products in period *d* ($/unit)	12.50
Fuzzy cost of overtime production per unit of products in period *d* ($/unit)	18.75
Fuzzy cost of shortage per unit of products at the plant in period *d* ($/unit)	37.50
Fuzzy cost of inventory per unit of raw materials at the plant in period *d* ($/unit)	0.10
Fuzzy cost of inventory per unit of products at the plant in period *d* ($/unit)	0.30
Fuzzy hiring cost for a labor in period *d* ($/man)	160
Fuzzy firing cost for a labor in period *d* ($/man)	280

Table 4.3 Crisp parameters.

	Values
Labor hours required per unit of products at the plant in period *d* (man-hours/unit)	0.016
Maximum capacity of producing products in the regular time at the plant in period *d* (units)	28,000
Maximum capacity of producing products in the overtime at the plant in period *d* (units)	7,000

The demand for plastic bottles exhibits seasonal fluctuations, primarily driven by variations in water consumption throughout the year. This supply chain consists of six retailers, each located in different regions and experiencing distinct seasonal demand levels. These demand fluctuations are represented using symmetrical Triangular Fuzzy Numbers (TFNs), with a variability of ±20% from the most likely values, as presented in Table 4.4. The production plant and all retailers are situated within close proximity in the central region of Thailand. The plant is located in Bangkok, while the retailers are distributed across the following areas: Bangkok, Nonthaburi, Pathum Thani, Nakhon Pathom, Samut Sakhon, and Samut Prakan. The geographical diversity of the retailers results in variations in several key supply chain costs, including transportation costs, holding costs, and penalty costs associated with lost sales. These costs are further elaborated in Figure 4.4 and Table 4.5, highlighting how location-specific factors influence overall supply chain efficiency and cost management.

Table 4.4 Fuzzy retailers' demands during 6-month periods.

Fuzzy retailer *r* demand of products in period *d* (units)							
Retailer		Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
1	Most Likely Values	5,796	4,971	4,200	3,711	5,004	6,287
2		4,849	3,996	2,646	2,546	4,418	5,362
3		3,967	2,505	1,510	1,293	2,728	3,779
4		7,904	7,016	6,222	6,370	7,386	8,865
5		9,716	8,902	7,462	7,251	8,305	9,396
6		7,809	6,520	5,037	5,528	6,453	7,328

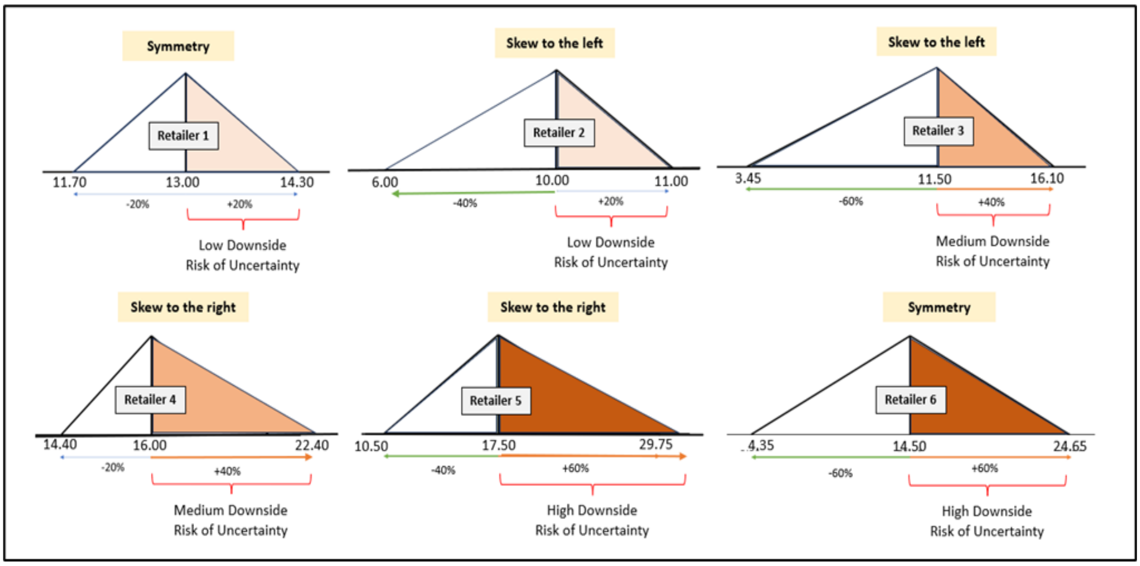

Figure 4.4. Types of skewness and risk of uncertainty of the transportation cost.

The transportation cost characteristics and the associated downside risks vary among the retailers, as depicted in Figure 4.4. Retailer 1 and Retailer 6 exhibit symmetrical cost distributions, but there are notable differences in their downside risks. Retailer 6 is exposed to a higher downside risk (+60% from its most likely value), while Retailer 1 faces a relatively lower downside risk (+20% from its most likely value). In contrast, Retailer 2 and Retailer 3 display left-skewed cost distributions, indicating a greater likelihood of lower transportation costs. Among these, Retailer 3 carries a medium downside risk (+40% from its most likely value), while Retailer 2 faces a lower downside risk (+20% from its most likely value). Retailers 4 and Retailer 5 have right-skewed cost distributions, but Retailer 5 is subject to a greater downside risk than Retailer 4. These variations in cost distributions and the corresponding risks illustrate the diverse logistical and financial challenges encountered by the supply chain in managing retailer demands. By addressing these differences, the study offers insights into optimizing transportation and inventory management, while also accounting for the inherent uncertainties and risks involved. Several operational factors can significantly increase the likelihood of elevated transportation costs for certain retailers. In this study, factors such as unusual traffic congestion play a critical role. High population density, especially in business districts during peak hours, can worsen traffic conditions. Additionally, events such as protests, roadblocks near government offices, or mass gatherings (mobile vulgus) can severely disrupt traffic flow. These disruptions not only lead to delays but also heighten driver frustration, increasing the risk of road rage and accidents, which, in turn, further extend the transportation delays. These circumstances contribute to a higher probability of increased transportation costs. A similar framework applies to holding costs and penalty costs for lost sales, which can also escalate due to adverse operational conditions. The interconnected nature of these factors emphasizes the need to account for such risks in supply chain planning. The specific impacts of these variables on transportation, holding, and penalty costs are summarized in Table 4.5, providing a comprehensive overview of how operational disruptions influence cost escalation.

The assumptions of SCAPP plan should be adhered as follows:

1. A list of qualified suppliers is provided, as detailed in Table 4.1.
2. Retailers' demands fluctuate dynamically throughout the 6-month planning period.
3. Retailer demand can either be fully satisfied or partially unmet, with any shortages incurring a penalty cost.
4. All supply chain costs are considered uncertain and exhibit varying types of skewness, with these characteristics remaining consistent over periods.
5. Subcontracting is not an option in this scenario and delivery lead time is assumed to be negligible.
6. Initial inventory levels and labor capacities are known at the starting period.

Table 4.5 Costs related to retailers.

Triangular Fuzzy Number	Fuzzy cost of transportation per unit of products from the plant to retailer r in period d ($/unit)					
	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5	Retailer 6
Optimistic Value	11.70	6.00	3.45	14.40	10.50	4.35
Most Likely Value	13.00	10.00	11.50	16.00	17.50	14.50
Pessimistic Value	14.30	11.00	16.10	22.40	29.75	24.65
Skewness Type	Symmetry	Skewness to the left	Skewness to the left	Skewness to the right	Skewness to the right	Symmetry
Risk of Uncertainty	Low	Low	Medium	Medium	High	High
Triangular Fuzzy Number	Fuzzy cost of inventory per unit of products at retailer r in period d ($/unit)					
	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5	Retailer 6
Optimistic Value	6.30	2.40	1.65	9.00	6.90	2.55
Most Likely Value	7.00	4.00	5.50	10.00	11.50	8.50
Pessimistic Value	7.70	4.40	7.70	14.00	19.55	14.45
Skewness Type	Symmetry	Skewness to the left	Skewness to the left	Skewness to the right	Skewness to the right	Symmetry
Risk of Uncertainty	Low	Low	Medium	Medium	High	High
Triangular Fuzzy Number	Fuzzy penalty cost of lost sales per unit of products at retailer r in period d ($/unit)					
	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5	Retailer 6
Optimistic Value	6.90	6.00	12.90	15.60	24.75	22.05
Most Likely Value	23.00	20.00	21.50	26.00	27.50	24.50
Pessimistic Value	36.80	28.00	23.65	44.20	38.50	26.95
Skewness Type	Symmetry	Skewness to the left	Skewness to the left	Skewness to the right	Skewness to the right	Symmetry
Risk of Uncertainty	High	Medium	Low	High	Medium	Low

4.4 Mathematical Notations and Model

The notations for indexes, parameters, and decision variables are detailed in Tables 4.6 to 4.10. Notably, all fuzzy parameters are distinguished by a tilde ($\tilde{}$) placed above their respective symbols to signify their fuzzy characteristics.

Table 4.6 Indexes of supply chain aggregate production planning problem.

Indexes	Meaning
s	Set of suppliers ($s = 1, 2, \dots, S$)
r	Set of retailers ($r = 1, 2, \dots, R$)
d	Set of planning periods ($d = 1, 2, \dots, D$)

Table 4.7 Crisp parameters of supply chain production planning problem.

Crisp Parameters	Meaning
LH_d	Labor hours required per unit of products at the plant in period d (person-hours/unit)
$MaxPCRT_d$	Maximum capacity of producing products in the regular time at the plant in period d (units)
$MaxPCOT_d$	Maximum capacity of producing products in the overtime at the plant in period d (units)
$MaxSupCap_{sd}$	Maximum capacity of supplier s for supplying raw materials in period d (units)
RMQ_{sd}	Number of raw materials supplied by supplier s in period d (units)
RTQ_d	Number of products produced in the regular time in period d (units)
OTQ_d	Number of products produced in the overtime in period d (units)
$ShortPPQ_d$	Number of shortage products at the plant in period d (units)
$IRMQ_d$	Remaining quantity of raw materials holding at the plant in period d (units)
IPQ_d	Remaining quantity of products holding at the plant in period d (units)
HL_d	Number of labors hired in period d (persons)
FL_d	Number of labors fired in period d (persons)
L_d	Number of labors in period d (persons)
$TranQ_{rd}$	Transportation quantity of products to retailer r in period d (units)
IRQ_{rd}	Remaining quantity of products holding at retailer r in period d (units)
$ShortPRQ_{rd}$	Number of shortage products at retailer r in period d (units)

Table 4.8 Uncertain parameters of supply chain aggregate production planning problem.

Uncertain Parameters	Meaning
$\widetilde{RMC}_{sd}$	Fuzzy price of a raw material supplied from supplier *s* in period *d* ($/unit)
$\widetilde{RC}_{d}$	Fuzzy cost of regular time production per unit of products in period *d* ($/unit)
$\widetilde{OC}_{d}$	Fuzzy cost of overtime production per unit of products in period *d* ($/unit)
$\widetilde{ShortPC}_{d}$	Fuzzy cost of shortage per unit of products at the plant in period *d* ($/unit)
$\widetilde{InCRM}_{d}$	Fuzzy cost of holding inventory per unit of raw materials at the plant in period *d* ($/unit)
$\widetilde{InCPM}_{d}$	Fuzzy cost of holding inventory per unit of products at the plant in period *d* ($/unit)
$\widetilde{HC}_{d}$	Fuzzy hiring cost for a labor in period *d* ($/person)
$\widetilde{FC}_{d}$	Fuzzy firing cost for a labor in period *d* ($/person)
$\widetilde{TranC}_{rd}$	Fuzzy cost of transportation per unit of products from the plant to retailer *r* in period *d* ($/unit)
$\widetilde{InCPR}_{rd}$	Fuzzy cost of holding inventory per unit of products at retailer *r* in period *d* ($/unit)
$\widetilde{PCLS}_{rd}$	Fuzzy penalty cost of lost sales per unit of products at retailer *r* in period *d* ($/unit)
$\widetilde{De}_{rd}$	Fuzzy retailer *r* demand of products in period *d* (units)

Table 4.9 Decision variables of supply chain planning problem.

Decision Variables	Meaning
RMQ_{sd}	Number of raw materials supplied by supplier *s* in period *d* (units)
RTQ_{d}	Number of products produced in the regular time in period *d* (units)
OTQ_{d}	Number of products produced in the overtime in period *d* (units)
$ShortPPQ_{d}$	Number of shortage products at the plant in period *d* (units)
$IRMQ_{d}$	Remaining quantity of raw materials holding at the plant in period *d* (units)
IPQ_{d}	Remaining quantity of products holding at the plant in period *d* (units)
HL_{d}	Number of labors hired in period *d* (persons)
FL_{d}	Number of labors fired in period *d* (persons)
L_{d}	Number of labors in period *d* (persons)
$TranQ_{rd}$	Transportation quantity of products to retailer *r* in period *d* (units)
IRQ_{rd}	Remaining quantity of products holding at retailer *r* in period *d* (units)
$ShortPRQ_{rd}$	Number of shortage products at retailer *r* in period *d* (units)

Table 4.10 Related notations of supply chain planning problem.

Notations	Meaning
γ	Credibility level
$\widetilde{TSCNC}$	Total supply chain network costs
$\widetilde{ProCC}$	Total procurement costs
$\widetilde{ProDC}$	Total production costs
$\widetilde{DisTC}$	Total distribution costs

The mathematical model developed for the Supply Chain Aggregate Production Planning (SCAPP) problem under uncertainty is designed to optimize key supply chain aspects, with a focus on minimizing total operational costs and the Mean-Conditional Value at Risk Gap (MCVaRG) associated with these costs. This model effectively integrates both deterministic and uncertain data, addressing the challenges posed by variability in critical parameters, such as demand and costs. To manage these uncertainties, the model employs fuzzy optimization techniques, which allow for the representation of the inherent imprecision and variability that characterize real-world supply chain operations. This approach ensures that the model remains adaptable to the complexities of supply chains, where precise data is often difficult to obtain or unreliable.

A defining feature of the model is its use of a fuzzy linear programming framework that specifically accounts for the unsymmetrical skewness of triangular fuzzy numbers. Unlike traditional models that typically assume symmetrical data distributions, this approach recognizes and incorporates the asymmetry frequently observed in real-world scenarios. By factoring in the skewness in triangular fuzzy numbers, the model provides a more accurate representation of the uncertainties and variability faced by supply chains. This advanced methodology enables more precise and informed decision-making in risk management and cost estimation, enhancing strategic planning and increasing the reliability of decisions in environments characterized by complexity and uncertainty.

- **Objective Functions**

1. **Minimizing total supply chain operation costs** is widely regarded as a primary objective in establishing an efficient supply chain network. The total supply chain network costs ($\widetilde{\mathrm{TSCNC}}$) are typically subject to uncertainty and consist of the combined procurement costs ($\widetilde{\mathrm{ProCC}}$), production costs ($\widetilde{\mathrm{ProDC}}$), and distribution costs ($\widetilde{\mathrm{DisTC}}$) over a specified period. Procurement costs include the expenses associated with purchasing raw materials, while production costs account for regular production costs, overtime production costs, product shortage costs, raw material and product inventory holding costs, and costs related to labor hiring and firing. Distribution costs encompass expenditures at the retail level, such as transportation costs, inventory holding costs, and penalty costs incurred from lost sales at retail locations.

$$
\begin{aligned}
&Minimize\ (\widetilde{TSCNC}) = \widetilde{ProCC} + \widetilde{ProDC} + \widetilde{DisTC} \\
&\quad = \textstyle\sum_s^S \sum_d^D \widetilde{RMC}_{sd} \times RMQ_{sd} + \left(\sum_d^D \widetilde{RC}_d \times RTQ_d\right) + \left(\sum_d^D \widetilde{OC}_d \times OTQ_d\right) \\
&\quad + \left(\textstyle\sum_d^D \widetilde{ShortPC}_d \times ShortPPQ_d\right) + \left(\sum_d^D \widetilde{InCRM}_d \times \frac{IRMQ_{d-1}+IRMQ_d}{2}\right) \\
&\quad + \left(\textstyle\sum_d^D \widetilde{InCPM}_d \times \frac{IPQ_{d-1}+IPQ_d}{2}\right) + \left(\sum_d^D \widetilde{HC}_d \times HL_d\right) + \left(\sum_d^D \widetilde{FC}_d \times FL_d\right) \\
&\quad + \left(\textstyle\sum_r^R \sum_d^D \widetilde{TranC}_{rd} \times TranQ_{rd}\right) + \left(\sum_r^R \sum_d^D \widetilde{InCPR}_{rd} \times \frac{IRQ_{rd-1}+IRQ_{rd}}{2}\right) \\
&\quad + \left(\textstyle\sum_r^R \sum_d^D \widetilde{PCLS}_{rd} \times ShortPRQ_{rd}\right) \qquad (4.7)
\end{aligned}
$$

2. **Minimizing the Mean-Conditional Value at Risk Gap (MCVaRG) of total supply chain operation costs** is a crucial objective in creating a resilient and effective supply chain network. This objective enables decision-makers to mitigate the risks associated with uncertainties in supply chain costs, with a particular focus on reducing downside risk. Downside risk represents the likelihood of incurring costs that exceed expected levels, ensuring that the supply chain remains cost-efficient even under adverse conditions.

$$
\begin{aligned}
&Minimize\ MCVaRG \\
&\quad = (MCVaR(\widetilde{ProCC}) + MCVaR(\widetilde{ProDC}) + MCVaR(\widetilde{DisTC})) - \widetilde{TSCNC} \\
&\quad = \textstyle\sum_s^S \sum_d^D [(1-\gamma)RMC_{sd}^m + (\gamma)RMC_{sd}^p] \times RMQ_{sd} \\
&\quad + \left(\textstyle\sum_d^D [(1-\gamma)RC_d^m + (\gamma)RC_d^p] \times RTQ_d\right) \\
&\quad + \left(\textstyle\sum_d^D [(1-\gamma)OC_d^m + (\gamma)OC_d^p] \times OTQ_d\right) \\
&\quad + \left(\textstyle\sum_d^D [(1-\gamma)ShortPC_d^m + (\gamma)ShortPC_d^p] \times ShortPPQ_d\right) \\
&\quad + \left(\textstyle\sum_d^D [(1-\gamma)InCRM_d^m + (\gamma)InCRM_d^p] \times \frac{IRMQ_{d-1}+IRMQ_d}{2}\right) \\
&\quad + \left(\textstyle\sum_d^D [(1-\gamma)HC_d^m + (\gamma)HC_d^p] \times HL_d\right) \\
&\quad + \left(\textstyle\sum_d^D [(1-\gamma)InCPM_d^m + (\gamma)InCPM_d^p] \times \frac{IPQ_{d-1}+IPQ_d}{2}\right) \\
&\quad + \left(\textstyle\sum_d^D [(1-\gamma)FC_d^m + (\gamma)FC_d^p] \times FL_d\right) \\
&\quad + \left(\textstyle\sum_r^R \sum_d^D [(1-\gamma)TranC_{rd}^m + (\gamma)TranC_{rd}^p] \times TranQ_{rd}\right) \\
&\quad + \left(\textstyle\sum_r^R \sum_d^D [(1-\gamma)InCPR_{rd}^m + (\gamma)InCPR_{rd}^p] \times \frac{IRQ_{rd-1}+IRQ_{rd}}{2}\right) \\
&\quad + \left(\textstyle\sum_r^R \sum_d^D [(1-\gamma)PCLS_{rd}^m + (\gamma)PCLS_{rd}^p] \times ShortPRQ_{rd}\right) \qquad (4.8)
\end{aligned}
$$

where γ is the credibility level that can be used to measure the degree of trustworthiness or believability. In this study, the credibility level (γ) is set at 80%.

- **Constraints**

1. **Suppliers' Capacity for Providing Raw Materials**: The maximum quantity of raw materials that suppliers can deliver within a specific period, reflecting their production and logistical capabilities.

$$RMQ_{sd} \leq MaxSupCap_{sd} \qquad \forall\, s, d \qquad (4.9)$$

2. **Raw Material Availability**: The extent to which required raw materials are accessible from suppliers, considering factors like supply chain disruptions, inventory levels, and lead times for procurement.

$$\sum_{s=1}^{S} RMQ_{sd} \geq (RTQ_d + OTQ_d) \qquad \forall\, d \tag{4.10}$$

3. **Product Shortages at the Plant**: Occurs when the production facility lacks sufficient raw materials or components to meet the planned production targets, potentially causing delays, increased costs, or missed delivery deadlines.

$$ShortPPQ_d = IPQ_{d-1} - ShortPPQ_{d-1} + RTQ_d + OTQ_d - IPQ_d - \widetilde{D}e_{rd} \quad \forall\, r, d \tag{4.11}$$

4. **Labor Capacity:** Refers to the availability and ability of the workforce to meet production demands. It includes the number of workers, their skills, working hours, and productivity levels, ensuring that the production plant can operate efficiently without shortages or excessive overtime.

$$LH_d \times RTQ_d \leq L_d * 9{,}600 \qquad \forall\, d \tag{4.12}$$

5. **Workforce Balancing:** Refers to the strategic allocation of labor resources to match production needs and workloads. It involves adjusting staffing levels across different shifts or production stages to ensure efficient operations, minimize downtime, and avoid overworking employees while maintaining optimal productivity.

$$L_d = L_{(d-1)} + HL_d - FL_d \qquad \forall\, d \tag{4.13}$$

6. **Limitation of Regular Time Production:** Refers to the maximum production capacity that can be achieved during standard working hours without requiring overtime. This limitation is typically determined by factors such as available labor, equipment capacity, and operational hours, and plays a key role in managing production schedules and costs.

$$RTQ_d \leq 28{,}000 \ \forall\, d \tag{4.14}$$

7. **Limitation of Overtime Production:** Refers to the maximum amount of additional production that can be achieved beyond regular working hours through overtime labor. This limitation is often constrained by factors such as labor laws, employee availability, and increased labor costs, and must be carefully managed to optimize production while controlling costs.

$$OTQ_d \leq 7{,}000 \qquad \forall\, d \tag{4.15}$$

8. **Raw Material Inventory**: Refers to the stock of raw materials held by a company for production purposes. It ensures that the production process can continue smoothly without interruptions due to shortages. The inventory level is managed to balance the cost of holding materials with the need to meet production demands, while also accounting for factors like lead time, demand fluctuations, and storage costs.

$$\mathrm{IRMQ_d} = \mathrm{IRMQ_{(d-1)}} + \sum_{s=1}^{S} \mathrm{RMQ_{sd}} - (\mathrm{RTQ_d} + \mathrm{OTQ_d}) \qquad \forall\, \mathrm{d} \qquad (4.16)$$

9. **Limitation of Transferring Products to Retailers:** This refers to constraints in the ability to deliver products from the production facility to retail locations. These limitations can include factors like transportation capacity, logistical challenges, delivery schedules, or regulatory restrictions. Efficient management of these constraints ensures that retailers receive products on time, preventing stockouts or delays that could negatively impact sales and customer satisfaction.

$$\sum_{r}^{R} \mathrm{TranQ_{rd}} \leq \mathrm{RTQ_d} + \mathrm{OTQ_d} \qquad \forall\, \mathrm{d} \qquad (4.17)$$

10. **Minimum Retailer Service Level for Satisfying Demand:** This refers to the minimum level of product availability that retailers must maintain to meet customer demand. It ensures that retailers have enough stock to avoid stockouts, aiming to satisfy customers' needs consistently. Meeting this service level is crucial for maintaining customer satisfaction, loyalty, and competitive advantage in the market.

$$\mathrm{TranQ_{rd}} \geq 0.8 \times \widetilde{\mathrm{D}}\mathrm{e_{rd}} \qquad \forall\, \mathrm{r, d} \qquad (4.18)$$

11. **Product Shortages at The Retailer**: This refers to the situation where a retailer does not have enough stock of a product to meet customer demand. It typically results in lost sales, customer dissatisfaction, and potential damage to the retailer's reputation. Managing product shortages is crucial for maintaining an efficient supply chain and ensuring customer needs are met on time.

$$ShortPQR_{rd} = IRQ_{rd-1} - ShortPQR_{rd-1} + TranQ_{rd} - IRQ_{rd} - \widetilde{D}e_{rd} \qquad \forall\, r, d \qquad (4.19)$$

12. **Non-Negativity:** Constraints (3.20) – (3.23) ensure that all decision variable values are non-negative, with certain values required to be integers.

$$\mathrm{L_d, HL_d, FL_d, IRMQ_d} \geq 0 \text{ and Integer} \qquad \forall\, \mathrm{d} \qquad (4.20)$$

$$\mathrm{RTQ_d,\ OTQ_d, ShortPPQ_d, IRMQ_d,\ IPQ_d} \geq 0 \text{ and Integer} \qquad \forall\, \mathrm{d} \qquad (4.21)$$

$$\mathrm{RMQ_{sd}} \geq 0 \text{ and Integer} \qquad \forall\, \mathrm{s, d} \qquad (4.22)$$

$$\mathrm{TranQ_{rd},\ IRQ_{rd},\ ShortPRQ_{rd}} \geq 0 \text{ and Integer} \qquad \forall\, \mathrm{r, d} \qquad (4.23)$$

4.5 Results

The analysis results explore three distinct approaches: minimizing costs exclusively, minimizing downside risk using the Mean-Conditional Value at Risk Gap (MCVaRG) independently, and simultaneously minimizing both costs and downside risk. Each approach is thoroughly examined and compared to assess its relative effectiveness, strengths, and trade-offs. Through this comparative analysis, the study aims to provide valuable insights into the efficacy of these strategies, emphasizing the potential benefits of integrated optimization approaches that balance financial objectives with risk management. This balanced approach contributes to a more robust and resilient decision-making framework, capable of adapting to complex and uncertain environments.

