Finding Efficiency and Inefficiency: Multi-Viewpoint Apporoach

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ABSTRACT

In order to evaluate the performance of DMU (Decision Making Unit), this paper proposes a new decision support technique referred to as Multi-Viewpoint DEA model. The proposed model is an extended model which integrates the DEA-efficiency model and the DEA-inefficiency model into one mathematical formulation, and allows us to analyze the performance of DMU between the strong points and weak points. A case study shows that the proposed model has desirable two features: (1) robustness of the evaluation value, and (2) unification between DEA-efficiency model and DEA-inefficiency model.

Keywords: Data Envelopment Analysis, Efficiency Approach, Inefficiency Approach.

1. Introduction

As initiated and developed by Charnes et al. [1], DEA is a nonparametric method for finding the relative efficiency of DMUs, each of which is a company responsible for converting multiple inputs into multiple outputs. DEA has been applied to a variety of managerial and economic problem situations in both public and private sectors [5, 9, 13, 14]. DEA assumes each DMU uses multiple inputs to yield multiple outputs, and defines the process which changes multiple inputs into multiple outputs as one evaluation value.

From the analysis concept, the decision method based on such evaluation value induces two kinds of approaches: One is the efficiency approach based on the Pareto optimal solution for the aspect only of the strong points [1, 5]. The other is the inefficiency approach based on the Pareto optimal solution for the aspect only of the weak points [7]. Then, the evaluation values in two approaches are inconsistent [8]. However, analysts have evaluated DMUs only by extreme aspect: either strong points or weak points. Thus, the traditional two approaches lack flexibility and robustness [17].

In fact, while there are many inputs and outputs in DEA framework, these items are not fully used in the previous approaches. This type of DEA problem has been usually tackled by multiplier restriction approaches [15] and cone ratio approaches [16]. While such multiplier restrictions usually reduce the number of zero weight, they often produce an infeasible solution in DEA. Therefore, new DEA model which has robustness on the evaluation values is required.

This paper proposes a new decision support technique referred to as Multi-Viewpoint DEA model. The remaining structure of this paper is organized as follows: the next section reviews the traditional DEA models. Section 3 proposes a new model. The proposed model integrates the DEA-efficiency model and the DEA-inefficiency model into one mathematical formulation, and allows us to analyze the performance of DMU by multi-viewpoint between the strong points and weak points. Section 4 verifies the proposed model through a case study. A case study shows that the proposed model has two desirable features: (1) robustness of the evaluation value, and (2) unification between DEA-efficiency model and DEA-inefficiency model. Finally, conclusion and future study are summarized in section 5.

2. DEA: Data Envelopment Analysis

2.1. Evaluation value

In order to describe the mathematical structure of the evaluation value, this paper assumes that there are n DMUs $(DMU_1, \cdots, DMU_k, \cdots, DMU_n)$, where each DMU is characterized by m inputs $(x_{1k}, \cdots, x_{ik}, \cdots, x_{mk})$ and s outputs $(y_{1k}, \cdots, y_{rk}, \ldots, y_{sk})$. Evaluation value of DMU_k is mathematically formulated by

$$\begin{matrix}\text{Evaluation value}\\ \text{of } DMU_k\end{matrix} = \frac{u_1 y_{1k} + u_2 y_{2k} + \cdots + u_s y_{sk}}{v_1 x_{1k} + v_2 x_{2k} + \cdots + v_m x_{mk}} \quad (1)$$

Here u_r is multiplier weight given to the r^{th} output, and v_i is multiplier weight given to the i^{th} input. From the analysis concept, there are two decision methods for calculating these weights. One is the efficiency approach based on the Pareto optimal solution for the aspect only of the strong points [1, 5]. The other is the inefficiency approach based on the Pareto optimal solution for the aspect only of the weak points [7, 8].

