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Random Walks by Local Topological Information on Scale-free Graphs

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The process that a token moves at random on a given graph are called "random walk." Random walks have been extensively studied as an essential technology with wide-ranging applications such as design and analysis of randomized algorithms.

Recently, a new model of random walk is introduced by Ikeda, et al. In a conventional simple random walk, the next vertex is selected from adjacent vertices at random with equal probability. Meanwhile, the new model employs topological information for adjacent vertices. In spite of the fact that random walk using local topological information is relatively simple extension of the usual model, the random walk is known as a model which covers vertices of a network so much more efficiently.

More precisely, the next vertex in the model is chosen with transition probability $p^{(\beta)}$ determined by a potential function for local topological information. Since Ikeda et al. are interested in the worst case on a general graph, they don't assume particular graph structures. The parameter β is a real constant to configure a ratio which evaluates degrees of adjacent vertices. When $\beta = 0.5$, the highest efficiency is obtained for the random walk.

Scale-free graph model which produces WWW graph and internet graph, etc. has attracted a lot of attention. This graph is considered that which can model various social networks because of its non-uniform structure,

rather than of a traditional Erdős-Rényi's Random Graph with its uniform structure. Scale-free graphs follow a law called "power law" and some social networks in real world are also stisfying this law.

The purpose of this research is to achieve more efficiency of Ikeda et al. new random walk using the non-uniformity of scale-free graph positively.

We adopt preferential attachment graph model which is the most acceptable among known scale-free graph models. This model can generate various graphs from sparse to dense and generated graphs are undirected graph.

Firstly, we evaluated relations among Ikeda's random walk parameter β, scale-free graph parameter m and graph size n in computational experiments. The results showed more efficiency of the random walk for optimal β than a conventional random walk and the fact that the improvement for a larger β is more efficient than that for a smaller β. It turned out that an optimal β is roughly a function of m, however independing on n.

Secondly, we attempted an improvement of the potential function. Since the distribution of vertex degree on scale-free graph follows a power law, the degree distribution is unbalanced extremely. The fact leads a prediction that the potential function can be designed for the random walk to behave better correcting the degree distribution linearly. According to the prediction, we have experimented for Ikeda's random walk with a potential function as a logarithm function, not as a normal degree function. In the result, the potential function improved a cover time for small m more than a normal degree potential function.

In preferential attachment graph model, a generated graph is undirected graphs that is inappropriate for a network model like Web graph. Bollobás's model, generalizing the preferential attachment graph model, is known as a graph model which generates directed graphs with power law. In general, however, the strong connectedness cannot be ensured on random directed graphs with power law. Since these graphs have many vertices with no in-degree, even reachability cannot be ensured. Fagin et al. introduced random walk with "back button" for the sake of this problem. In fact, in Google which is one of the most popular search engine, we can know which pages links to a Web page P by tracing links of P backward. For the above reason, we regard study of "random walks with moving backward

on directed edges" as important things for applications.

In this paper, we introduced random walks which allow Ikeda's random walks to move on backward edges with probability $\gamma(0 < \gamma \leq 1)$ on Bollobás's directed scale-free graphs with a power law and evaluated optimal parameters in experiments. In the result, the effect of improvements for optimal β is better than result of random walks not using local topological information ($\beta = 0$) as γ approaches to 0. Meanwhile, as γ approaches to 1, the difference between random walks not using local topological information and our random walks for optimal β is reduced relatively.