- **Result of purely minimizing total supply chain operational costs**

Fuzzy Linear Programming (FLP) is applied to the Supply Chain Aggregate Production Planning (SCAPP) problem with the objective of minimizing total operational costs throughout the supply chain. The optimal results, as detailed in Table 4.11, highlight critical decision variables and strategies for cost reduction. Polyethylene Terephthalate (PET) is sourced from Suppliers 1 and 2, selected for their competitive pricing on raw materials. Initially, plastic bottle production operates within regular working hours. However, once production reaches 28,000 units, overtime is implemented to produce an additional 7,000 units. Any demand beyond this results in shortages. These shortages are particularly noticeable during the high-demand season, with 3,025 units of shortage occurring in period 1 and 4,001 units in period 6. The workforce is adjusted between 40 and 55 employees over the six-month period to accommodate fluctuating production needs. Retailer service levels are prioritized based on cost efficiency, with Retailer 5 attaining a 100% service level, followed by Retailer 4 at 99.00%, Retailer 6 at 96.00%, Retailer 1 at 93.39%, Retailer 2 at 91.71%, and Retailer 3 at 89.59%. This prioritization centers solely on cost minimization, disregarding the downside risk linked to cost uncertainty. Consequently, total operational costs are minimized to $7,712,875, but this approach simultaneously increases downside risk to $2,147,100, potentially surpassing acceptable risk levels for decision-makers. This trade-off emphasizes the crucial need for integrating risk considerations alongside cost-minimization efforts to ensure that decisions reflect a balance between cost efficiency and the management of uncertainty.

Table 4.11 Result of purely minimizing total supply chain operational costs.

	Value						
Minimizing Total Supply Chain Operation Costs	$7,712,875.00						
Downside Risk of Total Supply Chain Operation Costs	$2,147,100.00						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Number of Products Produced in the Regular Time in Period *d* (units)	28,000	28,000	20,364	25,455	28,000	28,000	
Number of Products Produced in the Overtime in Period *d* (units)	7,000	4,566	0	0	4,950	7,000	
Number of Shortage Products at the Plant in Period *d* (units)	3,025	0	0	0	0	4,001	
Number of Labors in Period *d* (person)	55	55	40	50	55	55	
	Number of Raw Materials Supplied by Supplier *s* in Period *d* (units)						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Supplier 1	**25,000**	**25,000**	**20,364**	**25,000**	**25,000**	**25,000**	
Supplier 2	**10,000**	**7,566**	**0**	**455**	**7,950**	**10,000**	
Supplier 3	0	0	0	0	0	0	
Supplier 4	0	0	0	0	0	0	
	Transportation Quantity of Products to Retailer *r* in Period *d* (units)						Average Service Level (%)
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Retailer 1	4,685	4,747	3,080	3,935	4,780	4,879	93.39%
Retailer 2	3,711	3,772	1,526	2,770	4,194	4,021	91.71%
Retailer 3	2,805	2,281	838	1,841	2,504	2,755	89.59%
Retailer 4	**7,168**	**6,792**	**4,878**	**5,250**	**7,162**	**8,529**	**99.00%**
Retailer 5	**9,380**	**8,678**	**6,125**	**6,131**	**8,081**	**9,060**	**100.00%**
Retailer 6	**7,251**	**6,296**	**3,917**	**5,528**	**6,229**	**5,756**	**96.00%**

*Highlighted cells present the suppliers, who are chosen to supply Polyethylene Terephthalate (PET) and the retailers, who are satisfied with the highest percentage of the service level.

- **Result of purely minimizing Mean-Conditional Value at Risk Gap (MCVaRG) of total supply chain operational costs**

Fuzzy Linear Programming (FLP) is utilized to tackle the Supply Chain Aggregate Production Planning (SCAPP) problem, with a primary objective of minimizing the Mean-Conditional Value at Risk Gap (MCVaRG) associated with total operational costs across the supply chain. The optimal results, outlined in Table 4.12, identify key decision variables focused on reducing downside risk. Suppliers 3 and 4 are chosen to provide Polyethylene Terephthalate (PET) due to their lower downside risk, which reflects a reduced likelihood of cost escalations. Despite these supplier adjustments, production quantities, shortages, and workforce levels remain consistent with the cost-minimization strategy, as the production plan continues to satisfy retailer demand. Under this approach, Retailer 1 achieves a 100% service level, followed by Retailer 4 with an average service level of 99.00%, Retailer 2 at 97.61%, Retailer 5 at 94.80%, Retailer 3 at 94.19%, and Retailer 6 at 92.34% over the six-month period. This prioritization emphasizes minimizing downside risk from cost uncertainty, with the selection of suppliers and retailers driven by risk mitigation rather than pure cost considerations. As a result, the downside risk is notably reduced to $1,731,676.04, but the total supply chain operational costs rise to $8,429,300, potentially exceeding acceptable thresholds for decision-makers. This trade-off highlights the critical challenge of balancing cost efficiency with risk reduction in supply chain planning, emphasizing the need for decision-makers to carefully weigh both financial and risk management objectives.

Table 4.12 Result of purely minimizing Mean-Conditional Value at Risk Gap (MCVaRG) of total supply chain operational costs.

	Value						
Minimizing Downside Risk of total supply chain operation costs	$1,731,676.00						
Total Supply Chain Operation Costs	$8,429,300.00						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Number of Products Produced in the Regular Time in Period *d* (units)	28,000	28,000	20,364	25,455	28,000	28,000	
Number of Products Produced in the Overtime in Period *d* (units)	7,000	4,566	0	0	4,950	7,000	
Number of Shortage Products at the Plant in Period *d* (units)	3,025	0	0	0	0	4,001	
Number of Labors in Period *d* (person)	55	55	40	50	55	55	
	Number of Raw Materials Supplied by Supplier *s* in Period *d* (units)						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Supplier 1	0	0	0	0	0	0	
Supplier 2	0	0	0	0	0	0	
Supplier 3	**10,000**	**7,566**	**0**	**455**	**7,950**	**10,000**	
Supplier 4	**25,000**	**25,000**	**25,000**	**25,000**	**25,000**	**25,000**	
	Transportation Quantity of Products to Retailer *r* in Period *d* (units)						Average Service Level (%)
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Retailer 1	**5,460**	**4,747**	**3,080**	**3,935**	**4,780**	**5,951**	**100.00%**
Retailer 2	**4,113**	**3,772**	**1,526**	**2,770**	**4,194**	**4,905**	**97.61%**
Retailer 3	3,475	2,281	838	1,841	2,504	2,755	94.19%
Retailer 4	**7,168**	**6,792**	**4,878**	**5,250**	**7,162**	**8,529**	**99.00%**
Retailer 5	8,704	8,678	6,125	6,131	8,081	7,266	94.80%
Retailer 6	6,080	6,296	3,917	5,528	6,229	5,594	92.34%

*Highlighted cells present the suppliers, who are chosen to supply Polyethylene Terephthalate (PET) and the retailers, who are satisfied with the highest percentage of the service level.

- **Result of considering both costs and downside risk minimization**

Fuzzy Linear Programming (FLP) is applied to the Supply Chain Aggregate Production Planning (SCAPP) problem with the dual objective of minimizing both total operational costs and downside risk (MCVaRG) simultaneously. The optimal results from this model, shown in Table 4.13, illustrate the effectiveness of a multi-objective fuzzy linear programming approach, achieving an overall satisfaction level of 94.93% by maximizing the minimum satisfaction level. The minimum total operational costs are calculated at \$7,832,100, yielding a satisfaction level of 98.37%, while the minimum downside risk (MCVaRG) is \$1,921,500 with a satisfaction level of 94.93%. Table 4.13 presents the variable values, highlighting the optimal balance between minimizing both total costs and downside risk. In this model, Suppliers 2 and 3 are selected to provide Polyethylene Terephthalate (PET) to the production plant due to their balanced cost and moderate downside risk profiles. The production plant's operational strategy, including regular and overtime hours, remains the same as in previous strategies, aligning with the demand of the retailers. Retailer 4 is prioritized, achieving a 100% service level, followed by Retailer 2 at 98.29%, Retailer 6 at 96.67%, Retailer 1 at 95.33%, Retailer 5 at 93.71%, and Retailer 3 at 92.30% over the six-month period. This approach aims to strike a compromise between minimizing costs and reducing the risk from uncertainty, selecting suppliers and retailers that provide the best-balanced solution. In this case, Retailers 1, 3, and 5 are considered lower priorities for fulfillment. Retailer 1 is selected last due to its low cost but higher risk, Retailer 3 is deprioritized because of both higher cost and higher risk, and Retailer 5 is ranked last for its low risk but higher cost. This strategy ensures that a balanced solution is achieved while addressing both cost efficiency and risk mitigation.

Table 4.13 Result of considering both cost efficiency and risk mitigation.

	Value						
Overall Satisfaction Level	94.93%						
Satisfaction Level of Minimizing Total Supply Chain Operation Costs	98.37%						
Satisfaction Level of Minimizing Downside Risk of Total Supply Chain Operation Costs	94.93%						
Minimizing Total Supply Chain Operation Costs	$7,832,100.00						
Minimizing Downside Risk of Total Supply Chain Operation Costs	$1,921,500.00						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Number of Products Produced in the Regular Time in Period *d* (units)	28,000	28,000	20,364	25,455	28,000	28,000	
Number of Products Produced in the Overtime in Period *d* (units)	7,000	4,566	0	0	4,950	7,000	
Number of Shortage Products at the Plant in Period *d* (units)	3,025	0	0	0	0	4,001	
Number of Labors in Period *d* (person)	55	55	40	50	55	55	
	Number of Raw Materials Supplied by Supplier *s* in Period *d* (units)						
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	
Supplier 1	0	0	0	0	0	0	
Supplier 2	**25,000**	**25,000**	**25,000**	**25,000**	**25,000**	**25,000**	
Supplier 3	**10,000**	**7,566**	**0**	**455**	**7,950**	**10,000**	
Supplier 4	0	0	0	0	0	0	
	Transportation Quantity of Products to Retailer *r* in Period *d* (units)						Average Service

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Level (%)
Retailer 1	5,050	5,086	3,300	4,216	5,121	5,776	95.33%
Retailer 2	**4,535**	**4,041**	**1,635**	**2,968**	**4,493**	**5,285**	**98.29%**
Retailer 3	3,090	2,444	898	1,972	2,682	3,289	92.30%
Retailer 4	**8,108**	**7,277**	**5,226**	**5,625**	**7,673**	**9,138**	**100.00%**
Retailer 5	8,250	9,298	6,562	6,569	8,658	8,307	93.71%
Retailer 6	**7,207**	**6,746**	**4,197**	**5,922**	**6,674**	**6,991**	**96.67%**

*Highlighted cells present the suppliers, who are chosen to supply Polyethylene Terephthalate (PET) and the retailers, who are satisfied with the highest percentage of the service level.

4.5.1 Result Comparison

Table 4.14 and Figure 4.5 present a comparison of the outcomes from three fuzzy linear programming models, focusing on three key aspects: supplier selection, retailer selection, and the values of objective functions. In the first model, which minimizes total supply chain operational costs, the focus is solely on cost, disregarding downside risk. This approach leads to the selection of suppliers offering the lowest price for Polyethylene Terephthalate (PET) and prioritizes retailers with the lowest cost to fulfill their demands. These results align with traditional fuzzy programming methods, where symmetrical skewness is applied to represent fuzzy numbers, which are then defuzzified using standard techniques. The decision-making criterion here ultimately favors the lowest cost option. In the second model, which minimizes downside risk without considering costs, the objective is to reduce downside risk, leaving total operational costs unaccounted for. This results in selecting suppliers with the lowest downside risk in raw material prices and prioritizing retailers who contribute to minimizing risk. The focus in this model is on risk reduction rather than cost minimization. The third scenario examines the simultaneous minimization of both total operational costs and downside risk. This model seeks a balanced compromise between reducing costs and mitigating risk, a capability not fully captured by traditional fuzzy programming methods due to their inability to address the unsymmetrical skewness of risk, particularly downside risk. In this case, suppliers with moderate costs and downside risk are selected, and the production plant aims to meet the demands of retailers that help reduce both cost and risk. As shown in Figure 4.5, the model minimizes the total supply chain operation costs while accounting for potential risks. This balance is achieved by maximizing the minimum satisfaction level between cost minimization and risk reduction, ensuring that neither objective is excessively compromised. However, this balance is only possible when the impact of downside risk on costs is substantial enough to warrant its consideration in the decision-making process.

Table 4.14 Result comparison of three fuzzy linear programming models' outcomes.

	FLP Model (Minimizing total supply chain operation costs)	FLP Model (Minimizing downside risk of total supply chain operation costs)	MOFLP Model (Minimizing both total supply chain operation costs and downside risk of total supply chain operation costs)
Supplier Selection	Supplier 1 and Supplier 2	Supplier 3 and Supplier 4	Supplier 2 and Supplier 3
Retailer Selection: (Average Service Level)	Retailer 1: 93.39% Retailer 2: 91.71% Retailer 3: 89.59% ***Retailer 4: 99.00%*** ***Retailer 5: 100.00%*** ***Retailer 6: 96.00%***	***Retailer 1: 100.00%*** ***Retailer 2: 97.61%*** Retailer 3: 94.19% ***Retailer 4: 99.00%*** Retailer 5: 94.80% Retailer 6: 92.34%	Retailer 1: 95.33% ***Retailer 2: 98.29%*** Retailer 3: 92.30% ***Retailer 4: 100.00%*** Retailer 5: 93.71% ***Retailer 6: 96.67%***
Minimizing Total Supply Chain Operation Costs	$7,712,875.00	$8,429,300.00	$7,832,100.00
Minimizing Downside Risk (MCVaRG)	$2,147,100.00	$1,731,676.00	$1,921,500.00
Possible Range of Total Supply Chain Operation Costs	From $7,712,875.00 to $9,859,975.00	From $8,429,300.00 to $10,160,976.00	From $7,832,100.00 to $9,753,600.00

*Bold and italic retailers of each model represent the first three retailers, who are satisfied with the highest percentage of the service level.

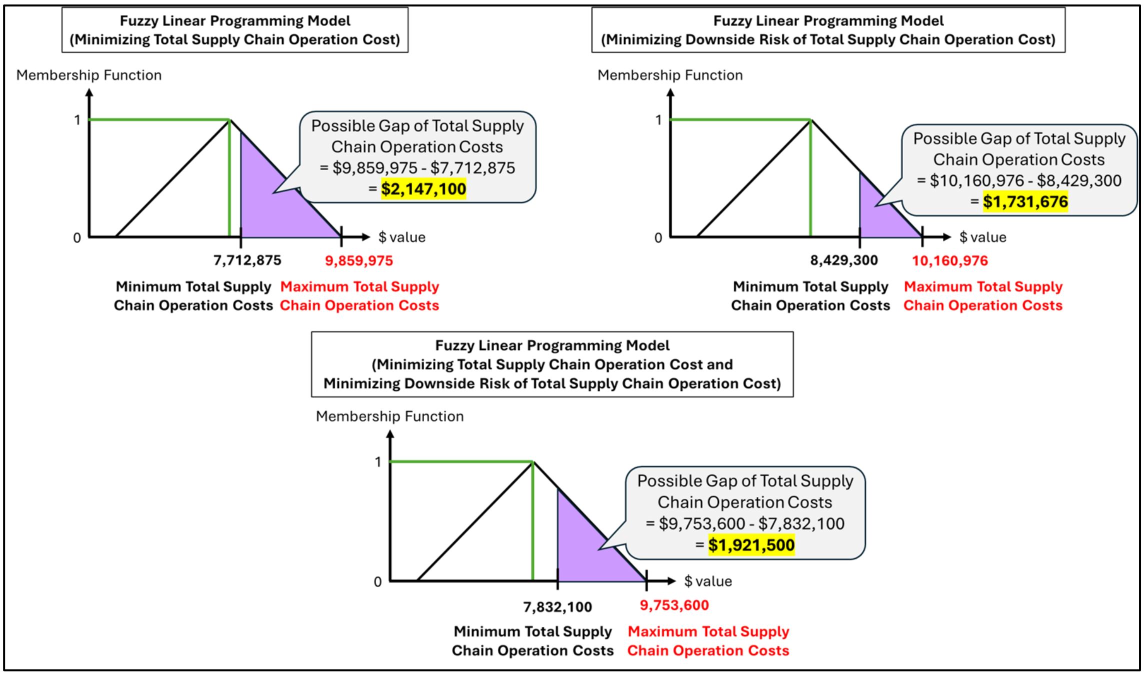

Figure 4.5. Demonstration of the calculation of the maximum total supply chain operation costs in the pessimistic case.

4.5.2 Result Validation

The proposed Multi-Objective Fuzzy Linear Programming (MOFLP) model incorporates a credibility parameter (γ), which can be adjusted based on the decision maker's perception of downside risk (MCVaRG). This parameter has the potential to impact on the outcomes of the selected plans and their comparative results. Therefore, a sensitivity analysis of this parameter will be performed, as detailed below.

- **Sensitivity analysis of percentage of credibility**

As previously mentioned, the credibility parameter represents the level of trust or belief in the results, with a higher credibility percentage signifying greater confidence in the outcomes. In this context, downside risk fluctuates as the confidence level (γ) ranges from 0.5 to 1. The sensitivity analysis explores how the credibility percentage (γ) affects the results within this range, as detailed in Tables 4.15 and 4.16. When γ = 0.5 (reflecting the most likely scenario), the model produces the lowest total supply chain operational costs and downside risk. Conversely, when γ = 1 (representing the most pessimistic scenario), the model results in the highest total supply chain operational costs and downside risk.

Table 4.15 Result of sensitivity analysis of percentage of credibility.

Percentage of Credibility (γ)	Minimizing Total Supply Chain Operation Costs	Minimizing Downside Risk (MCVaRG)
50%	$6,665,479.13	$888,131.70
60%	$7,097,234.97	$1,151,982.54
70%	$7,455,783.73	$1,530,145.78
80%	**$7,832,100.00**	**$1,921,500.00**
90%	$8,271,553.91	$2,365,962.17
100%	$8,649,609.54	$2,740,306.33

*Highlighted cell presents the results of applying γ at 80%, which was initially used in this case study as a benchmark.

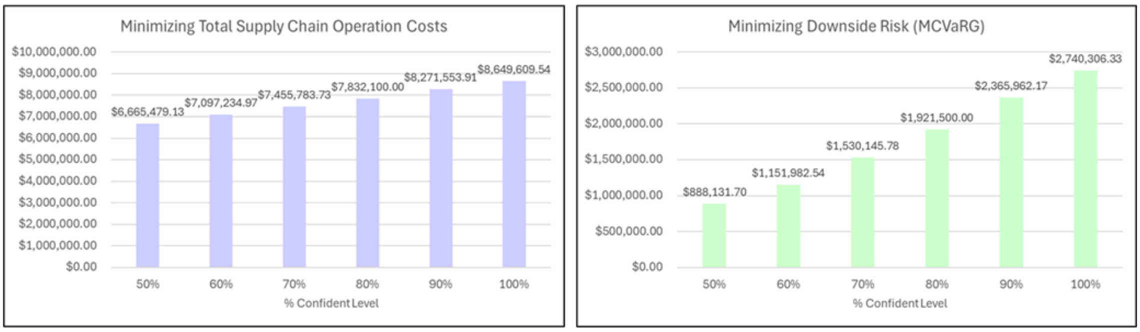

Figure 4.6 Outcomes of the obtained objective values while varying the percentage of credibility.

As illustrated in Table 4.15 and Figure 4.6, increasing the credibility percentage (γ) from 50% to 100% results in a rise in total supply chain operation costs, from $6,665,479.13 to $8,649,609.54. Likewise, the minimum downside risk (MCVaRG) grows from $888,131.70 to $2,740,306.33. This information is crucial for decision-makers, as it highlights the potential variability in outcomes, allowing for proactive adjustments. A higher credibility percentage (γ) represents a more pessimistic scenario, with both operational costs and downside risk increasing. Thus, a higher γ reflects a greater level of confidence in the reliability of the results.

Table 4.16 Result of percentages of average service level while varying percentage of credibility of each retailer.