Figure 1 visually represents the difference of two methods. Suppose that there are nine DMUs which have one input and two outputs where X-axis is output 1 over input and Y-axis is output 2 over input. So, if a DMU is located in upper–right region, it shows that the DMU has high productivity. DEA-efficiency model finds out the efficiency frontier which indicates the best practice line (B-C-D-E-F in Figure 1) and evaluates the relative evaluation value by the aspect only of the strong points. On the other hand, DEA-inefficiency model finds out the inefficiency frontier which indicates the worst practice line (B-I-H-G-F in Figure 1) and evaluates the relative evaluation value by the aspect only of the weak points.

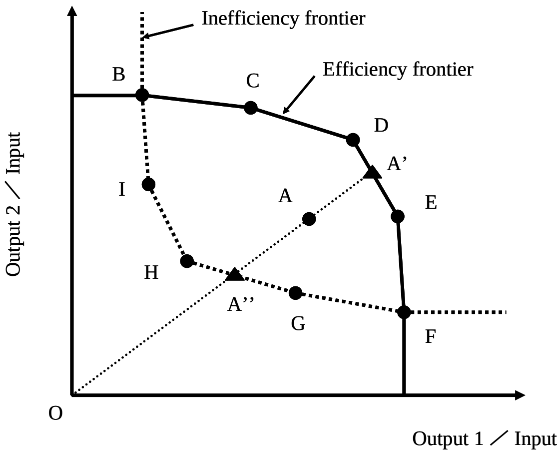

Figure 1. Efficiency model and Inefficiency model

2. 2. DEA-efficiency model

The efficiency approach measures the efficiency level of a specific DMU_k by relativity comparing its performance to the efficiency frontier. This paper is based on CCR model [1] while there are other models [5, 11]. The efficiency approach can be mathematically formulated by

$$\begin{aligned}
&\text{Max} && \sum_{r=1}^{s} u_r y_{rk} \; (=\theta_k^E) && (2\text{-}1)\\
&\text{s.t.} && -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} \le 0 && (2\text{-}2)\\
& && (j = 1, 2, \ldots, n) && \\
& && \sum_{i=1}^{m} v_i x_{ik} = 1 && (2\text{-}3)\\
& && v_i \ge 0,\; u_r \ge 0 &&
\end{aligned} \qquad (2)$$

Here formula (2-2) is a restriction condition because the productivity of all DMUs (formula (1)) becomes 100% or less. And the objective function (2-1) represents the maximization of the sum of virtual outputs of DMU_k, setting that the virtual inputs of DMU_k is equal to 1 (formula (2-3)). Therefore, the optimal solution of (v_i, u_r) represents the convenient weight for DMU_k. Especially, the optimal objective function value indicates the evaluation value (θ_k^E) for DMU_k. This evaluation value by the convenient weight is called "efficiency score" in the manner that $\theta_k^E = 1 (100\%)$ means the state of efficiency, while $\theta_k^E < 1 (100\%)$ means the state of inefficiency.

2. 3. DEA-inefficiency model

There is another approach which measures the inefficiency level of a specific DMU_k based on Inversed DEA model [7]. The inefficiency approach can be mathematically formulated by

$$\begin{aligned}
&\text{Min} && \sum_{r=1}^{s} u_r y_{rk} = \left(\frac{1}{\theta_k^{IE}}\right) && (3\text{-}1)\\
&\text{s.t.} && -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} \ge 0 && (3\text{-}2)\\
& && (j = 1, 2, \ldots, n) && \\
& && \sum_{i=1}^{m} v_i x_{ik} = 1 && (3\text{-}3)\\
& && v_i \ge 0,\; u_r \ge 0 &&
\end{aligned} \qquad (3)$$

Again, formula (3-2) is a restriction condition because the productivity of all DMU (formula (1)) becomes 100% or more. And the objective function (3-1) represents the minimization of the virtual outputs of DMU_k, setting that the virtual inputs of DMU_k is equal to 1 (formula (3-3)). Therefore, the optimal solution of (v_i, u_r) represents the inconvenient weight for DMU_k. Especially, the inverse number of optimal objective function value indicates the "inefficiency score" in the manner that $\theta_k^{IE} = 1\,(100\%)$ means the state of inefficiency, while $\theta_k^{IE} < 1\,(100\%)$ means the state of efficiency.