	Percentage of Credibility (γ)					
	50%	60%	70%	80%	90%	100%
Retailer 1	100.00%	100.00%	97.42%	95.33%	94.56%	93.03%
Retailer 2	100.00%	100.00%	100.00%	98.29%	97.64%	96.04%
Retailer 3	100.00%	94.69%	93.49%	92.30%	91.85%	88.99%
Retailer 4	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Retailer 5	100.00%	98.27%	94.52%	93.71%	92.82%	90.85%
Retailer 6	100.00%	100.00%	100.00%	96.67%	95.76%	94.56%

Table 4.16 shows that as the credibility percentage (γ) increases from 50% to 100%, the service level percentages for each retailer are affected. A higher γ signifies increased retailer demands, which are fuzzy and defuzzified using chance-constrained programming. In the most likely scenario ($\gamma = 0.5$), all retailers' demands can be fully met, resulting in a 100% service level. However, as γ increases, some retailers experience a gradual decline in service levels due to the plant's capacity limits. Specifically, service levels decrease in the following orders: Retailer 3, Retailer 5, Retailer 1, Retailer 6, and Retailer 2. Notably, Retailer 4 consistently maintains a 100% service level, even under the most pessimistic scenario ($\gamma = 1$). This is because Retailer 4 faces the highest penalty cost for lost sales and experiences the greatest uncertainty risk, as highlighted in Table 4.15. Allowing Retailer 4 to face shortages would result in significant costs and risks.

4.6 Discussion and Conclusion

This study provides significant managerial insights and implications for decision-makers involved in Supply Chain Aggregate Production Planning (SCAPP). Traditionally, SCAPP has primarily focused on minimizing total supply chain operational costs, often neglecting the associated risks that can significantly impact decision-making. The proposed methodology introduces an innovative approach for managing unsymmetrical skewness, offering a more comprehensive risk assessment framework that improves decision-making in uncertain environments. By addressing various types of skewness, the model enables more accurate risk evaluations, leading to better resource allocation and enhanced operational efficiency. Consequently, organizations can develop more tailored strategies that optimize performance in dynamic and complex supply chain scenarios, ensuring greater resilience against unpredictable situations.

However, the proposed model primarily addresses the downside risk of uncertainty, overlooking the potential optimistic side of cost outcomes. By focusing on minimizing the adverse effects of worst-case scenarios through the Mean-Conditional Value at Risk Gap (MCVaRG), the approach primarily caters to decision-makers who are risk-averse and prioritize mitigating negative impacts. While this method is effective for managing risk in scenarios where uncertainty poses significant threats, it does not fully capture the full spectrum of uncertainties, especially those that could lead to more favorable outcomes.

The integration of multiple objectives within SCAPP, particularly under conditions of uncertainty, is a crucial advancement. A single-objective approach often falls short in managing fluctuating market conditions and unpredictable situations. Adopting multiple objectives enables decision makers to develop more adaptable and resilient strategies, addressing vulnerabilities across the entire supply chain. This flexibility allows organizations to navigate uncertainties more effectively, ensuring long-term stability and sustainability. The framework proposed in this study equips decision-makers with the tools to assess the potential range of costs and manage risks proactively, contributing to the creation of more resilient and adaptive supply chain networks. However, further research is needed to explore how these multiple objectives can be optimized in increasingly complex and volatile supply chain environments.

The introduction of a novel multiple-objective fuzzy linear programming model in this research offers a flexible and robust framework for balancing cost minimization and downside risk mitigation. This model enhances decision-making capabilities in dynamic and uncertain environments, providing organizations with valuable insights to make more informed, risk-adjusted decisions. Despite its advantages, the study highlights several areas for future research. One critical avenue is the exploration of alternative fuzzy distributions, which could capture a broader range of uncertainties inherent in supply chain operations. Additionally, integrating advanced optimization techniques, such as meta-heuristic algorithms and machine learning, to further enhance the model's capabilities. These techniques could enable the development of more sophisticated and adaptable solutions, capable of adjusting to real-time changes in supply chain conditions. Additionally, future research should consider incorporating various risk measures and optimization methods that align with different risk attitudes and priorities of decision-makers. This would allow for a more personalized approach to supply chain optimization, accommodating the diverse needs of organizations.

Chapter 5
A New Integrated Fuzzy Optimization Approach for Sustainable Supply Chain Planning subjected to Sustainability and Uncertain Environments

This chapter presents a novel integrated fuzzy optimization approach designed to enhance sustainable supply chain planning in the face of increasing complexity and uncertainty. The proposed framework simultaneously addresses economic, environmental, and social dimensions of sustainability while incorporating the imprecise and ambiguous nature of real-world supply chain environments. This chapter begins with a comprehensive problem description that highlights the challenges of aligning sustainability goals with supply chain performance under uncertain conditions. The main contributions of the study are then outlined, emphasizing the development of an innovative model that integrates fuzzy logic with sustainability-driven decision-making. The methodology section details the techniques and tools employed to construct the proposed model, incorporating fuzzy set theory and multi-objective optimization to address uncertainty and competing priorities. A real-world case study is introduced to demonstrate the applicability and effectiveness of the proposed approach in a practical context. This is followed by a formal presentation of the mathematical notations and the formulation of the fuzzy optimization model. The results are analyzed in depth, including comparisons with traditional models, validation of the findings, and an exploration of the social and environmental implications of the planning decisions. Finally, this chapter concludes with a discussion of key insights, contributions to the field of sustainable supply chain management, and recommendations for future research. Through this comprehensive approach, this chapter aims to contribute to the development of more resilient, responsible, and adaptable supply chain systems in uncertain and sustainability-sensitive environments.

5.1 Problem Statement and Contributions

In today's globalized and highly interconnected world, supply chains are becoming increasingly complex and unpredictable. Factors such as market volatility, geopolitical tensions, fluctuating demand patterns, and climate change-driven disruptions frequently push supply chain operations beyond normal conditions. Traditional supply chain models, which often prioritize efficiency and cost minimization, struggle to adapt to these dynamic challenges. To ensure long-term sustainability and operational stability, modern supply chains must be designed with resilience, enabling them to anticipate, absorb, and recover from disruptions while maintaining performance and service levels. Sustainability in supply chain management extends beyond mere cost efficiency; it involves an approach that integrates economic, social, and environmental dimensions. By adopting sustainable practices, organizations can enhance not only their long-term profitability and competitiveness but also contribute to broader societal and environmental well-being. A well-designed sustainable supply chain mitigates risks associated with resource depletion, regulatory pressures, and ethical concerns while improving stakeholder trust and corporate reputation.

One of the critical challenges in achieving sustainable supply chain management is the presence of uncertainty. Supply chains operate in environments where decision variables such as supplier reliability, demand fluctuations, transportation delays, and production capacity are inherently uncertain. Addressing these uncertainties with traditional deterministic models often leads to suboptimal decisions that fail to capture the full range of potential risks and opportunities. To overcome this limitation, this study integrates sustainability principles with fuzzy set theory, providing a more flexible and adaptive approach to managing uncertainty. Fuzzy set theory offers a powerful framework for modeling imprecise and ambiguous data, enabling decision-makers to incorporate various degrees of uncertainty into supply chain planning. By applying fuzzy optimization techniques, this study enhances the robustness of supply chain decision-making, allowing for more informed and resilient planning. The proposed approach equips decision-makers with adaptable strategies that can dynamically adjust to evolving market conditions, disruptions, and sustainability requirements.

In today’s rapidly changing business environment, achieving a balance between sustainability and uncertainty management is essential for long-term success. This study provides valuable insights into how organizations can develop sustainable supply chain networks that are not only efficient but also capable of withstanding unforeseen challenges. By integrating sustainability with fuzzy set theory, this research contributes to the advancement of supply chain management, offering practical solutions for navigating complexity while fostering environmental and social responsibility.

This study makes notable contributions to the field of supply chain aggregate production planning by integrating sustainability and uncertainty into the decision-making framework. Traditional supply chain planning often prioritizes efficiency and cost minimization, overlooking the broader impacts of sustainability and the pervasive uncertainties affecting supply chain operations. This study expands the scope of supply chain planning by incorporating sustainability considerations, which encompass economic, environmental, and social dimensions. Furthermore, it addresses the critical challenge of uncertainty, which arises from unpredictable demand fluctuations, supply disruptions, and volatile market conditions. Conventional deterministic models are inadequate in handling these uncertainties, leading to rigid and suboptimal strategies. By explicitly integrating uncertainty into the supply chain planning framework, this study enhances decision-making robustness and adaptability, enabling organizations to maintain operational stability in dynamic environments. Additionally, this study introduces a novel integrated fuzzy optimization framework that combines Chance-Constrained Programming (CCP) and Intuitionistic Fuzzy Linear Programming (IFLP). This innovative approach improves the accuracy, flexibility, and reliability of supply chain decisions. CCP allows decision-makers to incorporate probabilistic constraints, ensuring that critical supply chain conditions are met with a predefined level of confidence, while IFLP accounts for considering both satisfaction and non-satisfaction levels, enhancing the model's ability to handle vagueness and ambiguity. This dual-layer uncertainty representation provides a more realistic and adaptable decision-making framework, advancing the field of supply chain optimization.

5.2 Methodology

This section presents the traditional fuzzy optimization approach and the new integrated fuzzy optimization approach, highlighting their respective methodologies. The traditional fuzzy optimization approach has been widely utilized in supply chain planning to handle uncertainty by representing uncertain parameters as fuzzy numbers and solving optimization problems through defuzzification techniques. However, this conventional approach often oversimplifies uncertainty, leading to suboptimal decision-making, especially in complex and dynamic supply chain environments. To address these challenges, this study introduces a new integrated fuzzy optimization approach that enhances decision-making by incorporating Chance-Constrained Programming (CCP) and Intuitionistic Fuzzy Linear Programming (IFLP). This novel framework provides a more robust, adaptive, and realistic optimization strategy.

- **Traditional Fuzzy Optimization Approach**

In this study, the traditional fuzzy optimization approach as shown in Figure 5.1 serves as a benchmark for evaluating the effectiveness of the proposed integrated fuzzy optimization method. By comparing both approaches, this study highlights the advantages and improvements introduced by the new methodology. The key procedural steps involved in the traditional fuzzy optimization approach are outlined as follows:

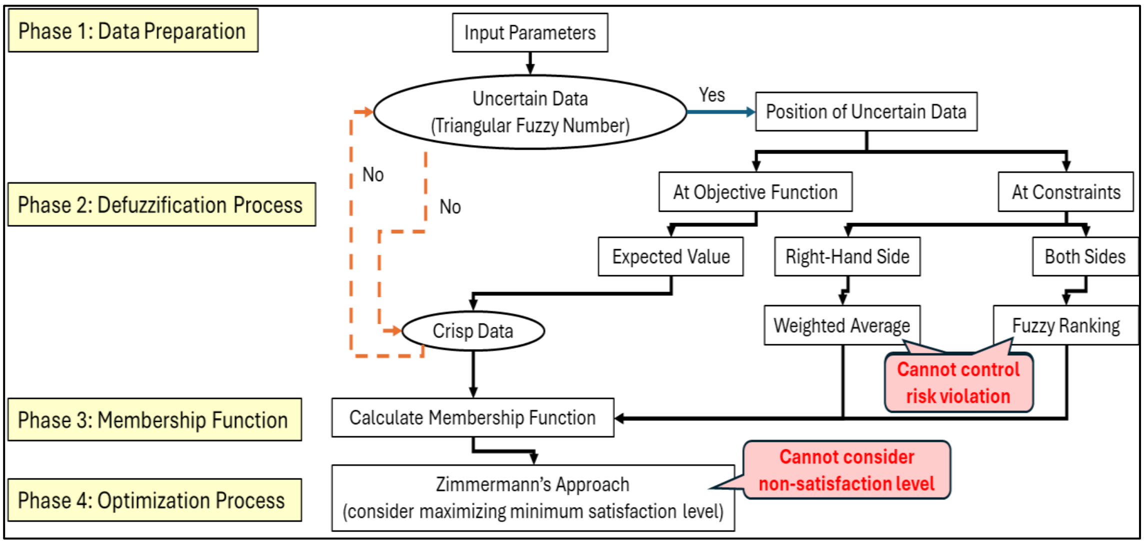

Figure 5.1. Methodology of traditional fuzzy optimization approach.

- **Phase 1: Data Preparation**

In the context of sustainable supply chain planning under uncertain environments, the input parameters can be classified into two categories: 1) Crisp data, which are accurately known with certainty, and 2) Uncertain data, typically modeled using Triangular Fuzzy Numbers (TFNs) to account for variability and imprecision.

- **Phase 2: Defuzzification Process**

At this stage, all fuzzy data are converted into crisp values through various defuzzification methods, which are selected based on the specific nature of the data's fuzziness within the model.

1. **Defuzzification approach for objective function**

The Expected Value (EV) approach is a traditional defuzzification method applied to the objective function, where it evaluates the overall average performance of the function.

$$EV(\tilde{Z}) = \frac{\frac{Z^{o}+Z^{m}}{2}+\frac{Z^{m}+Z^{p}}{2}}{2} = \frac{Z^{o}+2Z^{m}+Z^{p}}{4} \quad (5.1)$$

where Z^{o}, Z^{m}, and Z^{p} are values of objective functions in optimistic, most likely, and pessimistic situations, respectively.

2. **Defuzzification approach for constraint**

Weighted Average (WA) approach is a traditional defuzzification method employed for the right-hand side of constraints. It transforms fuzzy data by assigning appropriate weights to optimistic, most likely, and pessimistic scenarios, ensuring that the total weight sums to 1. However, it is crucial to recognize that the WA approach is not suitable for handling risk violations.

$$w^{o}b^{o} + w^{m}b^{m} + w^{p}b^{p} \quad (5.2)$$

$$w^{o} + w^{m} + w^{p} = 1 \quad (5.3)$$

where b^{o}, b^{m}, and b^{p} are values of available resource in optimistic, most likely, and pessimistic situations, respectively. w^{o}, w^{m}, and w^{p} are assigned weights in optimistic, most likely, and pessimistic situations, respectively.

- **Phase 3: Membership Function**

This process is used to normalize the different units of multiple objective functions onto a common scale, ranging from 0.0 to 1.0, and is referred to as the satisfaction level.

1. **Membership function for minimization of the objective function**

$$\mu_{Z_i} = \begin{cases} 1, & Z_i \le Z_i^{PIS} \\ \frac{Z_i^{NIS}-Z_i}{Z_i^{NIS}-Z_i^{PIS}}, & Z_i^{PIS} \le Z_i \le Z_i^{NIS} \\ 0, & Z_i \ge Z_i^{NIS} \end{cases} \quad (5.4)$$

2. Membership function for maximization of the objective function

$$\mu_{Z_i} = \begin{cases} 1, & Z_i \geq Z_i^{PIS} \\ \frac{Z_i - Z_i^{NIS}}{Z_i^{PIS} - Z_i^{NIS}}, & Z_i^{NIS} \leq Z_i \leq Z_i^{PIS} \\ 0, & Z_i \leq Z_i^{NIS} \end{cases} \tag{5.5}$$

where Z_i^{NIS} is the maximum bound for minimizing objective of the minimum bound for maximizing objective and Z_i^{PIS} is the maximum bound for maximizing objective or the minimum bound for minimizing objective.

- **Phase 4: Optimization Process**

This stage aims to identify the best compromise solution for multiple objective fuzzy linear programming problems. Zimmermann's approach is applied by maximizing the minimum satisfaction level across the multiple objective functions, with each objective function given equal importance.

Maximize μ_Z

Subjected to: $x \in F(x)$

$$\mu_Z \leq \mu_{Z_i}, \quad i = 1, 2, \ldots, I \tag{5.6}$$

where μ_Z is a minimum value of the satisfaction levels from the multiple objective functions and μ_{Z_i} is the satisfaction level of each objective function.

- **A New Integrated Fuzzy Optimization Approach**

The contribution of the new integrated fuzzy optimization approach lies in improving the effectiveness of the traditional fuzzy optimization method by providing better control over risk violations of constraints. It also allows decision-makers to simultaneously evaluate both satisfaction and non-satisfaction levels. This approach distinguishes itself from the traditional fuzzy optimization method in two key aspects, as outlined in Figure 5.2.

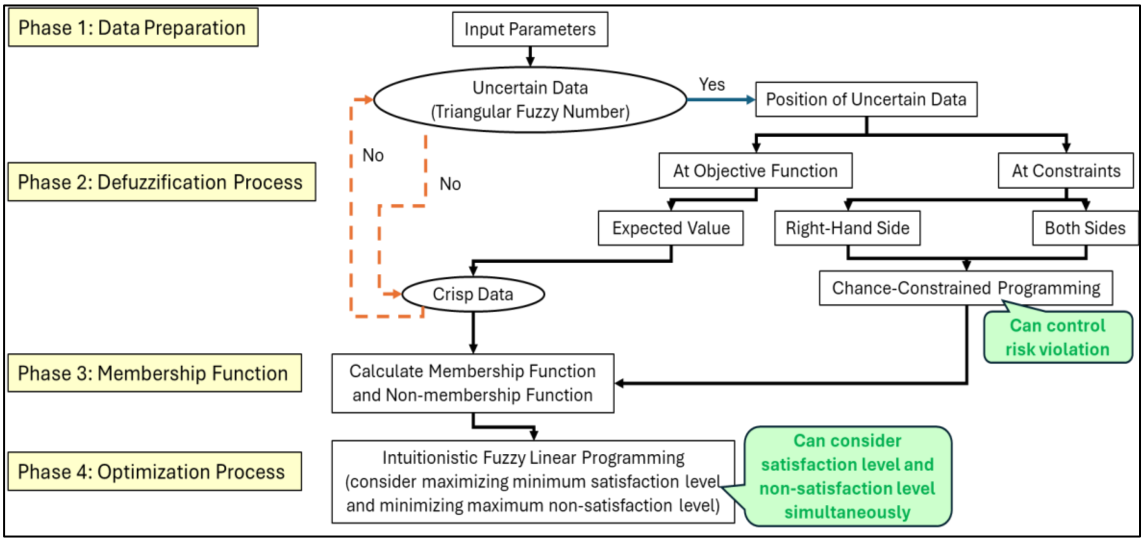

Figure 5.2. Methodology of a new integrated fuzzy optimization approach.

- **Phase 1: Data Preparation**

In the context of sustainable supply chain planning under uncertain environments, input parameters are divided into two categories: 1) Crisp data, which are precisely known, and 2) Uncertain data, typically represented by Triangular Fuzzy Numbers (TFNs).

- **Phase 2: Defuzzification Process**

At this stage, all fuzzy data are transformed into crisp values, allowing for effective control of risk violations in the constraints.

1. **Defuzzification approach for objective function**

The Expected Value (EV) approach is applied to defuzzify fuzzy data for the objective function, as illustrated in Equation (5.1).

2. **Defuzzification approach for constraint**

The Chance-Constrained Programming (CCP) approach is introduced as a defuzzification method for the right-hand side of the constraints. Equations (5.7) and (5.8) are applicable based on the assigned credibility level percentage (γ).

$$Cr\{\textstyle\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}_i\} \geq \gamma$$

$$when\ (0 \leq \gamma \leq 0.5): ax \leq (2\gamma)b^m + (1-2\gamma)b^p \qquad (5.7)$$

$$when\ (0.5 < \gamma \leq 1): ax \leq (2\gamma-1)b^o + (2-2\gamma)b^m \qquad (5.8)$$

where b^o, b^m, and b^p are values of available resource in optimistic, most likely, and pessimistic situations, respectively.

- **Phase 3: Membership Function and Non-Membership Function**

This process normalizes the different units of multiple objective functions to a common scale ranging from 0.0 to 1.0, known as the satisfaction level, as outlined in Equations (5.4) and (5.5). Moreover, the proposed approach allows decision-makers to consider both satisfaction and non-satisfaction levels simultaneously. As a result, Equations (5.9) and

(5.10) can be used to calculate the non-membership function.

1. Non-Membership Function for Minimization of the Objective Function

$$\tau_{Z_i} = \begin{cases} 1, & Z_i \geq Z_i^{PIS} \\ \frac{Z_i - Z_i^{NIS}}{Z_i^{PIS} - Z_i^{NIS}}, & Z_i^{NIS} \leq Z_i \leq Z_i^{PIS} \\ 0, & Z_i \leq Z_i^{NIS} \end{cases} \tag{5.9}$$

2. Non-Membership Function for Maximization of the Objective Function

$$\tau_{Z_i} = \begin{cases} 1, & Z_i \leq Z_i^{PIS} \\ \frac{Z_i^{NIS} - Z_i}{Z_i^{NIS} - Z_i^{PIS}}, & Z_i^{PIS} \leq Z_i \leq Z_i^{NIS} \\ 0, & Z_i \geq Z_i^{NIS} \end{cases} \tag{5.10}$$

where Z_i^{NIS} is the maximum bound for minimizing objective of the minimum bound for maximizing objective and Z_i^{PIS} is the maximum bound for maximizing objective or the minimum bound for minimizing objective.

- **Phase 4: Optimization Process**

The Intuitionistic Fuzzy Linear Programming (IFLP) approach is introduced to identify the optimal compromise solution for fuzzy linear programming problems with multiple objectives. This is accomplished by simultaneously maximizing the minimum satisfaction level and minimizing the maximum non-satisfaction level of the objective functions.

$$\begin{aligned} & Maximize\ \mu_Z - \tau_Z \\ & Subjected\ to: x \in F(x) \\ & \qquad \mu_Z \leq \mu_{Z_i}, \quad i = 1, 2, \ldots, I \\ & \qquad \tau_Z \leq \tau_{Z_i}, \quad i = 1, 2, \ldots, I \end{aligned} \tag{5.11}$$

where μ_Z and τ_Z are the membership function and the non-membership function of each objective function, respectively.

5.3 Case Study

A numerical case study on sustainable supply chain planning is conducted to assess the effectiveness of the integrated fuzzy optimization approach. In this case, two types of raw materials are provided by four suppliers to two manufacturers, each with different capacity levels. The finished products are then distributed through three distribution centers, each with its own capacity, to five customers. The structure of this sustainable supply chain network is shown in Figure 5.3.

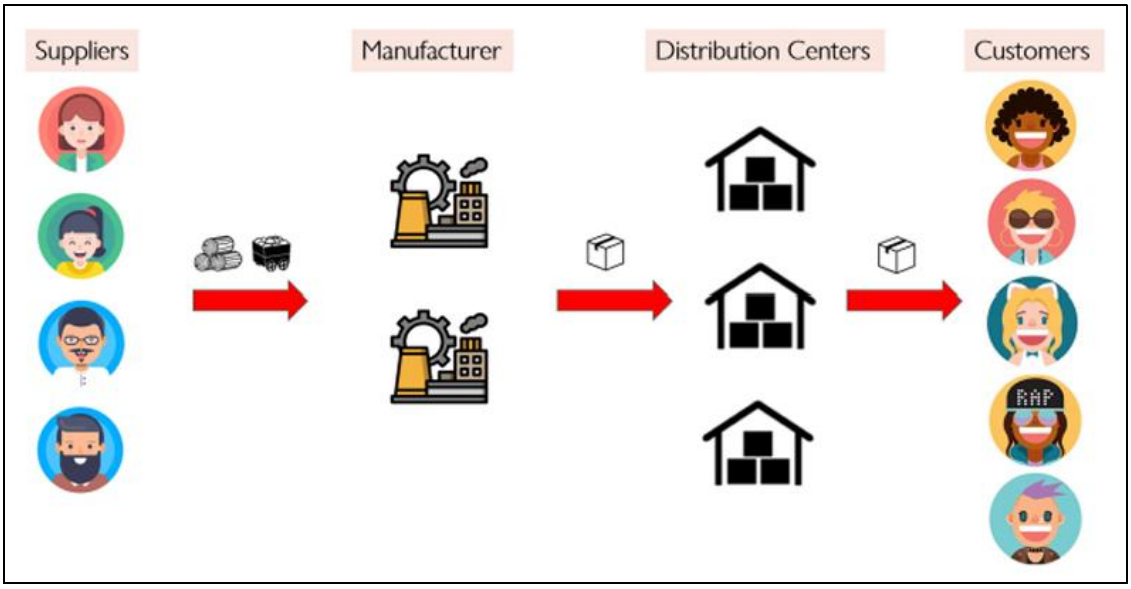

Figure 5.3 The structure of sustainable supply chain network.

Table 5.1 presents the values of the input parameters, with uncertain parameters being adjustable by up to ±20% to represent both optimistic and pessimistic scenarios.

Table 5.1 The value of input parameters of sustainable supply chain planning problem.

Parameters	**Values**
VR_r	[1, 1]
RPP_r	[1, 1]
SPS_{sr}	[[0.75, 0.80, 0.95, 0.62], [0.68, 0.65, 0.77, 0.88]]
SPM_m	[0.85, 0.92]
SPD_d	[0.79, 0.87, 0.96]
EPS_{sr}	[[0.83, 0.78, 0.91, 0.75], [0.72, 0.67, 0.85, 0.89]]
EPM_m	[0.91, 0.78]
EPD_d	[0.86, 0.72, 0.68]
$CapS_{sr}$	[[700, 700, 700, 700], [700, 700, 700, 700]]
$CapM_{mcm}$	[[750, 670], [880, 590]]
$SCapM_{mr}$	[[450, 380], [350, 470]]
$SCapD_{dcd}$	[[900, 900, 900], [900, 900, 900]]
$SSRM_{mr}$	[[0.15, 0.25], [0.22, 0.18]]
$SSRD_d$	[0.17, 0.23, 0.27]
$\widetilde{PCS}_{sr}$	[[1.25, 1.45, 0.85, 0.55], [0.75, 0.35, 1.15, 1.65]]
$\widetilde{FCS}_s$	[[75, 85, 98, 65], [58, 50, 70, 90]]

$\widetilde{FCM}_{mcm}$	[[8,500, 9,200], [7,900, 8,700]]
$\widetilde{FCD}_{dcd}$	[[5,600, 6,300, 5,900], [6,500, 5,200, 6,800]]
$\widetilde{PCM}_m$	[2.25, 2.75]
$\widetilde{HCM}_{rm}$	[2.46, 1.87]
$\widetilde{HCD}_d$	[2.87, 3.27, 3.65]
$\widetilde{TCS}_{smr}$	[[[1.23, 1.14, 1.72, 1.26], [1.41, 1.92, 1.34, 1.78]], [[1.56, 1.49, 1.85, 1.39], [1.17, 1.32, 1.68, 1.54]]]
$\widetilde{TCM}_{md}$	[[1.46, 1.38, 1.52], [1.94, 1.83, 1.75]]
$\widetilde{TCD}_{dc}$	[[1.54, 1.26, 1.43, 1.71, 1.64], [1.29, 1.35, 1.48, 1.52, 1.94], [1.58, 1.67, 1.21, 1.45, 1.76]]
$\widetilde{De}_c$	[225, 246, 289, 217, 263]

5.4 Mathematical Notations and Model

The notations for indexes, parameters, and decision variables are provided in Tables 5.2 to 5.5. Notably, all fuzzy parameters are indicated by a tilde ($\tilde{}$) placed over the corresponding symbols.

Table 5.2 Indexes of supply chain planning problem.

Indexes	**Meaning**
r	Set of raw materials ($r = 1, 2, \dots, R$)
s	Set of suppliers ($s = 1, 2, \dots, S$)
m	Set of manufacturers (m= $1, 2, \dots, M$)
d	Set of distribution centers ($d = 1, 2, \dots, D$)
c	Set of customers ($c = 1, 2, \dots, C$)
cm	Set of capacity levels of manufacturers ($cm = 1, 2, \dots, CM$)
cd	Set of capacity levels of distribution centers ($cd = 1, 2, \dots, CD$)

Table 5.3 Crisp parameters of supply chain planning problem.

Crisp Parameters	**Meaning**
VR_r	Volume of raw materials *r*
RPP_r	Amount of raw materials *r* for producing a unit of product
SPS_{sr}	Social performance score of suppliers *s* for raw materials *r*
SPM_m	Social performance score of manufacturers *m* for a product

SPD_d	Social performance score of distribution centers *d* for a product
EPS_{sr}	Environmental performance score of suppliers *s* for raw materials *r*
EPM_m	Environmental performance score of manufacturers *m* for a product
EPD_d	Environmental performance score of distribution centers *d* for a product
$CapS_{sr}$	Capacity of suppliers *s* for raw materials *r*
$CapM_{mcm}$	Production capacity *cm* of manufacturing *m* for a product
$SCapM_{mr}$	Capacity of manufacturers *m* for storing raw materials *r*
$SCapD_{dcd}$	Distribution capacity *cd* of distribution centers *d* for storing a product
$SSRM_{mr}$	Safety stock of raw materials *r* at manufacturers *m*
$SSRD_d$	Safety stock of a product at distribution centers *d*

Table 5.4 Uncertain parameters of supply chain planning problem.

Uncertain Parameters	**Meaning**
$\widetilde{PCS}_{sr}$	Unit cost of purchasing raw materials *r* from suppliers *s*
$\widetilde{FCS}_s$	Fixed cost of making a contract with suppliers *s*
$\widetilde{FCM}_{mcm}$	Fixed cost of installing manufacturing *m* process with capacity level *cm*
$\widetilde{FCD}_{dcd}$	Fixed cost of installing distribution centers *d* process with capacity level *cd*
$\widetilde{PCM}_m$	Unit cost of producing a product at manufacturers *m*
$\widetilde{HCM}_{rm}$	Unit holding cost of raw materials *r* at manufacturers *m*
$\widetilde{HCD}_d$	Unit holding cost of products at distribution centers *d*
$\widetilde{TCS}_{smr}$	Unit cost of transporting raw materials *r* from suppliers *s* to manufacturers *m*
$\widetilde{TCM}_{md}$	Unit cost of transporting products from manufacturers *m* to distribution centers *d*
$\widetilde{TCD}_{dc}$	Unit cost of transporting products from distribution centers *d* to customers *c*
$\widetilde{De}_c$	Demand of a product from customers *c*

Table 5.5 Decision variables of supply chain planning problem.

Decision Variables	**Meaning**
PR_{sr}	If suppliers *s* can provide raw materials *r* equal to 1, otherwise 0
SS_{sr}	If suppliers *s* is selected for providing raw materials *r* equal to 1, otherwise 0
IM_{mcm}	If manufacturing capacity *cm* is installed at manufacturers *m* equal to 1, otherwise 0
ID_{dcd}	If distribution capacity *cd* is installed at distribution centers *d* equal to 1, otherwise 0
$TRQS_{smr}$	Amount of transported raw materials *r* from suppliers *s* to manufacturers *m*

$TPQM_{md}$	Amount of transported products from manufacturers *m* to distribution centers *d*
$TRQD_{dc}$	Amount of transported products from distribution centers *d* to customers *c*
PQM_m	Amount of produced products at manufacturers *m*
$SSQM_{mr}$	Amount of safety stock of raw materials *r* at manufacturers *m*
$SSQD_d$	Amount of safety stock of products at distribution centers *d*
ARS_{sr}	Amount of raw materials *r* allocated to suppliers *s*

- **Objective Functions**

1. **Minimizing total costs** as an economic objective encompasses expenses related to facility establishment, supplier selection, transportation, raw material procurement, production, and the holding of safety stock for both raw materials and finished products.

$$\begin{aligned} Minimize\ Z_1 = & \sum_{s=1}^{S}\sum_{r=1}^{R}\widetilde{FCS}_{sr}\times SS_{sr} + \sum_{m=1}^{M}\sum_{cm=1}^{CM}\widetilde{FCM}_{mcm}\times IM_{mcm} \\ & + \sum_{d=1}^{D}\sum_{cd=1}^{CD}\widetilde{FCD}_{dcd}\times ID_{dcd} + \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{r=1}^{R}\widetilde{TCS}_{smr}\times TRQS_{smr} \\ & + \sum_{m=1}^{M}\sum_{d=1}^{D}\widetilde{TCM}_{md}\times TPQM_{md} + \sum_{d=1}^{D}\sum_{c=1}^{C}\widetilde{TCD}_{dc}\times TPQD_{dc} \\ & + \sum_{s=1}^{S}\sum_{r=1}^{R}\widetilde{PCS}_{sr}\times ARS_{sr} + \sum_{m=1}^{M}\widetilde{PCM}_m\times PQM_m \\ & + \sum_{m=1}^{M}\sum_{r=1}^{R}\widetilde{HCM}_r\times SSQM_{mr} + \sum_{d=1}^{D}\widetilde{HCD}_d\times SSQD_d \end{aligned} \quad (5.12)$$

In today's business landscape, companies are increasingly recognizing the importance of sustainability and ethical practices, while consumers are becoming more aware of the social and environmental impacts of their purchasing decisions. Integrating social and environmental considerations into the supply chain is essential for businesses to operate responsibly, comply with legal regulations, and enhance their reputation. From an ethical perspective, companies can minimize harm and contribute positively to society and the environment. In terms of legal and regulatory compliance, adherence to relevant laws helps mitigate the risk of fines, penalties, or legal disputes. Additionally, prioritizing social and environmental responsibility within the supply chain can strengthen a company's reputation, build customer trust, and enhance brand value.

In this study, social and environmental performance scores for each echelon of the supply chain are estimated using expert judgment. These methods involve calculating the weighted score of each key social or environmental factor at each echelon (W_i) and assessing the performance of each echelon in addressing these factors (R_i). Higher scores indicate stronger social and environmental practices within the supply chain.

$$Social\ Performance\ Score\ (SPS) = \sum_{i=1}^{I} W_i R_i \quad (5.13)$$

$$Environmental\ Performance\ Score\ (EPS) = \sum_{i=1}^{I} W_i R_i \quad (5.14)$$

Social performance assesses how effectively each echelon in the supply chain upholds social responsibility, impacts the community, respects labor rights, and adheres to ethical standards, with these aspects represented through weighted factors. A high social performance score signifies that raw materials and products are sourced responsibly, ensuring no violations of human rights or harm to communities. The total social performance score, calculated by integrating the social performance scores of each echelon with the quantity of transported raw materials or products, should be maximized, as expressed in Equation (5.15).

$$\begin{aligned} Maximize\ Z_2 = & \textstyle\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{r=1}^{R} SPS_{sr} \times TRQS_{smr} \\ & + \textstyle\sum_{m=1}^{M}\sum_{d=1}^{D} SPM_m \times TPQM_{md} \\ & + \textstyle\sum_{d=1}^{D}\sum_{c=1}^{C} SPD_d \times TPQD_{dc} \end{aligned} \quad (5.15)$$

Environmental performance measures how effectively each echelon in the supply chain adheres to sustainable practices, including energy consumption, carbon emissions, waste management, and responsible sourcing, with these aspects represented by weighted factors. A high environmental performance score signifies that raw materials and products are obtained and managed in an environmentally responsible and eco-friendly manner. The total environmental performance score, determined by integrating the environmental performance scores of each echelon with the quantity of transported raw materials or products, should be maximized, as expressed in Equation (5.16).

$$\begin{aligned} Maximize\ Z_3 = & \textstyle\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{r=1}^{R} EPS_{sr} \times TRQS_{smr} \\ & + \textstyle\sum_{m=1}^{M}\sum_{d=1}^{D} EPM_m \times TPQM_{md} \\ & + \textstyle\sum_{d=1}^{D}\sum_{c=1}^{C} EPD_d \times TPQD_{dc} \end{aligned} \quad (5.16)$$

- **Constraints**

$$\textstyle\sum_{m=1}^{M} TRQS_{smr} \leq ARS_{sr} \qquad \forall\, r \in R, s \in S \quad (5.17)$$
$$ARS_{sr} \leq CapS_{sr} \times PR_{sr} \times SS_{sr} \qquad \forall\, r \in R, s \in S \quad (5.18)$$
$$PQM_m \leq \textstyle\sum_{cm=1}^{CM} CapM_{mcm} \times IM_{mcm} \qquad \forall\, m \in M \quad (5.19)$$
$$RPP_r \times PQM_m + SSQM_{mr} = \textstyle\sum_{s=1}^{S} TRQS_{smr} \qquad \forall\, r \in R, m \in M \quad (5.20)$$
$$SSQM_{mr} \geq SSRM_{mr} \times RPP_r \times PQM_m \qquad \forall\, r \in R, m \in M \quad (5.21)$$
$$VR_r \times SSQM_{mr} \leq SCapM_{mr} \times IM_{mcm} \qquad \forall\, r \in R, m \in M, cm \in CM \quad (5.22)$$
$$PQM_m = \textstyle\sum_{d=1}^{D} TPQM_{md} \qquad \forall\, m \in M \quad (5.23)$$
$$\textstyle\sum_{m=1}^{M} VP \times TPQM_{md} \leq \textstyle\sum_{cd=1}^{CD} SCapS_{dcd} \times ID_{dcd} \qquad \forall\, d \in D \quad (5.24)$$
$$\textstyle\sum_{m=1}^{M} TPQM_{md} - SSQD_d = \textstyle\sum_{c=1}^{C} TPQD_{dc} \qquad \forall\, d \in D \quad (5.25)$$
$$SSQD_d \geq SSRD_d \times \textstyle\sum_{m=1}^{M} TPQM_{md} \qquad \forall\, d \in D \quad (5.26)$$
$$\textstyle\sum_{d=1}^{D} TPQD_{dc} = De_c \qquad \forall\, c \in C \quad (5.27)$$

Equation (5.17) ensures that the quantity of raw materials transported to manufacturers does not exceed the orders allocated to suppliers. Equation (5.18) specifies that the total raw material orders should not exceed the suppliers' capacity. Equation (5.19) restricts the production level of manufacturing processes to their installed capacity. Equation (5.20) addresses the supply of raw materials for production and safety stock by suppliers. Equation (5.21) defines the minimum required quantity of raw materials at the manufacturing stage. Equation (5.22) ensures the balance between production levels and outputs for each manufacturer. Equation (5.23) limits the maximum quantity of products transferred to the distribution centers' capacity. Equation (5.24) ensures a balance between input and output flows at each distribution center. Equation (5.25) requires that the total quantity of products transported from distribution centers meets customer demand.

5.5 Results

This section presents and analyzes the comparative results between the traditional fuzzy optimization approach and the proposed integrated fuzzy optimization framework. By examining these outcomes, the study highlights the improvements introduced by the integrated approach, which combines Chance-Constrained Programming (CCP) and Intuitionistic Fuzzy Linear Programming (IFLP).

5.5.1 Result Comparison

This section provides a detailed comparison of the optimal results achieved using two different approaches: the traditional fuzzy optimization method and the integrated fuzzy optimization approach. The comparison highlights the strengths and differences between the two methods in terms of performance, efficiency, and the ability to handle uncertainty within the system. The results, as summarized in Table 5.6, will allow for a better understanding of how the integrated approach enhances the outcomes compared to the traditional method, particularly in terms of improved decision-making, more accurate, and overall system optimization. The analysis aims to demonstrate the practical advantages of integrating advanced fuzzy optimization techniques into the decision-making process.

Table 5.6 Results comparison.

	Traditional Fuzzy Optimization Approach	Integrated Fuzzy Optimization Approach
Minimize Total Costs	\$77,385	\$70,631
Maximize Total Social Performance Scores	6,570.8 scores	7,100 scores
Maximize Total Environmental Performance Scores	6,750.5 scores	7,215.4 scores
Satisfaction Level of 1^{st} objective	57.698%	67.622%
Satisfaction Level of 2^{nd} objective	57.703%	69.720%
Satisfaction Level of 3^{rd} objective	57.698%	67.622%
Non-Satisfaction Level of 1^{st} objective	-	32.378%
Non-Satisfaction Level of 2^{nd} objective	-	30.280%
Non-Satisfaction Level of 3^{rd} objective	-	32.378%
Maximize minimum satisfaction value	57.697%	-

Maximize Minimum Satisfaction Value and Minimize Maximum Non-Satisfaction Value	-	35.245%

Table 5.6 illustrates that the integrated fuzzy optimization approach, which aims to simultaneously maximize the minimum satisfaction value and minimize the maximum non-satisfaction value, yields superior results. This approach achieves a minimum total cost of $70,631, a maximum total social performance score of 7,100, and a maximum total environmental performance score of 7,215.4. These results significantly surpass those obtained using the traditional fuzzy optimization method, which focuses only on maximizing the minimum satisfaction value. The traditional approach results in higher total costs of $77,385, lower total social performance scores of 6,570.8, and lower total environmental performance scores of 6,750.5. In conclusion, the integrated fuzzy optimization approach offers a more effective solution across all objective functions, providing improved performance while effectively controlling risk violations and simultaneously addressing both satisfaction and non-satisfaction levels.

5.5.2 Result Validation

- **Validation of risk violation**

The ability of the integrated fuzzy optimization approach to manage risk violations is evaluated by conducting experiments with different confidence levels (γ = 50%, 60%, 70%, 80%, 90%, and 100%), as shown in Table 5.7.

Table 5.7 Results of testing ability of controlling risk violation.

γ	Minimize Total Costs	Maximize Total Social Performance Score	Maximize Total Environment Performance Score
50%	$68,373	7,389.3 scores	7,476.7 scores
60%	$68,735	7,304.5 scores	7,399.6 scores
70%	$69,147	7,194.4 scores	7,300.2 scores
80%	$69,872	7,100.0 scores	7,215.4 scores
90%	$71,463	6,681.8 scores	6,869.1 scores
100%	$71,741	6,584.4 scores	6,765.5 scores

As presented in Table 5.7, an increase in the confidence level percentage leads to higher values across all objective functions, which suggests a decrease in their desirability. This demonstrates the trade-off between the confidence level used to control risk violations and the optimal values of the objective functions. As a result, a higher confidence level (indicating a lower risk of constraint violations) narrows the feasible solution set, which in turn results in less favorable optimal objective function values.

- **Validation of considering both satisfaction and non-satisfaction levels simultaneously**

To assess the effectiveness of simultaneously considering both satisfaction and non-satisfaction levels, experiments are conducted using both the traditional fuzzy optimization approach and the integrated fuzzy optimization approach, across different objective function scenarios as presented in Tables 5.8 to 5.10.

Table 5.8 Results comparison of optimization of one objective function.

Optimization of One Objective Function			
	Value	Satisfaction Level	Non-Satisfaction Level
Traditional Fuzzy Optimization Approach			
Minimizing Total Costs	$65,026	99.996%	100%-99.996% = 0.0037%
A New Integrated Fuzzy Optimization Approach			
Minimizing Total Costs	$65,026	99.996%	0.0037%

Note: The traditional fuzzy optimization approach considers only the satisfaction level. Therefore, the non-satisfaction level can also be calculated from the opposite of the satisfaction level.

Table 5.8 shows that there is no significant difference between the traditional fuzzy optimization approach and the newly integrated fuzzy optimization approach when only a single objective function is considered. In cases where a single objective is the focus of the optimization process, both approaches produce similar results, indicating that the added complexity of the integrated approach does not provide additional benefits in such scenarios. This suggests that for problems with only one objective function, both approaches perform equivalently, and the integrated approach may not offer distinct advantages over the traditional method. However, this outcome highlights the potential for the integrated approach to shine in more complex situations involving multiple objectives, where its ability to balance satisfaction and non-satisfaction levels becomes more impactful.

Table 5.9 Results comparison of optimization of two objective functions.

Optimization of Two Objective Functions				
	Value	Satisfaction Level	Non-Satisfaction Level	Maximize (Satisfaction Level – Non-Satisfaction Level)
Case 1				
Traditional Fuzzy Optimization Approach				
Minimizing Total Costs	$77,163	57.78%	100% - 57.78% = 42.22%	57.78% - 42.22% = 15.56%
Maximize Total Social Performance Scores	6,573.40 scores	57.78%	100% - 57.78% = 42.22%	
A New Integrated Fuzzy Optimization Approach				
Minimizing Total Costs	$70,584	68.20%	31.79%	36.41%
Maximize Total Social Performance Scores	7,057.30 scores	68.20%	31.79%	
Case 2				
Traditional Fuzzy Optimization Approach				
Minimizing Total Costs	$77,304	57.95%	100% -57.95% = 42.05%	57.95% - 42.05% = 15.90%
Maximize Total Environmental Performance Scores	6,578.50 scores	57.95%	100% -57.95% = 42.05%	
A New Integrated Fuzzy Optimization Approach				
Minimizing Total Costs	$70,486	67.83%	32.17%	35.66%
Maximize Total Environmental Performance Scores	6,907.50 scores	67.83%	32.17%	

Case 3				
Traditional Fuzzy Optimization Approach				
Maximize Total Social Performance Scores	7,885.80 scores	97.63%	100% - 97.63% = 2.37%	97.62% - 2.38% = 95.24%
Maximize Total Environmental Performance Scores	8,004.40 scores	97.62%	100% - 97.62% = 2.38%	
A New Integrated Fuzzy Optimization Approach				
Maximize Total Social Performance Scores	7,886.30 scores	97.97%	2.03%	95.94%
Maximize Total Environmental Performance Scores	8,004.70 scores	97.99%	2.01%	

Note: The traditional fuzzy optimization approach considers only satisfaction level. Therefore, the value of considering both satisfaction and non-satisfaction levels simultaneously can be calculated by Maximize (Satisfaction Level – Non-Satisfaction Level)

Table 5.10 Results comparison of optimization of three objective functions.

Optimization of Three Objective Functions				
	Value	Satisfaction Level	Non-Satisfaction Level	Maximize (Satisfaction Level – Non-Satisfaction Level)
Traditional Fuzzy Optimization Approach				
Minimizing Total Costs	$77,385	57.96%	100% - 57.96% =42.04%	57.96% - 42.04% =15.92%
Maximize Total Social Performance Scores	6,683.2 scores	57.97%	100% - 57.97% = 42.03%	
Maximize Total Environmental Performance Scores	6,870.6 scores	57.97%	100% - 57.97% = 42.03%	
A New Integrated Fuzzy Optimization Approach				
Minimizing Total Costs	$70,631	67.62%	32.37%	35.25%
Maximize Total Social Performance Scores	7,100 scores	69.72%	30.28%	
Maximize Total Environmental Performance Scores	7,215.40 scores	67.62%	32.37%	

Tables 5.9 and 5.10 highlight a key limitation of the traditional fuzzy optimization approach, which primarily focuses on improving the lowest satisfaction level of a single objective. While this approach may achieve better performance for one objective, it often overlooks the potential increase in the non-satisfaction levels of other objectives. This can lead to suboptimal outcomes, where the overall balance across objectives is not fully optimized, potentially resulting in trade-offs that are not favorable to decision-makers. To overcome this limitation, the integrated fuzzy optimization approach is introduced. This approach allows decision-makers to simultaneously consider both satisfaction and non-satisfaction levels for multiple objectives, providing a more balanced solution. By incorporating both factors into the optimization process, the integrated approach ensures

that improvements in one objective do not disproportionately negatively affect others. As a result, decision-makers can achieve a more equitable and effective optimization outcome, addressing multiple objectives in a way that enhances overall system performance and reduces the risk of undesirable trade-offs.