2.4. Problems in Traditional Approaches

As shown in Figure 1, DMU_B and DMU_F are evaluated as both states of "efficiency ($\theta_k^E = 1$)" and "inefficiency $(\theta_k^{IE} = 1)$ ". This result clearly shows mathematical difference in two approaches. For the example, DMU_B has the best productivity for the Output 2 / input, while it has worst productivity for the Output 1 / input. In efficiency approach, the weight of DMU_B is evaluated by the aspect of the strong points. Therefore, the weight of Output 2 / input becomes a positive value and the weight of Output 1 / input becomes zero. On the other hand, in inefficiency approach, the weight of DMU_B is evaluated by the aspect of the weak points. Therefore, the weight of Output 2 / input becomes zero and the weight of Output 1 / input becomes a positive value. This difference of the weight estimation causes the mathematical problems as follow:

(a) No robustness of evaluation value

Both approaches may produce zero weights for most inputs and outputs. The zero weight indicates that the corresponding inputs or outputs are not used for the evaluation value. Moreover, if the specific inputs or output items are removed from the analysis, the evaluation value may change greatly [17]. This type of DEA problem is usually tackled by multiplier restriction approaches [15] and cone ratio approaches [16]. Such multiplier restrictions usually reduce the number of zero weight, and these approaches often produce an infeasible solution. The development of DEA model which has robustness of the evaluation value is required.

(b) Lack of unification between DEA-efficiency and DEA-inefficiency model

Fundamentally, efficient DMU can not be inefficient while inefficient DMU can not be efficient. However, the evaluation value may be not consistent like the DMU_B and DMU_F in the Figure 1 where they are in the both states of "efficiency" and "inefficiency". Thus, it is not easy for a decision maker to understand the difference between evaluation values. The basis of the evaluation value which has unification between DEA-efficiency model and DEA-inefficiency model is required.

3. Multi-Viewpoint DEA Model

Let us propose a new decision support technique referred to as Multi-Viewpoint DEA model. The proposed model is a re-formulation of the DEA-efficiency and DEA-inefficiency model into one mathematical formulation.

3.1. DEA-efficiency model based on GP technique

This paper applies the following formula (4) which added the variable (d_j^+, d_j^-) to formula (2-2):

$$-\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + d_j^+ - d_j^- = 0 \;\; (j = 1, 2, \ldots, n) \qquad (4)$$

Here d_j^+ indicates the slack variables, and d_j^- indicates the artificial variables. Therefore, the objective function (2-1) can be replaced by mathematically using several big M as follows:

$$\sum_{r=1}^{s} u_r y_{rk} - M \sum_{j=1}^{n} d_j^- \qquad (5)$$

From the formula (4) and formula (2-3), the objective function (5) can be rewritten as follows:

$$\begin{aligned} \sum_{r=1}^{s} u_r y_{rk} - M \sum_{j=1}^{n} d_j^- &= (\sum_{i=1}^{m} v_i x_{ik} - d_k^+ + d_k^-) - M \sum_{j=1}^{n} d_j^- \\ &= 1 - d_k^+ + d_k^- - M \sum_{j=1}^{n} d_j^- \qquad (6) \\ &= 1 - d_k^+ + (1 - M) d_k^- - M \sum_{j=1, j \neq k}^{n} d_j^- \end{aligned}$$

Using GP (Goal Programming) technique, the DEA-efficiency-model (formula (2)) can be replaced by the following Linear Programming:

$$\text{Max} \quad 1 - d_k^+ + (1-M)d_k^- - M \sum_{j=1,\, j\neq k}^{n} d_j^-$$

$$\text{s.t.} \quad -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + d_j^+ - d_j^- = 0 \quad (j = 1, 2, \ldots, n) \tag{7}$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$v_i \geq 0,\, u_r \geq 0,\, d_j^+,\, d_j^- \geq 0$$

The efficiency score (θ_k^E) of DMU_k as follows:

$$\theta_k^E = 1 - d_k^{+*} \left(= \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ik} (=1)} \right) \tag{8}$$

Where superscript "*" indicate the optimal solution of formula (7).