5.5.3 Considering the Importance of Social and Environmental Aspects

Historically, social and environmental considerations were often seen as secondary factors in supply chain planning, with non-compliance mainly resulting in legal penalties, fines, or, in some cases, the suspension of business operations. However, today's consumers are increasingly aware of the social and environmental impacts associated with their purchasing decisions. As a result, supply chains that prioritize sustainability and social responsibility have the potential to significantly enhance a company's reputation, build customer trust, and set it apart from competitors. By adopting robust ethical practices and fostering a positive brand image, companies can not only drive customer loyalty but also increase their market share. Table 5.11 compares the outcomes of scenarios where social and environmental concerns are not given primary importance with those in which these factors are central to the decision-making process. This comparison highlights the tangible benefits of integrating social and environmental objectives into supply chain strategies.

Table 5.11 Comparison of Results with and without prioritizing social and environmental objectives.

Not Consider Social and Environmental Issues as Main Objectives						
Minimum Total Costs = $65,026						
Supplier	Providing Raw Material (units)		Social Performance Scores		Environmental Performance Scores	
	1	2	Raw Material		Raw Material	
			1	2	1	2
1	700	700	0.75	0.68	0.83	0.72
2	533	700	0.80	0.65	0.78	0.67
3	-	638	-	0.77	-	0.85
4	700	-	0.62	-	0.75	-
Consider Social and Environmental Issues as Main Objectives						
Minimum Total Costs = $70,631						
Supplier	Providing Raw Material (units)		Social Performance Scores		Environmental Performance Scores	

	1	2	Raw Material		Raw Material	
			1	2	1	2
1	700	700	0.75	0.68	0.83	0.72
2	700	475	0.80	0.65	0.78	0.67
3	700	700	0.95	0.77	0.91	0.85
4	401	700	0.62	0.88	0.75	0.89

Table 5.11 demonstrates that when social and environmental considerations are prioritized as primary objectives, there is an increase in the minimum total costs, rising from \$65,026 to \$70,631. This increase reflects the inherent trade-off between cost efficiency and enhanced social and environmental performance. As companies strive to improve their social and environmental impact, they often face higher costs associated with sourcing more sustainable materials, selecting suppliers with better ethical practices, and implementing eco-friendly processes. These costs are offset by the long-term benefits of better social and environmental outcomes. The impact of this trade-off is particularly evident in the supplier selection process. When social and environmental performance are emphasized, suppliers with the highest performance scores in these areas are selected first, even if their costs may be higher. Suppliers that rank lower in social and environmental criteria are chosen later, typically at a higher cost, to balance performance with cost efficiency. This approach is consistently applied across all echelons of the supply chain, ensuring that sustainability and ethical practices are integrated throughout the entire supply chain process, from raw material procurement to final product distribution. By prioritizing social and environmental issues, companies are not only fostering positive societal and environmental change but also positioning themselves to meet consumer demand for responsible and sustainable practices.

5.6 Discussion and Conclusion

This study introduces a novel integrated fuzzy optimization approach, which significantly advances traditional fuzzy optimization methods by addressing two key aspects: risk management and multi-objective optimization. The proposed approach presents substantial improvements over existing techniques, offering a more flexible and comprehensive framework for decision-making, particularly in uncertain environments.

One of the key innovations of this approach is its ability to define the credibility level (γ), which enables decision-makers to control the risk of violating fuzzy constraints. By integrating Chance-Constrained Programming (CCP), the method effectively tackles the uncertainties inherent in data within fuzzy constraints. This is particularly beneficial for managing risk systematically, as it provides a controlled way to balance the feasibility of solutions with the risk of constraint violations. This feature enhances the robustness of the decision-making process, allowing for more informed trade-offs between risk and practicality.

Another major contribution of the proposed approach is the integration of Intuitionistic Fuzzy Linear Programming (IFLP). This allows for the simultaneous optimization of both satisfaction and non-satisfaction levels, addressing trade-offs between the two. Traditional fuzzy optimization methods often focus on optimizing a single aspect, such as satisfaction levels, while neglecting the negative implications of non-satisfaction. By considering both aspects simultaneously, the integrated approach offers a more balanced and comprehensive framework for tackling multi-objective problems, especially in complex decision-making scenarios.

The effectiveness of the proposed approach was demonstrated in its application to a fuzzy multi-objective sustainable supply chain planning problem, which involved three critical objectives: minimizing total costs, maximizing social performance, and maximizing environmental performance. The first objective, minimizing total costs, reflects the economic dimension of supply chain planning, helping companies reduce unnecessary expenses and enhance financial stability. The second objective, maximizing social performance, addresses the growing societal demand for businesses to contribute positively to their communities, strengthen brand reputation, and build customer trust. Finally, the third objective, maximizing environmental performance, underscores the need to minimize environmental harm, support sustainable business practices, and comply with increasingly stringent environmental regulations. The results demonstrate that the integrated fuzzy optimization approach can effectively optimize these objectives, providing valuable insights for developing a balanced and sustainable supply chain strategy.

Despite these promising results, the study acknowledges several limitations. First, there are no explicit restrictions on the degree of fuzziness, which could impact the reliability and accuracy of the final solutions. Second, while triangular distributions are used to represent fuzzy parameters, alternative distributions, such as trapezoidal or Gaussian distributions, could improve the accuracy of the model, particularly in more complex or specific scenarios. Third, the approach could benefit from further refinement to address more complex and realistic scenarios in sustainable supply chain planning, thus expanding its applicability to a wider range of industries and contexts.

In conclusion, the proposed integrated fuzzy optimization approach represents a significant step forward in addressing the uncertainties inherent in sustainable supply chain planning. By combining advanced optimization techniques, such as Chance-Constrained Programming and Intuitionistic Fuzzy Linear Programming, the approach offers a comprehensive framework that balances economic, social, and environmental objectives. While the results are promising, future research should focus on refining the approach to enhance its robustness and practical utility. This could include addressing the limitations related to fuzziness, exploring alternative distributions for fuzzy parameters, and extending the model to incorporate more complex and realistic scenarios. Ultimately, the proposed approach has the potential to provide valuable insights for companies striving to create supply chains that are not only cost-efficient but also socially responsible and environmentally sustainable.

Chapter 6
Advanced Fuzzy Mathematical Modeling with Monte Carlo Simulation: A Comprehensive Framework for Analyzing Fuzzy Supply Chain Aggregate Production Planning Problem

This chapter introduces an advanced analytical framework that integrates fuzzy mathematical modeling with Monte Carlo simulation to address the complexities of Supply Chain Aggregate Production Planning (SCAPP) under uncertainty. Recognizing the limitations of conventional deterministic models in capturing the inherent vagueness and variability in supply chain environments, this study proposes a robust approach that combines the strengths of fuzzy set theory and Monte Carlo simulation. This chapter begins with a detailed description of the problem, emphasizing the challenges posed by imprecise data. It further outlines the main contributions of the research, highlighting the innovation in blending fuzzy optimization with probabilistic simulation to enhance decision-making accuracy and reliability. The methodology section presents fuzzy optimization approach and Monte Carlo simulation in detail, explaining how both techniques are used in tandem to model uncertainty and evaluate alternative production planning strategies. A real-world case study is included to demonstrate the practical applicability and effectiveness of the proposed framework. This is followed by a formal presentation of the mathematical notations and the model formulation, which provide the foundation for the subsequent analysis. The results section presents key findings, including a comparative evaluation, to validate the performance and benefits of the model. Finally, this chapter concludes with a discussion of the implications of the results, the advantages of integrating fuzzy modeling with simulation techniques, and directions for future research. This chapter aims to offer a comprehensive and adaptable solution for improving the resilience and efficiency of SCAPP in uncertain supply chain environments.

6.1 Problem Description and Contributions

Uncertainty in supply chain aggregate production planning (SCAPP) poses significant challenges for decision-makers, as unpredictable fluctuations in demand, resource availability, and operational constraints can disrupt production efficiency and cost-effectiveness. Traditional deterministic optimization models often fall short in handling such variability, leading to suboptimal decisions and increased operational risks. While conventional optimization approaches rely on fixed input parameters, they lack the flexibility to incorporate uncertainty and imprecise data, making them inadequate for real-world dynamic environments.

To address these limitations, this study integrates Monte Carlo simulation with fuzzy mathematical programming, creating a robust framework for decision-making under uncertainty. Monte Carlo simulation enables the generation of probabilistic outcomes, allowing for detailed variability analysis and the identification of potential risks across multiple scenarios. The fuzzy-based model complements this by incorporating imprecise and ambiguous information, enhancing model flexibility and improving adaptability to uncertain conditions. This hybrid approach allows decision-makers to assess the resilience of production plans under diverse circumstances, ensuring more realistic and informed decision-making.

A key component of this study is the validation of the proposed approach through a comparative analysis of simulated and fuzzy-based outcomes, confirming its reliability in real-world applications. By integrating probabilistic and fuzzy-based decision-making techniques, the study enhances SCAPP by offering practical benefits such as improved adaptability, effective scenario analysis, and greater precision in navigating uncertainty. The findings contribute to the development of a comprehensive and dynamic SCAPP framework, equipping decision-makers with the necessary tools to optimize production planning while mitigating risks associated with uncertain environments.

This study makes several key contributions to the field of supply chain aggregate production planning through the integration of advanced analytical techniques and optimization models. First, the incorporation of Monte Carlo simulation enhances variability analysis by providing a probabilistic approach to model the impact of uncertainty within a fuzzy-based optimization framework. This integration enables a more comprehensive examination of risk, allowing for data-driven decision-making that is more resilient to uncertainty compared to traditional deterministic models. Second, the study identifies critical decision variable behavior and systemic variability patterns, offering valuable insights into how uncertainty affects production planning, resource allocation, and cost structures. Through extensive scenario analysis, the research demonstrates how the model captures and accounts for fluctuations, empowering decision-makers to develop adaptive and risk-aware strategies. Lastly, the study rigorously validates the reliability and practical applicability of the proposed fuzzy-based optimization model by comparing simulated outcomes with real-world fuzzy-based results. This validation ensures the model’s robustness in uncertain environments, providing a practical decision-support framework that can be effectively applied across various industrial settings to improve strategic planning, operational efficiency, and risk mitigation in supply chain management.

6.2 Methodology

This section introduces a fuzzy-based optimization methodology, integrated with Monte Carlo Simulation, to design an efficient and reliable Supply Chain Aggregate Production Planning (SCAPP) system. The combination of these two advanced techniques allows for a more robust approach to decision-making in the face of uncertainty, enabling supply chain managers to optimize production planning and resource allocation while accounting for the inherent variability in demand, supply, and operational constraints. The fuzzy-based optimization model enhances the system's adaptability, allowing it to respond effectively to unpredictable changes and dynamic conditions within the supply chain. In parallel, Monte Carlo Simulation is employed to generate probabilistic outcomes by simulating a wide range of possible scenarios. This method provides a comprehensive analysis of variability, highlighting potential risks and fluctuations in key parameters that may affect production planning decisions. By running numerous iterations of the model under different random inputs, Monte Carlo Simulation allows for a better understanding of the range of possible outcomes and supports scenario-based planning for more informed decision-making.

- **Fuzzy-Based Optimization Approach**

The framework presents a multi-objective fuzzy linear programming approach for Supply Chain Aggregate Production Planning (SCAPP), structured into four distinct phases: 1) Data Preparation, 2) Defuzzification, 3) Membership Function, and 4) Optimization Process. These phases are detailed and visually represented in Figure 6.1. Each phase plays a crucial role in ensuring the effective application of fuzzy logic and optimization techniques to address the complexities and uncertainties inherent in supply chain planning.

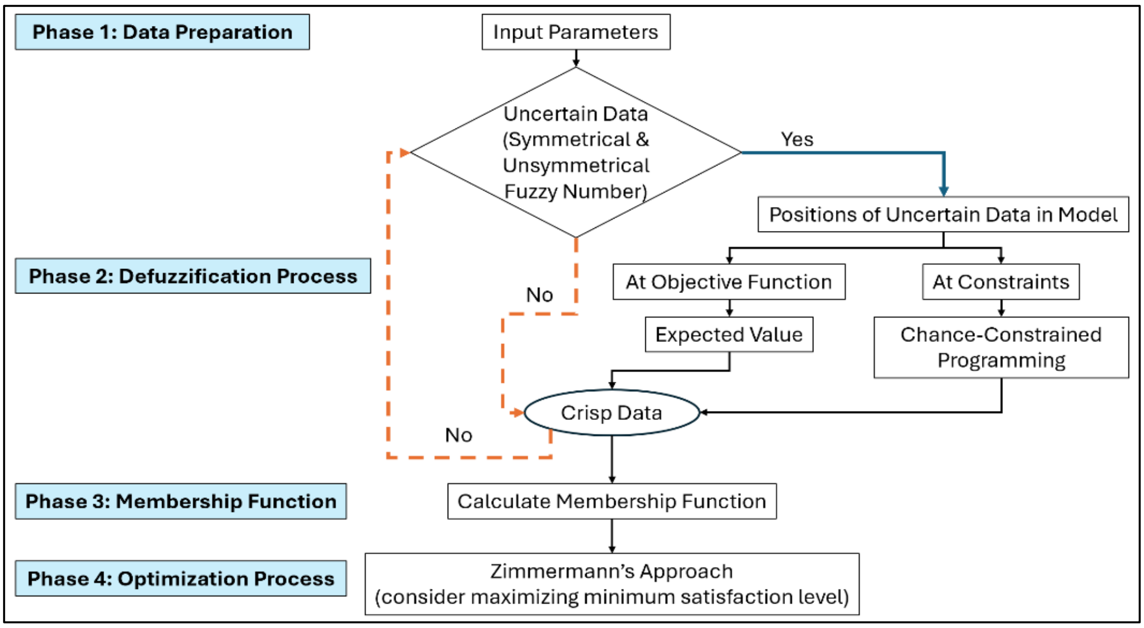

Figure 6.1. Methodology of fuzzy-based optimization approach.

- **Phase 1: Data Preparation**

In this phase, parameters are classified into two categories: crisp, which are precise and well-defined, and uncertain, which are ambiguous or difficult to specify. The uncertain parameters are represented using Triangular Fuzzy Numbers (TFNs) to capture the inherent vagueness and variability.

- **Phase 2: Defuzzification Process**

In this stage, uncertain parameters are transformed into crisp values through defuzzification. The method of defuzzification is selected based on whether the fuzzy parameters are incorporated into the objective functions or the constraints of the model.

1. **Defuzzification approach for objective function**

The Expected Value (EV) approach is introduced as a standard defuzzification method for objective functions, designed to evaluate their average overall performance.

$$EV = \frac{(C^o + (2 \times C^m) + C^p)}{4} \tag{6.1}$$

where C^o, C^m, and C^p represent the objective coefficients in optimistic, most likely, and pessimistic scenarios, respectively.

2. **Defuzzification approach for constraint**

Chance-Constrained Programming (CCP) employs a fuzzy measure known as credibility to transform fuzzy data and ensure a specified confidence level (γ) for the constraints. The credibility metric represents the reliability of a fuzzy event, where higher values indicate stronger confidence and a lower likelihood of constraint violation. CCP has been widely utilized in various studies to manage uncertain parameters by adjusting the confidence levels. A higher confidence level leads to more dependable outcomes, while a 100% confidence level encompasses all possible scenarios. Conversely, lower confidence levels ($\gamma < 0.5$) are rarely applied, as they tend to offer limited reliability and greater uncertainty.

$$Cr\{\textstyle\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}_i\} \geq \gamma$$

$$when\ (0 \leq \gamma \leq 0.5): ax \leq (2\gamma)b^m + (1 - 2\gamma)b^p \tag{6.2}$$

$$when\ (0.5 < \gamma \leq 1): ax \leq (2\gamma - 1)b^o + (2 - 2\gamma)b^m \tag{6.3}$$

where b^o, b^m, and b^p are values of available resource in optimistic, most likely, and pessimistic situations, respectively. γ is the percentage of credibility level, which is assigned to 80% in this study.

- **Phase 3: Membership Function**

In Supply Chain Aggregate Production Planning (SCAPP), multiple objective functions emerge due to differing stakeholder priorities, often involving diverse units and scales. To manage this complexity, a membership function is used to normalize these objectives on a common scale ranging from 0.0 to 1.0, referred to as the satisfaction level. This standardization facilitates consistent evaluation and ensures a balanced approach to addressing the needs and expectations of all stakeholders.

1. Membership function for minimization of the objective function

$$\mu_{Z_i} = \begin{cases} 1, & Z_i \leq Z_i^{PIS} \\ \frac{Z_i^{NIS} - Z_i}{Z_i^{NIS} - Z_i^{PIS}}, & Z_i^{PIS} \leq Z_i \leq Z_i^{NIS} \\ 0, & Z_i \geq Z_i^{NIS} \end{cases} \tag{6.4}$$

where Z_i^{NIS} is the maximum bound for minimizing objective of the minimum bound for maximizing objective and Z_i^{PIS} is the maximum bound for maximizing objective or the minimum bound for minimizing objective.

- **Phase 4: Optimization Process**

This phase focuses on determining an optimal solution within Multi-Objective Fuzzy Linear Programming (MOFLP). The Zimmermann approach, a widely recognized weightless fuzzy linear programming method, treats all objective functions and constraints with equal importance. The primary goal is to maximize the minimum satisfaction level across multiple objectives, ensuring a balanced and optimal outcome for all involved criteria.

$$\begin{aligned} &\text{Maximize } \mu_Z \\ &\text{Subjected to: } x \in F(x) \\ &\qquad \mu_Z \leq \mu_{Z_i}, \quad i = 1, 2, \ldots, I \end{aligned} \tag{6.5}$$

where μ_Z is a minimum value of the satisfaction levels from the multiple objective functions and μ_{Z_i} is the satisfaction level of each objective function.

- **A Monte Carlo Approach**

Monte Carlo Simulation is a powerful statistical method employed to model complex systems under uncertainty by generating random samples from input variables. These input variables can represent uncertain parameters, such as fluctuating demand, varying resource availability, or unpredictable operational constraints. By simulating a large number of random scenarios, Monte Carlo simulation captures the inherent randomness and variability in these systems, allowing decision-makers to explore a wide range of possible outcomes.

Through repeated simulations, the method generates a comprehensive distribution of results, which can be used to estimate probabilities associated with different outcomes. This process not only provides a deeper understanding of the variability in the system but also offers valuable insights into the likelihood of different results. Monte Carlo simulation helps identify potential risks, uncertainties, and sensitivities in the system, enabling better decision-making under uncertain conditions. By incorporating this technique, organizations can gain a clearer picture of the potential range of outcomes and make more informed choices that account for the full scope of uncertainty and variability.

The following procedure is used for generating a single value of fuzzy variable [77].

- **Step 1**: Generate a value t of the uniform random variable T over (0, 1].
- **Step 2**: Generate a value x of the uniform random variable U over the t-level set $G_t := \{x|\mu_G(x) \geq t\}$ of the fuzzy set G.
- **Step 3**: Assume that $V = x$, i.e. that x is a single value of the fuzzy variable V.

This procedure is also correct for discrete fuzzy variables, i.e. the case when G is a fuzzy set in the space of integers. Then, in Step 2, G_t is a finite discrete set and x should be generated according to the probability distribution $P(U = x) = 1/card(G_t)$. It is visible that the assumptions accepted for G are essential in Step 2 of the procedure. Due to these, G_t is a non-empty, bounded set. In the continuous case G_t becomes a bounded interval. Discussion at this point will be confined to the continuous case, i.e. the case when V takes on values from the set of real numbers (G is a fuzzy set in the real line). With the accepted assumptions concerning G, the procedure is the same as generating a value of the random variable $U_G(T, S)$ defined in the following way:

$$U_G(T, S) = g^-(T) + S(g^+(T) - g^-(T)) \\ = (1 - S)g^-(T) + Sg^+(T) \quad (6.6)$$

where T and S are independent uniform random variables over (0, 1] and [0, 1], respectively, and the functions g^- and g^+ are defined as follows: $g^-(T) = infG_t$ and $g^+(T) = supG_t$, $t \in (0, 1]$.

This study analyzes three cases of applying fuzzy input parameters for T and S: (1) both T and S are the same fuzzy input parameters, (2) T values are the same while S values differ, and (3) both T and S values are differ fuzzy input parameters. These cases are examined by simulating 40 fuzzy input parameters to determine the most appropriate case, as illustrated in Figure 6.2.

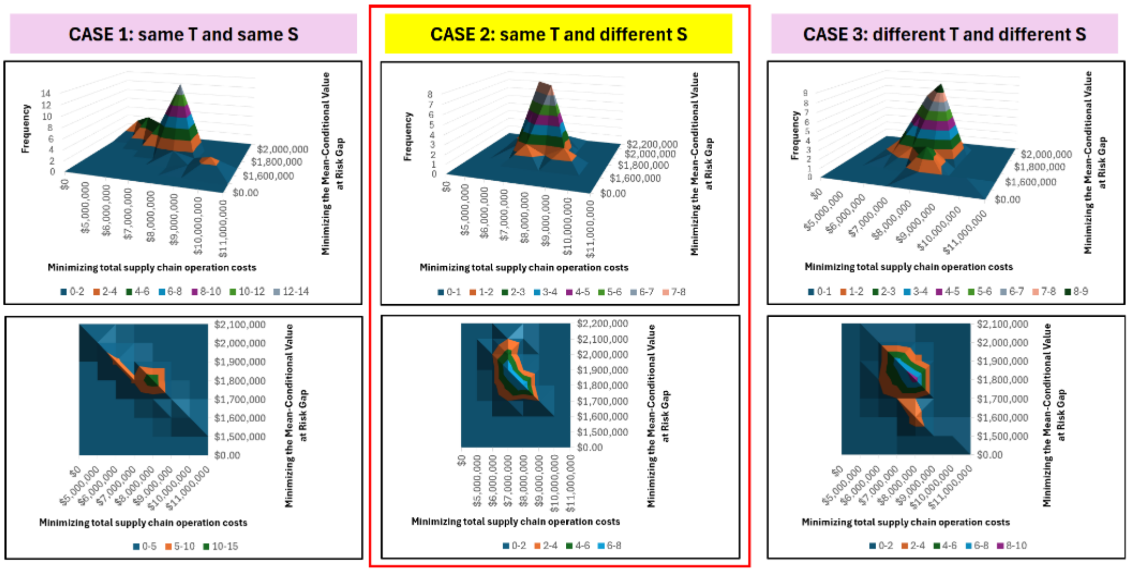

Figure 6.2. Comparison among three cases.

According to the results in Figure 6.2, it can be interpreted as follows:

- **Case 1**: Same *T* and *S*. It means all variables are correlated. This occurs when the uncertainty of every variable arises from the same reason. This is not the case in the considering situation.
- **Case 2**: Same *T* but Different *S*. It means the same cut level and different choices from intervals. In other words, random values are generated independently under the same level of possibility. This assumption seems reasonable in the Monte Carlo Simulation and is applied in this study.
- **Case 3**: Different *T* and *S*. It may need more trials to obtain stable results.

In this study, simulations are conducted by running the model with 100 randomly generated inputs derived from Case 2 (Same T but Different S). These inputs are designed to reflect a range of possible values, capturing the variability inherent in the system. By executing the model with these random inputs, a diverse set of outcomes are generated by representing various potential scenarios under the specified conditions. Once the simulations are completed, the resulting data is thoroughly analyzed by compiling statistical summaries to interpret and evaluate the performance of the system across different scenarios. This comprehensive statistical analysis enables a deeper interpretation of the likelihood of various scenarios, providing valuable insights into how the system behaves under different conditions. It allows for a more informed understanding of the potential risks, uncertainties, and decision-making trade-offs that may arise in real-world applications.

6.3 Case Study

A Supply Chain Aggregate Production Planning (SCAPP) problem is demonstrated in a small Thai plastic bottle industry, showcasing the application of a fuzzy-based optimization model. This model involves multiple stakeholders, including four PET resin suppliers, a capacity-limited production plant, and six regional retailers. The uncertainties and complexities within the system are addressed by modeling critical parameters such as supplier prices, demand patterns, and various costs (including transportation, holding, and penalties) using Triangular Fuzzy Numbers (TFNs) and skewed fuzzy distributions. These fuzzy models effectively capture the inherent uncertainty and downside risks, which are common in supply chain planning, particularly in environments with fluctuating market conditions and ambiguous data. The primary objective of the production plant is to meet at least 80% of the demand from the regional retailers, with penalties imposed for any sales that cannot be fulfilled due to capacity constraints. The model explicitly excludes the option of subcontracting, ensuring that production is strictly constrained by the plant's capacity. Additionally, delivery lead times are assumed to be negligible, simplifying the model by eliminating time-related uncertainties in the transportation process. This fuzzy-based optimization approach allows for more accurate decision-making in the face of uncertain supply chain parameters, providing a robust framework for managing the trade-offs between costs, capacity limitations, and demand fulfillment. The model is particularly useful for small and medium-sized industries, such as this Thai plastic bottle manufacturer, where traditional deterministic models may struggle to address the complexities and uncertainties of the real-world supply chain.

Table 6.1 Fuzzy raw material cost.

Triangular Fuzzy Number	$\widetilde{RMC}_{st}$ ($/unit)			
	Supplier 1	Supplier 2	Supplier 3	Supplier 4
Optimistic	5.23	6.50	8.44	10.84
Most Likely	9.50	10.00	11.25	12.75
Pessimistic	13.78	13.50	14.06	14.66

Table 6.2 Fuzzy costs related to production ($/unit).

	Optimistic	Most Likely	Pessimistic		Optimistic	Most Likely	Pessimistic
$\widetilde{RC}_t$	10	12.50	15	$\widetilde{IPC}_t$	0.24	0.30	0.36
$\widetilde{OC}_t$	15	18.75	22.5	$\widetilde{HC}_t$	128	160	192
$\widetilde{SPC}_t$	30	37.50	45	$\widetilde{FC}_t$	224	280	336
$\widetilde{IRC}_t$	0.08	0.10	0.12				

Table 6.3 Fuzzy demands.

	$\widetilde{D}_{rt}$ ($/unit)					
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Retailer 1	5,796	4,971	4,200	3,711	5,004	6,287
Retailer 2	4,849	3,996	2,646	2,546	4,418	5,362
Retailer 3	3,967	2,505	1,510	1,293	2,728	3,779
Retailer 4	7,904	7,016	6,222	6,370	7,386	8,865
Retailer 5	9,716	8,902	7,462	7,251	8,305	9,396
Retailer 6	7,809	6,520	5,037	5,528	6,453	7,328

Note: Fuzzy retailer's demand is varied ±20% from the most likely values as shown in Table 6.3.

Table 6.4 Crisp parameters.

	LH_t (man-hours/units)	MRT_t (units)	MOT_t (units)	MSC_{st} (units)
Value	0.016	28,000	7,000	25,000

Table 6.5 Fuzzy costs related to raw materials.

	$\widetilde{RMC}_{st}$ ($/unit)					
	Retailer 1	Retailer 2	Retailer 2	Retailer 4	Retailer 5	Retailer 6
Optimistic	11.70	6.00	3.45	14.40	10.50	4.35
Most likely	13.00	10	11.50	16.00	17.50	14.50
Pessimistic	14.30	11.00	16.10	22.40	29.75	24.65
Risk of Uncertainty	Low	Low	Medium	Medium	High	High
	$\widetilde{IPRC}_{rt}$ ($/unit)					
	Retailer 1	Retailer 2	Retailer 2	Retailer 4	Retailer 5	Retailer 6
Optimistic	6.30	2.40	1.65	9.00	6.90	2.55
Most likely	7.00	4.00	5.50	10.00	11.50	8.50
Pessimistic	7.70	4.40	7.70	14.00	19.55	14.45
Risk of Uncertainty	Low	Low	Medium	Medium	High	High
	$\widetilde{PC}_{rt}$ ($/unit)					
	Retailer 1	Retailer 2	Retailer 2	Retailer 4	Retailer 5	Retailer 6
Optimistic	6.90	6.00	12.90	15.60	24.75	22.05
Most likely	23.00	20.00	21.50	26.00	27.50	24.50
Pessimistic	36.80	28.00	23.65	44.20	38.50	26.95
Risk of Uncertainty	High	Medium	Low	High	Medium	Low

6.4 Mathematical Notations and Model

The notations for indexes, parameters, and decision variables are provided in Tables 6.6 to 6.7. Notably, all fuzzy parameters are indicated by a tilde ($\sim$) placed over the corresponding symbols.

Table 6.6 Mathematical notations of input parameters.

Notations	Meaning
s	Index of suppliers, $s = 1, \ldots, S$
r	Index of retailers, $r = 1, \ldots, R$
t	Index of periods, $t = 1, \ldots, T$
$\widetilde{RMC}_{st}$	Fuzzy raw material cost of supplier s in period t
$\widetilde{RC}_t$	Fuzzy regular time production cost per unit of product in period t
$\widetilde{OC}_t$	Fuzzy overtime production cost per unit of product in period t
$\widetilde{SPC}_t$	Fuzzy shortage cost per unit of product at the plant in period t
$\widetilde{IRC}_t$	Fuzzy inventory cost per unit of raw material at the plant in period t
$\widetilde{IPC}_t$	Fuzzy inventory cost per unit of product at the plant in period t
$\widetilde{HC}_t$	Fuzzy hiring cost for a labor in period t
$\widetilde{FC}_t$	Fuzzy firing cost for a labor in period t
$\widetilde{TranC}_{rt}$	Fuzzy transportation cost per unit of a product from the plant to retailer r in period t
$\widetilde{IPRC}_{rt}$	Fuzzy inventory cost per unit of product at retailer r in period t
$\widetilde{PC}_{rt}$	Fuzzy penalty cost of lost sales per unit of a product at retailer r in period t
$\widetilde{D}_{rt}$	Fuzzy retailer r demand of products in period t
$\widetilde{TC}$	Total supply chain costs
$\widetilde{PCC}$	Total procurement costs
$\widetilde{PDC}$	Total production costs
$\widetilde{DTC}$	Total distribution costs
LH_t	Labor hours required per unit of products at the plant in period t
MRT_t	Maximum capacity of producing products in the regular time at the plant in period t
MOT_t	Maximum capacity of producing products in the overtime at the plant in period t
MSC_{st}	Maximum capacity of supplier s for supplying raw materials in period t

Table 6.7 Mathematical notations of decision variables.

Notations	Meaning
RMQ_{st}	Number of raw materials supplied by supplier *s* in period *t*
RTQ_t	Number of products produced in the regular time in period *t*
OTQ_t	Number of products produced in the overtime in period *t*
SPQ_t	Number of shortage products at the plant in period *t*
IRQ_t	Remaining quantity of raw materials holding at the plant in period *t*
IPQ_t	Remaining quantity of products holding at the plant in period *t*
HL_t	Number of labors hired in period *t*
FL_t	Number of labors fired in period *t*
L_t	Number of labors level in period *t*
$TranQ_{rt}$	Transportation quantity of products to retailer *r* in period *t*
$IPRQ_{rt}$	Remaining quantity of products holding at retailer *r* in period *t*
$SPRQ_{rt}$	Number of shortage products at retailer *r* in period *t*
MRT_t	Maximum capacity of producing products in the regular time at the plant in period *t*
MOT_t	Maximum capacity of producing products in the overtime at the plant in period *t*
MSC_{st}	Maximum capacity of supplier *s* for supplying raw materials in period *t*

- **Objective Functions**

1. **Minimizing total supply chain operation costs** is a fundamental objective in the design and optimization of an effective supply chain network. As supply chains become increasingly complex, costs are often influenced by a variety of unpredictable factors, such as fluctuations in raw material prices, labor costs, transportation expenses, and inventory holding costs. These costs may not only vary over time but can also be affected by external factors like market demand, supply disruptions, and regulatory changes.

$$
\begin{aligned}
&Minimize\ \widetilde{TC} = \widetilde{PCC} + \widetilde{PDC} + \widetilde{DTC} \\
&= \sum_s^S \sum_t^T \widetilde{RMC}_{st} \times RMQ_{st} + \left(\sum_t^T \widetilde{RC}_t \times RTQ_t\right) + \left(\sum_t^T \widetilde{OC}_t \times OTQ_t\right) \\
&+\left(\sum_t^T \widetilde{SPC}_t \times SPQ_t\right) + \left(\sum_t^T \widetilde{IRC}_t \times \frac{IRQ_{t-1}+IRQ_t}{2}\right) + \left(\sum_t^T \widetilde{IPC}_t \times \frac{IPQ_{t-1}+IPQ_t}{2}\right) \\
&+\left(\sum_t^T \widetilde{HC}_t \times HL_t\right) + \left(\sum_t^T \widetilde{FC}_t \times FL_t\right) + \left(\sum_r^R \sum_t^T \widetilde{TranC}_{rt} \times TranQ_{rt}\right) \\
&+ \left(\sum_t^T \widetilde{IPRC}_{rt} \times \frac{IPRQ_{rt-1}+IPRQ_{rt}}{2}\right) + \left(\sum_r^R \sum_t^T \widetilde{PC}_{rt} \times SPRQ_{rt}\right) \qquad (6.1)
\end{aligned}
$$

2. **Minimizing the Mean-Conditional Value at Risk Gap (MCVaRG) of supply chain operation costs** is crucial for designing effective networks, as it reduces the likelihood of higher-than-expected costs and mitigates downside risks from uncertainties.

$$
\begin{aligned}
&Minimize\ MCVaRG = MCVaRG\left(\widetilde{PCC}\right) + MCVaRG\left(\widetilde{PDC}\right) + MCVaRG\left(\widetilde{DTC}\right) \\
&= \textstyle\sum_s^S \sum_t^T \left[(1-\gamma)RMC_{st}^m + (\gamma)RMC_{st}^p\right] \times RMQ_{st} \\
&+\left(\textstyle\sum_t^T \left[(1-\gamma)RC_t^m + (\gamma)RC_t^p\right] \times RTQ_t\right) \\
&+\left(\textstyle\sum_t^T \left[(1-\gamma)OC_t^m + (\gamma)OC_t^p\right] \times OTQ_t\right) \\
&+\left(\textstyle\sum_t^T \left[(1-\gamma)SPC_t^m + (\gamma)SPC_t^p\right] \times SPQ_t\right) \\
&+\left(\textstyle\sum_t^T \left[(1-\gamma)IRC_t^m + (\gamma)IRC_t^p\right] \times \frac{IRQ_{t-1}+IRQ_t}{2}\right) \\
&+\left(\textstyle\sum_t^T \left[(1-\gamma)IPC_t^m + (\gamma)IPC_t^p\right] \times \frac{IPQ_{t-1}+IPQ_t}{2}\right) \\
&+\left(\textstyle\sum_t^T \left[(1-\gamma)HC_t^m + (\gamma)HC_t^p\right] \times HL_t\right) \\
&+\left(\textstyle\sum_t^T \left[(1-\gamma)FC_t^m + (\gamma)FC_t^p\right] \times FL_t\right) \\
&+(\textstyle\sum_r^R \sum_t^T \left[(1-\gamma)TranC_t^m + (\gamma)TranC_t^p\right] \times TranQ_{rt}) \\
&+(\textstyle\sum_r^R \sum_t^T \left[(1-\gamma)IPRC_t^m + (\gamma)IPRC_t^p\right] \times \frac{IPRQ_{rt-1}+IPRQ_{rt}}{2}) \\
&+\left(\textstyle\sum_r^R \sum_t^T [(1-\gamma)PC_{rt}^{\widetilde{m} + (}\gamma)PC_{rt}^p]PC_{rt} \times SPRQ_{rt}\right) \qquad (6.2)
\end{aligned}
$$

- **Constraints**

$RMQ_{st} \leq MSC_{st} \quad \forall s,t$ (6.3)

$\sum_{s=1}^S RMQ_{st} \geq (RTQ_t + OTQ_t) \quad \forall t$ (6.4)

$SPQ_{rt} = IPQ_{rt-1} - SPQ_{rt} + RTQ_t + OTQ_t - IPQ_{rt} - D_t \quad \forall r,t$ (6.5)

$LH_t \times RTQ_t \leq L_t \times 9{,}600 \quad \forall t$ (6.6)

$L_t = L_{t-1} + HL_t - FL_t \quad \forall t$ (6.7)

$RTQ_t \leq 28{,}000 \quad \forall t$ (6.8)

$OTQ_t \leq 7{,}000 \quad \forall t$ (6.9)

$IRQ_t = IRQ_{t-1} + \sum_{s=1}^S RMQ_{st} - (RTQ_t + OTQ_t) \quad \forall t$ (6.10)

$\sum_{r=1}^S TranQ_{st} \leq RTQ_t + OTQ_t \quad \forall t$ (6.11)

$TranQ_{st} \geq 0.8 \times \widetilde{D_{rt}} \quad \forall r,t$ (6.12)

$SPRQ_{rt} = IPRQ_{rt-1} - SPRQ_{rt} + RTQ_t + OTQ_t - IPRQ_{rt} - D_t \quad \forall r,t$ (6.13)

$RMQ_{st}, RTQ_t, OTQ_t, SPQ_t, IRQ_t, IPQ_t, HL_t, FL_t, L_t \geq 0\ and\ integer\ \forall t$ (6.14)

$TranQ_{rt}, IPRQ_{rt}, SPRQ_{rt} \geq 0\ and\ integer \quad \forall r,t$ (6.15)

6.5 Results Comparison

Table 6.8 presents a comparative analysis of fuzzy-based and probability-based optimization approaches for the SCAPP problem, demonstrating similar objective function values. The Fuzzy Linear Programming (FLP) method is employed to minimize both total supply chain costs and downside risk (MCVaRG), resulting in an optimal total cost of $7,832,100 and an optimal MCVaRG of $1,921,500. When FLP is integrated with Monte Carlo simulation, the peak results from 100 simulation runs yield an optimal total cost of $7,870,772 and an optimal MCVaRG of $1,921,232.20. Despite the close results, fuzzy-based optimization is preferred due to its lower computational complexity and faster processing, delivering reliable outcomes with fewer iterations.

Table 6.8 Result of comparing objectives values.

	Fuzzy-Based Optimization	Probability-Based Optimization
Minimizing total supply chain costs	$7,832,100.00	$7,870,772.00
Minimizing total downside risk	$1,921,500.00	$1,921,232.20

Minimum total supply chain costs	Minimum total downside risk								
	$0.00	$1,500,000	$1,600,000	$1,700,000	$1,800,000	$1,900,000	$2,000,000	$2,100,000	$2,200,000
$0	0	0	0	0	0	0	0	0	0
$5,000,000	0	0	0	0	0	0	0	0	0
$6,000,000	0	0	0	0	2	3	2	2	0
$7,000,000	0	0	0	4	9	27	10	4	0
$8,000,000	0	0	0	2	17	11	0	1	0
$9,000,000	0	0	0	2	0	0	0	0	0
$10,000,000	0	0	0	0	0	0	0	0	0
$11,000,000	0	0	0	0	0	0	0	0	0

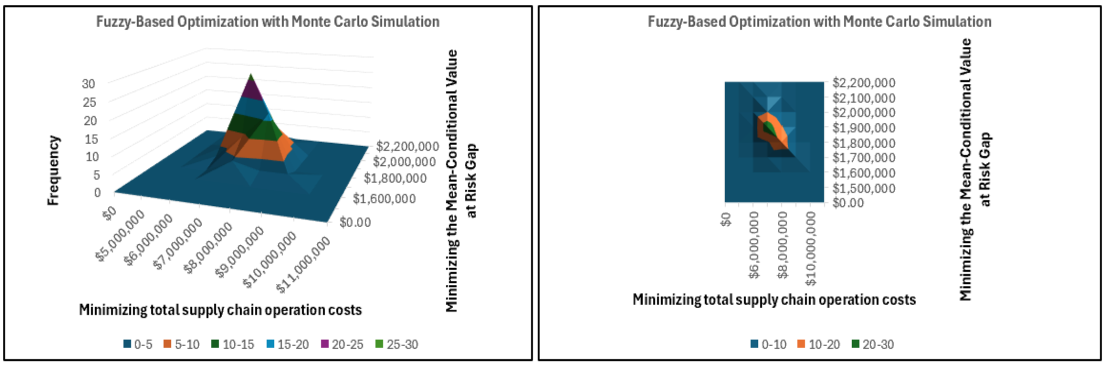

Figure 6.3 The peak of the distribution.

Table 6.9 Result of comparing average percentage of raw materials supplied.

	RMQ_{st} (units)	
	Fuzzy-Based Optimization	Probability-Based Optimization
Supplier 2	100%	95.61%
Supplier 3	23.98%	28.53%

Table 6.10 Result of comparing average percentage of decision variables related to production.

	RTQ_t	OTQ_t	SPQ_t	L_t
Fuzzy-Based Optimization	93.94%	55.99%	2.88%	52 persons
Probability-Based Optimization	94.18%	66.67%	3.10%	49 persons

Table 6.11 Result of comparing percentage of satisfying demands.

	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5	Retailer 6
Fuzzy-Based Optimization	95.33%	98.29%	92.30%	100.00%	93.71%	96.67%
Probability-Based Optimization	95.48%	96.56%	94.61%	97.50%	94.82%	95.95%
Rank of Satisfied Demands	4	2	6	1	5	3

Tables 6.9, 6.10, and 6.11 present a comparative analysis of key performance metrics over six months for both approaches. Table 6.9 indicates that suppliers 2 and 3 contribute nearly identical percentages of raw materials in both methods. Table 6.10 examines average production percentages during regular time and overtime, as well as shortages and labor levels, highlighting comparable results across the two approaches. Similarly, Table 6.11 evaluates the average percentage of retailer demand satisfaction, demonstrating closely aligned outcomes for both approaches.

The Monte Carlo simulation explores the relationships and correlations among critical decision variables by generating a range of possible scenarios based on input uncertainties. This analysis provides valuable insights into how fluctuations in one variable influence others, enabling the identification of patterns, trends, risks, and opportunities. Particularly useful in complex decision-making, this method supports strategic planning and operational optimization. Figure 6.4 illustrates strong linear correlations among raw material demand, product shortages, and production levels, highlighting the impact of shortages on product shipments to retailers. In contrast, weaker correlations between labor levels and shipments stem from production capacity constraints, underscoring the importance of addressing these limitations to enhance system flexibility and efficiency.

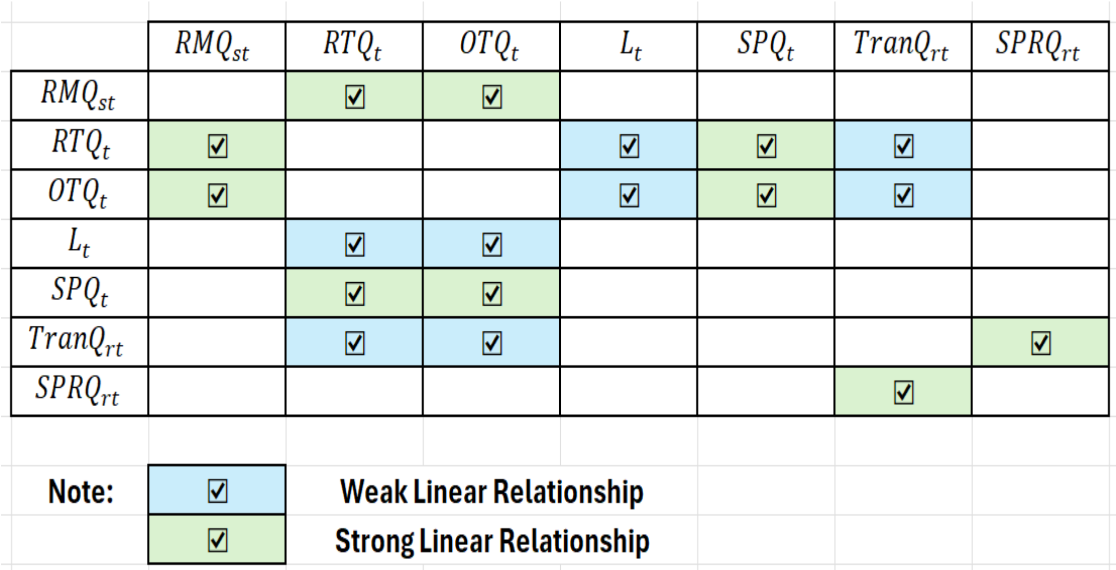

Figure 6.4 The correlation among key decision variables.

6.6 Discussion and Conclusion

This study highlights the significance of integrating Monte Carlo simulation with a fuzzy-based optimization model for supply chain aggregate production planning under uncertainty. The key findings emphasize that this hybrid approach not only enhances risk management by analyzing variability and identifying risks but also provides flexibility in decision-making. By incorporating imprecise information, the model enables decision-makers to adapt effectively to uncertainty, addressing the limitations of traditional deterministic or probabilistic models.

One of the primary advantages of this integrated approach is its ability to bridge the gap between variability analysis and uncertainty modeling. Monte Carlo simulation offers a detailed assessment of variability, allowing for a deeper understanding of potential risks and opportunities. Meanwhile, fuzzy mathematical programming accommodates ambiguity by modeling imprecise information, thus improving the robustness of strategic decisions. This combination results in a framework that is both analytically rigorous and practically adaptable, making it highly suitable for real-world supply chain planning.

Furthermore, the validation of the model demonstrates its reliability in aligning production and resource allocation with fluctuating costs and uncertain demand. Unlike conventional approaches, which often struggle to balance efficiency and flexibility, this model provides a structured adaptable methodology for decision-making. The ability to assess downside risk through Mean-Conditional Value at Risk Gap (MCVaRG) further strengthens the framework by ensuring that financial risks are systematically accounted for in production planning.

Beyond its theoretical contributions, this study offers practical insights for supply chain managers and planners. The framework enhances strategic planning by enabling organizations to anticipate and mitigate uncertainties more effectively. It also improves operational efficiency by aligning production capacity with real-world constraints, thereby reducing waste and optimizing resource utilization. The findings suggest that adopting such an integrated approach could lead to more resilient and cost-effective supply chain operations.

However, despite its strengths, the model presents some limitations that warrant further research. While the current approach effectively integrates fuzzy optimization with stochastic simulation, it does not explicitly consider dynamic external factors such as geopolitical disruptions, sudden market shifts, or supply chain shocks. Future studies could enhance the model's applicability by incorporating adaptive mechanisms that respond to real-time data and evolving external conditions. Additionally, refining computational methods could improve the model's scalability, making it more suitable for large-scale, complex supply chain networks.

Chapter 7
Fuzzy Optimization with Resilience Metrics for Sustainable Supply Chain Planning under Uncertain and Disruption Environments

This chapter presents a novel fuzzy optimization framework that incorporates resilience metrics to support sustainable supply chain planning in environments characterized by uncertainty and disruption. As modern supply chains face increasing exposure to risks such as natural disasters, geopolitical tensions, and global pandemics, the ability to maintain operational continuity while adhering to sustainability goals has become critically important. This study addresses these challenges by integrating fuzzy logic to handle imprecise information and embedding resilience metrics to evaluate the supply chain's capacity to absorb and recover from disruptions. This chapter begins with a detailed problem description, outlining the complexity of achieving sustainability in the presence of unpredictable and disruptive events. It also highlights the main contributions of the research, particularly the development of a resilience-oriented fuzzy optimization model that offers both theoretical advancement and practical utility. The methodology section elaborates on the approach used to construct the model, combining fuzzy set theory with resilience assessment techniques to support robust decision-making. A case study is presented to demonstrate the applicability of the proposed model in a real-world context, followed by a comprehensive explanation of the mathematical notations and model formulation. The results are then analyzed, including comparative assessments to validate the model's performance and effectiveness. This chapter concludes with a discussion of the findings, emphasizing the benefits of integrating resilience into sustainable supply chain planning, and offers suggestions for future research. Overall, this chapter aims to enhance the strategic capabilities of supply chain systems by promoting resilience and sustainability under uncertain and disruptive conditions.

7.1 Problem Description and Contributions

The growing complexity and uncertainty in modern supply chains pose significant obstacles to achieving a balance between sustainability and resilience. Traditional optimization models often fall short in addressing the dynamic nature of supply chain disruptions, fluctuating customer demands, and environmental concerns, making it increasingly difficult for decision-makers to formulate adaptive and sustainable strategies. Despite the theoretical advancements in optimization, there remains a critical gap in integrating risk management techniques that can simultaneously handle imprecise data, satisfaction levels, and the potential for constraint violations.

This study introduces a novel fuzzy optimization technique that aims to overcome these limitations by combining Chance-Constrained Programming (CCP) and Intuitionistic Fuzzy Linear Programming (IFLP). While this framework offers a promising approach to optimizing supply chain planning, it still faces challenges in accounting for the intricate interplay between sustainability, resilience, and the multifaceted uncertainties of real-world supply chains. The incorporation of resilience metrics to evaluate the system's ability to recover from disruptions adds value but requires further refinement in handling the dynamic and evolving nature of risks. Moreover, the trade-offs between minimizing costs and maximizing social and environmental performance need deeper exploration, particularly under highly uncertain conditions.

The case study presented provides practical insights into the model's applicability, but its scalability and robustness in addressing broader industry-wide challenges, such as volatile markets or global supply chain disruptions, remain unclear. This research underscores the need for further investigation into the integration of fuzzy optimization with real-time decision-making processes to truly enhance the efficiency, sustainability, and resilience of supply chains in unpredictable environments.

This study makes significant contributions to the field of fuzzy optimization and sustainable supply chain management. First, it advances fuzzy optimization for risk and resilience management by integrating Chance-Constrained Programming (CCP) with Intuitionistic Fuzzy Linear Programming (IFLP), creating a novel framework that effectively manages constraint violation risks. Unlike traditional optimization methods, this integrated framework considers both satisfaction and non-satisfaction levels, enabling decision-makers to better balance trade-offs under uncertainty. Furthermore, by incorporating resilience metrics such as flexibility, redundancy, and recovery capability, the study provides a structured methodology for assessing and improving the robustness of supply chain operations in uncertain environments. Second, the study offers empirical validation of sustainable supply chain optimization through a detailed case study, showcasing the model's real-world applicability. By simultaneously minimizing total costs and maximizing social and environmental performance, while accounting for imprecise costs and uncertain customer demands, the study establishes a comprehensive decision-support tool for sustainable supply chain management. This practical validation underscores the framework's ability to enhance decision-making, ensuring economic efficiency while promoting environmental and social responsibility in supply chain operations.

7.2 Methodology

This section presents a comprehensive fuzzy optimization methodology that incorporates resilience metrics to enhance the efficiency and adaptability of sustainable supply chain production planning. By integrating advanced fuzzy optimization techniques with key resilience factors such as flexibility, redundancy, and recovery capability, the proposed framework provides a robust decision-support tool for managing uncertainties in supply chain operations. This approach ensures that production planning remains both cost-effective and resilient to disruptions, enabling businesses to maintain operational stability while achieving sustainability objectives. Furthermore, the methodology accounts for the dynamic nature of supply chain environments, addressing challenges related to fluctuating demand, resource availability, and unforeseen risks. Through this integration, the framework not only optimizes production efficiency but also strengthens the supply chain's ability to withstand and recover from disruptions, contributing to long-term sustainability and reliability.

- **Fuzzy Optimization Approach**

The framework utilizes a multi-objective fuzzy linear programming approach, systematically organized into four distinct phases: (1) Data Preparation, where relevant input parameters and uncertainty factors are collected and analyzed; (2) Defuzzification, which transforms fuzzy data into precise values to facilitate computational processing; (3) Membership Function, where fuzzy sets are formulated to represent different levels of uncertainty and decision preferences; and (4) Optimization Process, where the formulated model is solved to achieve optimal trade-offs among multiple objectives. These phases, as illustrated in Figure 7.1, collectively enhance the model's ability to handle uncertainty while ensuring balanced decision-making in sustainable supply chain production planning.

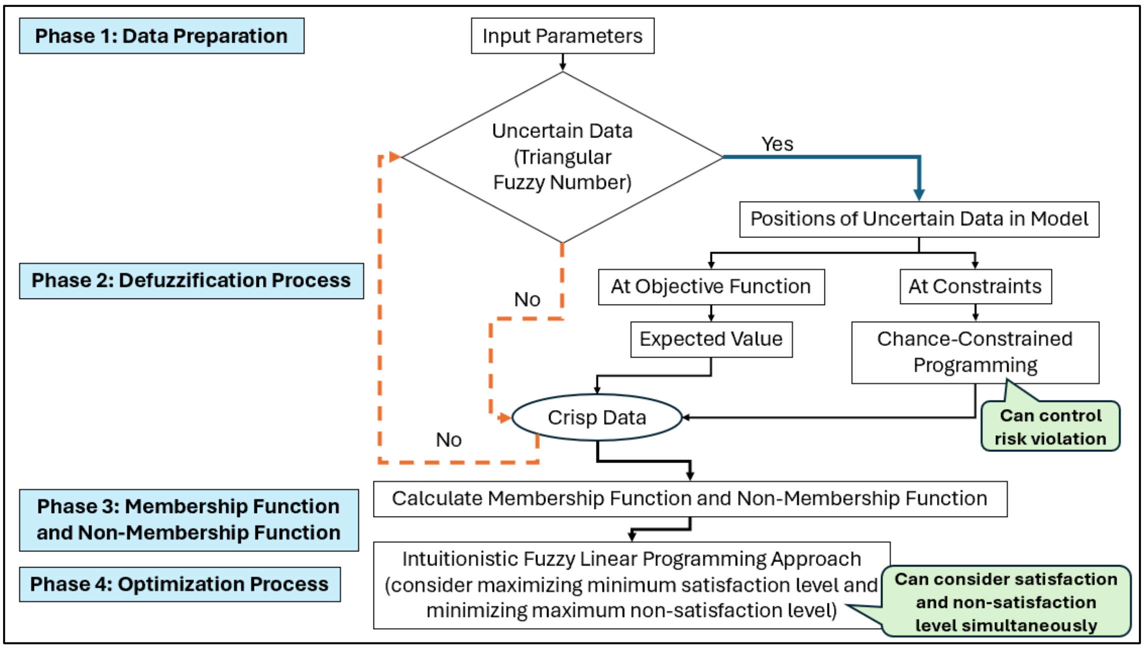

Figure 7.1. Methodology of fuzzy optimization approach.

- **Phase 1: Data Preparation**

In this phase, parameters are categorized into two types: crisp parameters, which are precise and well-defined, and uncertain parameters, which exhibit ambiguity or are challenging to quantify accurately. To effectively model uncertainty, these ambiguous parameters are represented using Triangular Fuzzy Numbers (TFNs), allowing for a more flexible and realistic characterization of imprecise data in the optimization process.

- **Phase 2: Defuzzification Process**

In this stage, uncertain parameters are converted into crisp values through the process of defuzzification. The specific defuzzification method chosen depends on whether the fuzzy parameters are incorporated into the objective functions or the constraints of the model, ensuring that the transformation aligns with the overall optimization structure.

1. **Defuzzification approach for objective function**

The Expected Value (EV) approach is presented as a standard defuzzification method for objective functions, designed to assess their average performance by calculating the expected value of the fuzzy parameters. This method provides a clear and quantitative representation of the overall performance of the objective functions.

$$EV = \frac{(C^o + (2 \times C^m) + C^p)}{4} \tag{7.1}$$

where C^o, C^m, and C^p represent the objective coefficients in optimistic, most likely, and pessimistic scenarios, respectively.

2. **Defuzzification approach for constraint**

Chance-Constrained Programming (CCP) employs a fuzzy measure known as credibility to transform fuzzy data while ensuring that constraints are satisfied with a specified confidence level. This approach can be applied to any fuzzy parameter within the constraints by selecting an appropriate credibility percentage (γ). The credibility metric represents the degree of reliability of a fuzzy event, with higher values indicating greater confidence and a lower risk of constraint violation. CCP has been widely utilized in various studies to manage uncertain parameters, allowing for the adjustment of confidence levels to enhance outcome reliability. A confidence level of 100% encompasses all possible scenarios, ensuring complete certainty, while lower confidence levels ($\gamma < 0.5$) are infrequently used due to their reduced reliability.

$$Cr\{\textstyle\sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}_i\} \geq \gamma$$

$$when\ (0 \leq \gamma \leq 0.5): ax \leq (2\gamma)b^m + (1 - 2\gamma)b^p \tag{7.2}$$

$$when\ (0.5 < \gamma \leq 1): ax \leq (2\gamma - 1)b^o + (2 - 2\gamma)b^m \tag{7.3}$$

where b^o, b^m, and b^p are values of available resource in optimistic, most likely, and pessimistic situations, respectively. γ is the percentage of credibility level, which is assigned to 80% in this study.

- **Phase 3: Membership Function**

In supply chain aggregate production planning, multiple objective functions emerge due to the diverse priorities and complexities of various stakeholders. These objectives are often difficult to compare directly because of differences in units and scales. To address this challenge, membership function and non-membership function are used to normalize these objectives on a common scale, ranging from 0.0 to 1.0. This standardized scale is referred to as the satisfaction level and non-satisfaction level, respectively, allowing for a unified comparison of the objectives.

1. **Membership function for minimization of the objective function**

$$\mu_{Z_i} = \begin{cases} 1, & Z_i \le Z_i^{PIS} \\ \frac{Z_i^{NIS} - Z_i}{Z_i^{NIS} - Z_i^{PIS}}, & Z_i^{PIS} \le Z_i \le Z_i^{NIS} \\ 0, & Z_i \ge Z_i^{NIS} \end{cases} \tag{7.4}$$

2. **Membership function for maximization of the objective function**

$$\mu_{z_i} = \begin{cases} 1, & z_i \ge z_i^{PIS} \\ \frac{z_i - z_i^{NIS}}{z_i^{PIS} - z_i^{NIS}}, & z_i^{NIS} \le z_i \le z_i^{PIS} \\ 0, & z_i \le z_i^{NIS} \end{cases} \tag{7.5}$$

where Z_i^{NIS} is the maximum bound for minimizing objective of the minimum bound for maximizing objective and Z_i^{PIS} is the maximum bound for maximizing objective or the minimum bound for minimizing objective.

1. **Non-membership function for minimization of the objective function**

$$\tau_{z_i} = \begin{cases} 1, & z_i \ge z_i^{PIS} \\ \frac{z_i - z_i^{NIS}}{z_i^{PIS} - z_i^{NIS}}, & z_i^{NIS} \le z_i \le z_i^{PIS} \\ 0, & z_i \le z_i^{NIS} \end{cases} \tag{7.6}$$

2. **Non-membership function for maximization of the objective function**

$$\tau_{z_i} = \begin{cases} 1, & z_i \le z_i^{PIS} \\ \frac{z_i^{NIS} - z_i}{z_i^{NIS} - z_i^{PIS}}, & z_i^{PIS} \le z_i \le z_i^{NIS} \\ 0, & z_i \ge z_i^{NIS} \end{cases} \tag{7.7}$$

where z_i^{PIS} is the maximum bound for minimizing the objective or the minimum bound for maximizing the objective, and z_i^{NIS} is the maximum bound for maximizing the objective or the minimum bound for minimizing the objective.

- **Phase 4: Optimization Process**

This phase centers on finding the optimal solution within Multi-Objective Fuzzy Linear Programming (MOFLP). The Intuitionistic Fuzzy Linear Programming (IFLP) approach is employed to identify the best compromise solution for fuzzy linear programming involving multiple objectives. This is accomplished by simultaneously maximizing the minimum satisfaction level and minimizing the maximum non-satisfaction level of the objective functions, ensuring an effective balance between conflicting goals.

$$
\begin{aligned}
&Maximize\ (\mu_z - \tau_z)\\
&Subject\ to: x \in F(x)\\
&\qquad \mu_z < \mu_{z_i}, \qquad i = 1, 2, \dots, I\\
&\qquad \tau_z < \tau_{z_i}, \qquad i = 1, 2, \dots, I
\end{aligned} \tag{7.8}
$$

where μ_z represents the minimum satisfaction level among multiple objective functions, while μ_{z_i} denotes the satisfaction level of each individual objective function. τ_z represents the maximum non-satisfaction level among multiple objective functions, while τ_{z_i} denotes the non-satisfaction level of each individual objective function.

- **Resilience metrics**

Resilience metrics are quantitative tools used to assess a system's capacity to endure and recover from disruptions or adverse events. In the context of supply chains, these metrics evaluate critical factors such as flexibility, redundancy, and recovery capability, enabling operations to persist despite uncertainties or disruptions. By utilizing key resilience metrics, organizations can identify vulnerabilities and enhance their strategies to ensure operational stability under challenging circumstances.

The following procedure is used calculating resilience metrics.

- **Step 1**: Define resilience metrics
  1. **Flexibility metric**: It assesses how easily the system can adjust to variations in demand, production capacity, supply disruptions, or other unforeseen changes.
  2. **Redundancy metric**: It measures the availability of alternative suppliers, production lines, or logistics options to ensure continuity when primary resources are unavailable or compromised.
  3. **Recovery capability metric**: It measures efficiency with which the system can restore operations, recover lost performance, and minimize the impact of disruptions.

- **Step 2**: Normalize the indicators
  Normalize each indicator to a scale (0 to 1) for comparability.

$$Normalized\ Value = \frac{Actual\ Value - Minimum\ Value}{Maximum\ Value - Minimum\ Value} \tag{7.9}$$

- **Step 3**: Assign weights to each metric
  In this study, each metric is assigned an equal weight due to the lack of prior knowledge regarding their relative importance. The total weight is set to 1.

- **Step 4**: Calculate resilience metrics
  It is employed to combine the normalized indicators and weights to calculate the overall resilience metrics.

$$RM = \sum_{i=1}^{I} W_i \times NS_i \tag{7.10}$$

where W_i is weight of each resilience metric (in this study, weight is assigned equally 33%) and NS_i is normalized score of each resilience metric.

7.3 Case Study

A numerical case study is presented, involving the production of a final product that requires two distinct raw materials. These raw materials are sourced from four different suppliers, each offering unique supply capabilities. The materials are then delivered to two manufacturers, each with varying production capacity levels, where they are processed and transformed into the final product. Once produced, the finished products are distributed to customers through three distribution centers, each of which operates with its own distinct capacity limitations. Following the distribution phase, the products are delivered to five customers, each with their own specific demand requirements, further adding complexity to the distribution network. The entire supply chain structure, encompassing suppliers, manufacturers, distribution centers, and customers, is illustrated in Figure 7.2. This setup demonstrates the interconnectedness and complexity of managing multiple stakeholders, capacities, and demand scenarios within a supply chain network.

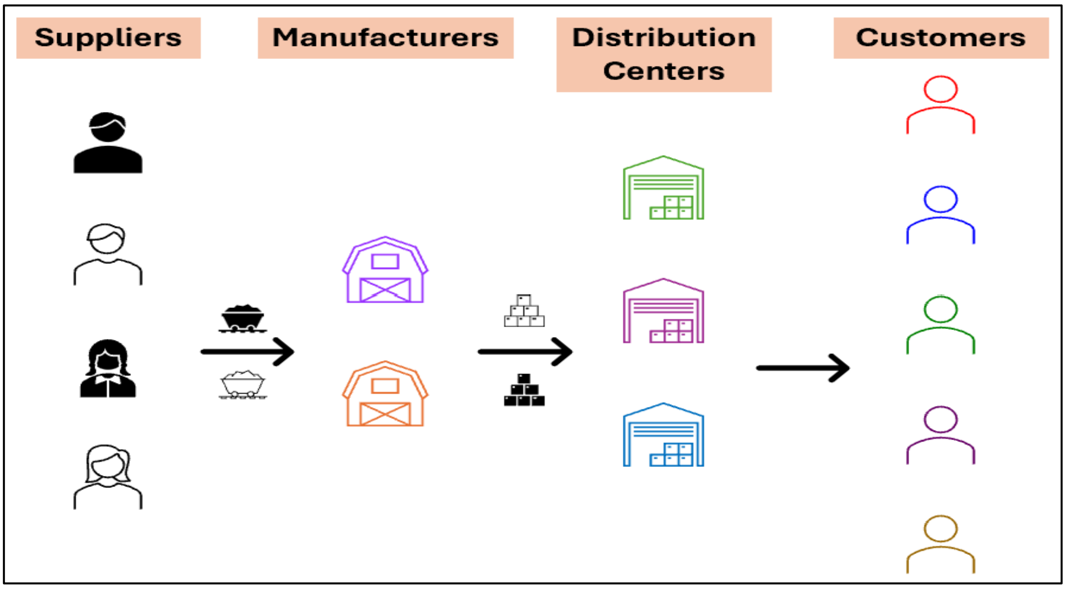

Figure 7.2 Supply chain network.

Table 7.1 displays the values of the input parameters, with uncertain parameters subjected to a variation of up to ±20% to capture both optimistic and pessimistic scenarios.

Table 7.1 Values of input parameters.

Parameters	Values
RPP_r	(1, 1)
$SPCS_{sr}$	(0.75, 0.80, 0.95, 0.62), (0.68, 0.85, 0.77, 0.88)
$SPCM_m$	(0.85, 0.92)
$SPCD_d$	(0.79, 0.87, 0.96)
$EPCS_{sr}$	(0.83, 0.78, 0.91, 0.75), (0.72, 0.67, 0.85, 0.89)
$EPCM_m$	(0.91, 0.78)
$EPCD_d$	(0.86, 0.72, 0.68)
$SCapRM_{sr}$	(700, 700, 700, 700), (700, 700, 700, 700)
$MCapP_{mcm}$	(750, 670), (880, 590)
$SMCap_{mr}$	(450, 380), (350, 470)
$SDCap_{dcd}$	(900, 900, 900), (900, 900, 900)
$SSRM_{mr}$	(0.15, 0.25), (0.22, 0.18)
$SSRD_d$	(0.17, 0.23, 0.27)
$\widetilde{PurC}_{sr}$	(1.25, 1.45, 0.85, 0.55), (0.75, 0.35, 1.15, 1.65)
$\widetilde{FCS}_{sr}$	(75, 85, 98, 65), (58, 50, 70, 90)
$\widetilde{FCM}_{mcm}$	(8,500, 9,200), (7,900, 8,700)
$\widetilde{FCD}_{dcd}$	(5,600, 6,300, 5,900), (6,500, 5,200, 6,800)
$\widetilde{ProdC}_m$	(2.25, 2.75)
$\widetilde{MHC}_{rm}$	(2.46, 1.87)
$\widetilde{DHC}_d$	(2.87, 3.27, 3.65)
$\widetilde{TCSM}_{smr}$	[(1.23, 1.14, 1.72, 1.26), (1.41, 1.92, 1.34, 1.78)] [(1.56, 1.49, 1.85, 1.39), (1.17, 1.32, 1.68, 1.54)]
$\widetilde{TCMD}_{md}$	(1.46, 1.38, 1.52), (1.94, 1.83, 1.75)
$\widetilde{TCDC}_{dc}$	(1.54, 1.26, 1.43, 1.71, 1.64), (1.29, 1.35, 1.48, 1.52, 1.94), (1.58, 1.67, 1.21, 1.45, 1.76)
$\widetilde{De}_c$	(225, 246, 289, 217, 263)

7.4 Mathematical Notations and Model

The notations for indexes, parameters, and decision variables are provided in Tables 7.2 to 7.4. Notably, all fuzzy parameters are indicated by a tilde ($\sim$) placed over the corresponding symbols.

Table 7.2 Indexes.

Indexes	Meaning
r	Index of raw materials, $r = 1, \ldots, R$
s	Index of suppliers, $s = 1, \ldots, S$
m	Index of manufacturers, $m = 1, \ldots, M$
d	Index of distribution centers, $d = 1, \ldots, D$
c	Index of customers, $c = 1, \ldots, C$
cm	Index of capacity levels of manufacturers, $cm = 1, \ldots, CM$
cd	Index of capacity levels of distribution centers, $cd = 1, \ldots, CD$

Table 7.3 Mathematical notations of parameters.

Parameters	Meaning
RPP_r	Amount of required raw materials r for producing a unit of product
SPC	Social performance score
$SPCS_{sr}$	Social performance score of supplier s for raw material r
$SPCM_m$	Social performance score of manufacturers m for a product
$SPCD_d$	Social performance score of distribution centers d for a product
EPC	Environmental performance score
$EPCS_{sr}$	Environmental performance score of supplier s for raw material r
$EPCM_m$	Environmental performance score of manufacturers m for a product
$EPCD_d$	Environmental performance score of distribution centers d for a product
$SCapRM_{sr}$	Capacity of suppliers s for raw materials r
$MCapP_{mcm}$	Production capacity cm of manufacturers m for manufacturing products
$SMCap_{mr}$	Capacity of manufacturers m for storing raw materials r
$SDCap_{dcd}$	Capacity of distribution centers d for storing products
$SSRM_{mr}$	Safety stock of raw materials r at manufacturers m
$SSRD_d$	Safety stock of products at distribution centers d
$\widetilde{PurC}_{sr}$	Fuzzy purchasing cost of raw materials r from suppliers s
$\widetilde{FCS}_{sr}$	Fuzzy fixed cost of making a contract with suppliers s
$\widetilde{FCM}_{mcm}$	Fuzzy fixed cost of installing factory of manufacturers m with different

	manufacturers' capacity levels *cm*
$\widetilde{FCD}_{dcd}$	Fuzzy fixed cost of installing distribution centers *d* with different distribution centers' capacity levels *cd*
$\widetilde{ProdC}_{m}$	Fuzzy producing cost for a product at manufacturers *m*
$\widetilde{MHC}_{rm}$	Fuzzy holding cost of raw materials *r* at manufacturers *m*
$\widetilde{DHC}_{d}$	Fuzzy holding cost of products at distribution centers *d*
$\widetilde{TCSM}_{smr}$	Fuzzy transporting cost of raw materials *r* from suppliers *s* to manufacturers *m*
$\widetilde{TCMD}_{md}$	Fuzzy transporting cost of products from manufacturers *m* to distribution centers *d*
$\widetilde{TCDC}_{dc}$	Fuzzy transporting cost of products from distribution centers *d* to customers *c*
$\widetilde{De}_{c}$	Fuzzy demand of customers *c*

Table 7.4 Mathematical notations of decision variables.

Decision Variables	Meaning
PR_{sr}	If suppliers *s* can provide raw materials *r* equal to 1, otherwise 0.
SS_{sr}	If suppliers *s* is selected for providing raw materials *r* equal to 1, otherwise 0.
IM_{mcm}	If factory of manufacturers *m* is installed with capacity level *cm* equal to 1, otherwise 0.
ID_{dcd}	If distribution centers *d* is installed with capacity level *cd* equal to 1, otherwise 0.
$TRQS_{tmr}$	Amount of transported raw materials *r* from suppliers *s* to manufacturers *m*
$TPQM_{md}$	Amount of transported products from manufacturers *m* to distribution centers *d*
$TPQD_{dc}$	Amount of transported products from distribution centers *d* to customers *c*
PQM_{m}	Amount of produced products at manufacturers *m*
$SSQM_{mr}$	Amount of safety stock of raw materials *r* at manufacturers *m*
$SSQD_{d}$	Amount of safety stock of products at distribution centers *d*
ARM_{sr}	Amount of raw materials *r* allocated to suppliers *s*

- **Objective Functions**

1. **Minimizing total supply chain operational costs** is a key objective in developing an effective and sustainable supply chain plan, particularly when costs are subject to inherent uncertainty.

$$Minimize\ \widetilde{TSCOC} = \sum_{s=1}^{S}\sum_{r=1}^{R}\widetilde{FCS_{sr}} \times SS_{sr} + \sum_{m=1}^{M}\sum_{cm=1}^{CM}\widetilde{FCM_{mcm}} \times IM_{mcm} + \sum_{d=1}^{D}\sum_{cd=1}^{CD}\widetilde{FCD_{dcd}} \times ID_{dcd} + \sum_{m=1}^{M}\sum_{d=1}^{D}\widetilde{TCMD_{md}} \times TPQM_{md} + \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{r=1}^{R}\widetilde{TCSM_{smr}} \times TRQS_{tmr} + \sum_{d=1}^{D}\sum_{c=1}^{C}\widetilde{TCDC_{dc}} \times TPQD_{dc} + \sum_{s=1}^{S}\sum_{r=1}^{R}\widetilde{PurC_{sr}} \times \widetilde{PurC_{sr}} + \sum_{m=1}^{M}\widetilde{ProdC_{m}} \times PQM_{m} + \sum_{m=1}^{M}\sum_{r=1}^{R}\widetilde{MHC_{rm}} \times SSQM_{mr} + \sum_{d=1}^{D}\widetilde{DHC_{d}} \times SSQD_{d} \quad (7.11)$$

In recent years, many companies have increasingly recognized the importance of sustainability and ethical practices. Consumers are becoming more conscious of the social and environmental impacts of the products and services they purchase. Integrating social and environmental considerations into the supply chain has become essential for businesses to operate responsibly, comply with legal standards, and strengthen their reputation.

In this study, the social and environmental performance of each supply chain echelon is assessed using expert judgment. This process involves calculating a weighted score for each significant social or environmental factor within an echelon (W_i) and evaluating the performance rating of each echelon in addressing these factors (R_i). The highest scores reflect the best performance in social and environmental terms.

$$Social/Environmental\ Performance\ Score = \sum_{i}^{I} W_i R_i \quad (7.12)$$

2. **The total social performance score**, which is calculated by multiplying each echelon's social performance score by the quantity of raw materials or products transported, must be maximized, as indicated in Equation (7.13).

$$Maximize\ SPC = \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{r=1}^{R} SPCS_{sr} \times TRQS_{tmr} + \sum_{m=1}^{M}\sum_{d=1}^{D} SPCM_{m} \times TPQM_{md} + \sum_{d=1}^{D}\sum_{c=1}^{C} SPCD_{d} \times TPQD_{dc} \quad (7.13)$$

3. **The total environmental performance score**, which is calculated by multiplying each echelon's environmental performance score by the quantity of raw materials or products transported, must be maximized, as shown in Equation (7.14).

$$Maximize\ EPC = \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{r=1}^{R} EPCS_{sr} \times TRQS_{tmr} + \sum_{m=1}^{M}\sum_{d=1}^{D} EPCM_{m} \times TPQM_{md} + \sum_{d=1}^{D}\sum_{c=1}^{C} EPCD_{d} \times TPQD_{dc} \quad (7.14)$$

- **Constraints**

$$\sum_{m=1}^{M} TRQS_{tmr} \leq ARM_{sr} \quad \forall r, s \quad (7.15)$$
$$ARM_{sr} \leq SCapRM_{sr} \times PR_{sr} \times SS_{sr} \quad \forall r, s \quad (7.16)$$

$$PQM_m \leq \sum_{cm=1}^{CM} MCapP_{mcm} \times IM_{mcm} \quad \forall m \tag{7.17}$$
$$RPP_r \times PQM_m + SSQM_{mr} = \sum_{s=1}^{S} TRQS_{tmr} \quad \forall r, m \tag{7.18}$$
$$SSQM_{mr} \geq SSRM_{mr} \times RPP_r \times PQM_m \quad \forall r, m \tag{7.19}$$
$$SSQM_{mr} \leq SMCap_{mr} \times IM_{mcm} \quad \forall r, m, cm \tag{7.20}$$
$$PQM_m = \sum_{d=1}^{D} TPQM_{md} \quad \forall m \tag{7.21}$$
$$\sum_{m=1}^{M} TPQM_{md} \leq \sum_{cd=1}^{CD} SDCap_{dcd} \times ID_{dcd} \quad \forall d \tag{7.22}$$
$$\sum_{m=1}^{M} TPQM_{md} - SSQD_d = \sum_{c=1}^{C} TPQD_{dc} \quad \forall d \tag{7.23}$$
$$SSQD_d \geq SSRD_d \times \sum_{m=1}^{M} TPQM_{md} \quad \forall d \tag{7.24}$$
$$\sum_{c=1}^{C} TPQD_{dc} = \widetilde{De_c} \quad \forall c \tag{7.25}$$

7.5 Results

- **Result of fuzzy optimization approach**

Fuzzy Linear Programming (FLP) is employed to address the SCAPP problem, aiming to simultaneously minimize total supply chain operational costs, maximize the social performance score, and maximize the environmental performance score. The optimal results reveal a minimum total supply chain operational cost of $70,631, a maximum social performance score of 7,100, and a maximum environmental performance score of 7,215.4, as summarized in Table 7.5.

Table 7.5 Result of fuzzy optimization approach.

	Minimizing total supply chain operational costs	Maximizing social performance scores	Maximizing environmental performance scores
Objective Values	$70,631	7,100 scores	7,215.4 scores
Satisfaction Level	67.62%	69.72%	67.62%
Non-Satisfaction Level	32.37%	30.28%	32.37%

- **Result of measuring resilience metrics of model under disruption**

In this study, the probability distribution of disruption levels is categorized into six cases, as illustrated in Table 7.6.

Table 7.6 The probability distribution of disruption level.

	% Disruption	Probability	Samples
CASE1: Very High	50%	4%	2
CASE2: High	40%	8%	4
CASE3: Moderate	30%	10%	5
CASE4: Low	20%	12%	6
CASE5: Very Low	10%	16%	8
CASE6: None	0%	50%	25

In this study, the proposed fuzzy linear programming model is assessed by simultaneously varying the capacities of suppliers for providing raw materials, manufacturers for producing products, and distribution centers for distributing products. Variations in supplier capacity are influenced by factors such as raw material shortages, natural disasters, financial difficulties faced by suppliers, or unforeseen operational challenges. Manufacturer capacity may fluctuate due to issues like machinery breakdowns, labor shortages, operational inefficiencies, or natural disasters. Distribution center capacity can vary because of product shortages, labor shortages, operational inefficiencies, or natural disasters. The combined variations in supply chain capacity encompass all disruptions in supplier, manufacturer, and distribution center capacities within the supply chain. By integrating these disruptions into the model, the study evaluates how effectively the system adapts to different disruption levels, ensuring continued efficiency and sustainability of the supply chain under challenging conditions. This evaluation provides decision-makers with valuable insights into the impact of supply chain reliability and highlights the importance of a flexible, resilient supply chain design.

Table 7.7 The results under disruption cases.

Supplier Disruption					
Case	Total supply chain costs	Total social scores	Total environmental scores	Probability distribution of handling disruption	Resilience Index
1	0	0	0	0%	70.37%
2	0	0	0	0%	
3	$64,672.33	5,446.60	5,680.10	60%	
4	$66,520.00	6,021.40	6,212.08	100%	
5	$73,150.25	6,404.85	6,570.09	100%	
6	$75,002.84	6,990.12	7,117.54	100%	
Manufacturer Disruption					
Case	Total supply chain costs	Total social scores	Total environmental scores	Probability distribution of handling disruption	Resilience Index
1	0	0	0	0%	75.89%
2	$71,942.00	6,504.30	6,665.35	50%	
3	$74,618.60	6,821.96	7,036.82	100%	
4	$75,131.83	7,092.70	7,159.37	100%	
5	$76,997.75	7,112.66	7,196.39	100%	
6	$75,002.84	6,990.12	7,117.54	100%	
Distribution Center Disruption					
Case	Total supply chain costs	Total social scores	Total environmental scores	Probability distribution of handling disruption	Resilience Index
1	$89,165.00	6,835.75	6,890.35	100%	81.46%
2	$85,115.00	7,360.30	7,354.13	100%	
3	$79,915.80	6,999.96	7,105.54	100%	
4	$75,363.17	7,146.38	7,211.35	100%	
5	$77,433.13	7,081.65	7,153.50	100%	

6	75,002.84	6,990.12	7,117.54	100%	
Supply Chain Disruption					
Case	Total supply chain costs	Total social scores	Total environmental scores	Probability distribution of handling disruption	Resilience Index
1	0	0	0	0%	67.36%
2	0	0	0	0%	
3	$74,040.50	5,496.50	5,630.75	40%	
4	$72,541.17	6,237.85	6,345.98	100%	
5	$75,509.25	6,681.79	6,764.33	100%	
6	$75,002.84	6,990.12	7,117.54	100%	

As shown in Table 7.7, the supply chain experiences a significant impact when disruptions affect suppliers by over 30% and manufacturers by over 40% of the probability distribution of disruption levels. Consequently, decision-makers must develop strategies to mitigate these challenges by adapting their supply chain operations. In this study, one backup supplier with a capacity to provide 700 units of raw materials and 500 units of subcontracted production is integrated into the original supply chain to enhance resilience.

Table 7.8 The implement results under disruption cases.

Supplier Disruption					
Case	Total supply chain costs	Total social scores	Total environmental scores	Probability distribution of handling disruption	Resilience Index
1	$74,227.00	5745.1	5948.1	100%	79.77%
2	$77,442.00	6562.0	6718.2	100%	
3	$70,582.60	6049.7	6303.9	60%	
4	$66,520.00	6021.4	6212.1	100%	
5	$73,150.25	6404.9	6570.1	100%	
6	$75,002.84	6990.1	7117.5	100%	
Manufacturer Disruption					
Case	Total supply chain costs	Total social scores	Total environmental scores	Probability distribution of handling disruption	Resilience Index
1	$73,674.50	6123.9	6241.8	100%	80.39%
2	$73,275.00	6672.3	6839.3	100%	
3	$74,618.60	6822.0	7036.8	100%	
4	$75,131.83	7092.7	7159.4	100%	
5	$76,997.75	7112.7	7196.4	100%	
6	$75,002.84	6990.1	7117.5	100%	
Supply Chain Disruption					
Case	Total supply chain costs	Total social scores	Total environmental scores	Probability distribution of handling disruption	Resilience Index
1	$85,894.00	5739.0	5768.1	100%	78.77%
2	$84,170.25	6552.0	6579.6	100%	
3	$78,294.20	6376.2	6429.5	100%	
4	$72,541.17	6237.9	6346.0	100%	
5	$75,509.25	6681.8	6764.3	100%	

6	$75,002.84	6990.1	7117.5	100%	

As shown in Table 7.8, all disruption scenarios (Case1-Case6) involving supplier, manufacturer, distribution center, and overall supply chain disruptions can be effectively managed after implementation. Furthermore, the flexibility and redundancy indices have improved, as 100% of disruption cases are now addressable. However, the recovery capability index has slightly decreased, as the costs associated with the backup supplier and subcontracted production are higher than normal when disruptions occur.

Table 7.9 Comparing resilience index.

Before Implementation				
Resilience Index	Supplier	Manufacturer	Distribution Center	Supply Chain
1.Flexibility	27.72%	30.36%	33.00%	27.06%
2.Redundancy	27.72%	30.36%	33.00%	27.06%
3.Recovery Capability	14.93%	15.17%	15.46%	13.24%
Total	70.37%	75.89%	81.46%	67.36%
After Implementation				
Resilience Index	Supplier	Manufacturer	Distribution Center	Supply Chain
1.Flexibility	33.00%	33.00%	33.00%	33.00%
2.Redundancy	33.00%	33.00%	33.00%	33.00%
3.Recovery Capability	13.80%	14.40%	15.46%	12.80%
Total	79.77%	80.39%	81.46%	78.77%

Table 7.9 presents a comparative analysis of the resilience index for before and after the implementation of the supply chain configuration under disruption. The results clearly indicate that the implementation of the proposed supply chain configuration leads to a notable improvement in the resilience index across all evaluated scenarios. This enhancement demonstrates the effectiveness of the configuration in strengthening the supply chain's ability to absorb, adapt to, and recover from unexpected disturbances, thereby contributing to greater operational stability and robustness.

7.6 Discussion and Conclusion

The increasing complexity and unpredictability of modern supply chains present significant challenges in achieving both sustainability and resilience. While conventional approaches often fail to account for the dynamic nature of uncertainties in costs, customer demands, and disruptions, this study introduces an advanced fuzzy optimization framework that integrates Chance-Constrained Programming (CCP) with Intuitionistic Fuzzy Linear Programming (IFLP) to address these issues. The framework offers decision-makers a systematic approach to managing risk, balancing multiple objectives, and ensuring operational stability under uncertain conditions.

A key contribution of this study is its demonstration of enhanced decision-making under uncertainty. By incorporating fuzzy optimization techniques, decision-makers can navigate imprecise data with greater flexibility and accuracy, allowing for more robust planning in fluctuating supply chain environments. The trade-offs between cost, resilience, and sustainability are explicitly modeled, enabling a structured approach to risk-informed decision-making. However, while the framework provides a structured methodology for managing uncertainty, its reliance on specific assumptions regarding fuzzy parameters and resilience metrics may limit its adaptability across diverse industries and rapidly evolving supply chain contexts.

Additionally, the study aligns sustainability and resilience goals, illustrating that these objectives are not mutually exclusive but can be optimized simultaneously. The methodology demonstrates how organizations can integrate environmental and social considerations into their supply chain strategies while reinforcing their ability to withstand disruptions. However, a deeper analysis of the long-term trade-offs between sustainability investments and resilience-building costs is necessary, as different industries may require varying levels of investment in redundancy and flexibility.

From a risk management perspective, the framework's incorporation of flexibility, redundancy, and recovery capability provides a structured mechanism for identifying and mitigating risks proactively. By assessing a supply chain's capacity to adapt to disruptions and recover efficiently, the model supports the development of more resilient supply chain strategies. Nevertheless, practical implementation challenges remain, particularly in industries with high volatility and limited historical data for modeling uncertainties effectively. Further refinement is required to enhance real-time adaptability and integration with emerging technologies such as machine learning for predictive risk management.

The case study reinforces the practical viability of the proposed framework, demonstrating its applicability in addressing real-world sustainability and resilience challenges. The results highlight the importance of employing fuzzy optimization techniques in supply chain management, particularly in mitigating the risks associated with fluctuating costs and variable customer demands. However, scalability and generalizability of the model across different industries and global supply chain networks require further investigation.

Ultimately, this study advances the field of sustainable and resilient supply chain management by integrating fuzzy optimization with resilience metrics, providing a structured flexible decision-making framework. However, future research should focus on enhancing computational efficiency, expanding the applicability of the model to larger-scale supply chains, and exploring real-time decision-making adaptations to improve responsiveness in volatile environments. The findings emphasize the critical role of advanced fuzzy optimization in shaping the next generation of supply chain planning methodologies, paving the way for further academic inquiry and practical advancements in an ever-changing global landscape.

Chapter 8
Discussion and Conclusion

This final chapter provides a comprehensive discussion of the findings and conclusions drawn from the preceding chapters. It synthesizes the key insights gained from the research, offering a critical analysis of the proposed models and methodologies in the context of real-world supply chain challenges. This chapter begins by highlighting the managerial implications of the study, focusing on how the proposed fuzzy optimization approaches can be applied to improve decision-making in supply chain planning, particularly under uncertain and disruptive conditions. Then, the limitations of the study are provided. Subsequently, the limitations of the study are discussed in various aspects. This chapter concludes with recommendations for future research, suggesting areas for further exploration and improvement to enhance the applicability of the proposed frameworks in supply chain contexts. By reflecting on both the strengths and limitations of the study, this chapter aims to provide a balanced perspective on the contributions of the research and its potential impact on the field of supply chain management.

8.1 Managerial Implications

The findings of this study present significant managerial implications, offering decision-makers a robust framework to enhance supply chain performance under uncertainty. The proposed integration of fuzzy optimization provides a comprehensive approach for optimizing supply chain operations. The following key implications highlight how managers can utilize these insights to improve decision-making and strategic planning.

- **Development of Innovative Fuzzy Optimization Models**

The introduction of fuzzy optimization models provides managers with a sophisticated tool to navigate uncertainties in supply chain operations. Unlike traditional deterministic models that struggle with imprecise data, fuzzy optimization techniques allow managers to incorporate vagueness and ambiguity in critical parameters such as fluctuating costs, uncertain customer demands, and variable production capacities. This capability enables managers to develop more adaptive and responsive decision-making strategies, ensuring that supply chain plans remain effective even in volatile conditions.

- **Enhancing Resilience and Sustainability**

Supply chain resilience and sustainability have become top priorities for organizations seeking long-term operational stability. This study emphasizes the importance of integrating resilience metrics such as flexibility, redundancy, and recovery capability into supply chain planning. By adopting this approach, managers can proactively prepare for disruptions, whether caused by natural disasters, economic instability, or sudden demand shifts. Additionally, the study underscores how sustainability and resilience can be mutually reinforced rather than conflicting goals. By optimizing cost structures while maximizing environmental and social performance, managers can develop supply chain strategies that not only improve profitability but also enhance corporate responsibility and regulatory compliance. The proposed approaches ensure that supply chains remain sustainable and resilient in an increasingly uncertain business environment.

- **Bridging Theory and Practice**

A persistent challenge in supply chain management is translating complex mathematical models into practical, actionable insights. This study bridges that gap by demonstrating how advanced fuzzy optimization techniques can be effectively implemented in real-world scenarios. Through a detailed case study, this study illustrates how managers can apply data-driven decision-making to enhance supply chain efficiency. By utilizing this framework, managers can move beyond intuition-based decision-making and instead rely on quantitative, model-driven strategies that align with business goals. This practical applicability ensures that advanced optimization techniques are not confined to academic research but are integrated into everyday supply chain management practices, ultimately improving decision accuracy and operational robustness.

- **Advancing Knowledge in Supply Chain Optimization**

The proposed integrated of fuzzy optimization in supply chain planning represents a significant advancement in supply chain optimization methodologies. By incorporating fuzzy optimization techniques, managers can stay ahead of industry trends and enhance their strategic capabilities. This study provides valuable insights into how decision-makers can adopt innovative approaches to supply chain planning, moving beyond traditional deterministic models that often fail under real-world uncertainties. The findings of this study offer a foundation for continuous improvement and innovation, enabling businesses to develop agile, data-driven, and future-proof supply chains that remain competitive in a rapidly evolving market.

By implementing these insights, managers can significantly improve their ability to develop resilient, sustainable, and optimized supply chain operations. The study provides a clear pathway for integrating advanced optimization techniques into strategic planning and decision-making, ensuring that businesses can thrive in the face of uncertainty while achieving both operational excellence and long-term sustainability.

8.2 Limitations and Further Study

While this study offers a novel fuzzy optimization framework for supply chain planning, there are several limitations that must be considered when interpreting the findings. These limitations highlight areas for future research to further refine and extend the model's applicability and effectiveness in real-world scenarios.

- **Limitations**

One of the primary limitations of the proposed methodology is its computational complexity. The integration of fuzzy optimization techniques inherently requires substantial computational resources, particularly when applied to large-scale supply chain networks with multiple stages and decision variables. This can lead to challenges in real-time decision-making, particularly for organizations with limited access to advanced computational infrastructure. Furthermore, while the study accounts for imprecise parameters, it does not fully explore the impact of external uncertainties, such as political instability, supply chain disruptions from global events, or changes in regulatory environments. These factors could influence the model's effectiveness in specific industries or regions, limiting its generalizability to broader supply chain contexts. Lastly, the assumptions related to resilience metrics (such as flexibility, redundancy, and recovery) may vary significantly depending on the industry, and the model might not fully capture the unique dynamics of different supply chain environments, requiring adaptation and customization.

- **Further Study**

Given these limitations, several avenues for further research emerge. First, future studies could focus on reducing the computational complexity of the proposed fuzzy optimization framework, potentially by developing more efficient algorithms or incorporating machine learning techniques to improve decision-making speed and scalability. This could help organizations with limited computational power to adopt these advanced methods in practical settings. Additionally, expanding the model to include a broader range of external uncertainties, such as geopolitical risks, market fluctuations, and environmental disruptions, would enhance its relevance and applicability across different industries.

Another area for future exploration is the empirical validation of the model in diverse supply chain environments. While the study demonstrates the model’s efficacy through a case study, additional real-world applications across different sectors could provide further insights into the model's robustness and adaptability. By testing the framework in varied industry contexts, researchers can uncover additional factors influencing the interplay between resilience, sustainability, and optimization. Furthermore, researchers could explore the integration of supply chain risk management techniques into the proposed framework to enhance its ability to respond to unforeseen disruptions.

Finally, future research could examine the long-term effects of sustainable supply chain practices, including the role of circular economy principles, eco-design, and sustainable sourcing strategies in enhancing both resilience and sustainability. Understanding how these long-term practices interact with the fuzzy optimization framework could provide valuable insights for organizations aiming to develop sustainable supply chains that can adapt to future challenges while maintaining profitability and social responsibility. By addressing these research gaps, the field of supply chain optimization can continue to evolve and offer more effective tools for managers in increasingly complex and uncertain environments.

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