3.2. DEA-inefficiency model based on GP technique

Let us apply the formula (4) which added the variable (d_j^+, d_j^-) to formula (3-2). This paper notes that d_j^+ indicates the artificial variables and d_j^- indicates the slack variables in DEA-inefficiency model. Using GP technique, the DEA-inefficiency-model (formula (3)) can be replaced by the following Linear Programming:

$$\text{Min} \quad 1 + (M-1)d_k^+ + d_k^- + M \sum_{j=1,\, j\neq k}^{n} d_j^+$$

$$\text{s.t.} \quad -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + d_j^+ - d_j^- = 0 \quad (j = 1, 2, \ldots, n) \tag{9}$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$v_i \geq 0,\, u_r \geq 0,\, d_j^+,\, d_j^- \geq 0$$

The inefficiency score (θ_k^{IE}) of DMU_k as follows:

$$\theta_k^{IE} = \frac{1}{1 + d_k^{-*}} \tag{10}$$

Where superscript "*" indicate the optimal solution of formula (9).

3.3. Mathematical integration of the efficiency and inefficiency model

In order to integrate two DEA models into one formula mathematically, this paper introduces slack variables. As seen in formula (7) and (9), it is understood that the both models have the same restriction conditions. Then, this paper applies the following formula (11) which added any constant (α, β) to the objective function of formula (7) and (9).

$$\begin{aligned} &\alpha\{1 - d_k^+ + (1-M)d_k^- - M \sum_{j=1,\, j\neq k}^{n} d_j^-\} \\ &\quad - \beta\{1 + (M-1)d_k^+ + d_k^- + M \sum_{j=1,\, j\neq k}^{n} d_j^+\} \\ &= (\alpha - \beta) - \{\alpha - \beta(1-M)\}d_k^+ + \{\alpha(1-M) - \beta\}\, d_k^- \\ &\quad - (\alpha M \sum_{j=1,\, j\neq k}^{n} d_j^+ + \beta M \sum_{j=1,\, j\neq k}^{n} d_j^-) \end{aligned} \tag{11}$$

When formula (11) is divided by several big M mathematically, it can be developed as follows:

$$\begin{aligned} &-(\beta d_k^+ + \alpha\, d_k^-) - (\beta \sum_{j=1,\, j\neq k}^{n} d_j^+ + \alpha \sum_{j=1,\, j\neq k}^{n} d_j^-) \\ &= -\beta \sum_{j=1}^{n} d_j^+ - \alpha \sum_{j=1}^{n} d_j^- \end{aligned} \tag{12}$$

Where these constants can be estimated $\alpha + \beta = 1$, because the constants (α, β) indicate relative ratios of the DEA-efficient and the DEA-inefficiency model. Then the proposed model is formulated as the following Linear Programming:

$$\text{Max} \quad -(1-\alpha)\sum_{j=1}^{n} d_j^+ - \alpha \sum_{j=1}^{n} d_j^-$$

$$\text{s.t.} \quad -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + d_j^+ - d_j^- = 0 \quad (j = 1, 2, \ldots, n) \tag{13}$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$v_i \geq 0,\, u_r \geq 0,\, d_j^+,\, d_j^- \geq 0$$

Where x_{ij} : i^{th} input value of j^{th} DMU,
y_{rj} : r^{th} input value of j^{th} DMU,
v_i , u_r : input and output weight,
d_i^+ , d_r^- : slack variables.

The formula (13) includes the viewpoint's parameter α, and allows us to analyze the performance of DMU by changing the parameter α between the strong points (especially, if $\alpha = 1$ then the optimal solutions is the same with one of DEA-efficiency model) and weak points (if $\alpha = 0$ then the optimal solutions is the same with one of DEA-inefficiency model).

And if $\alpha = \alpha'$ then this paper defines the evaluation value ($\theta_k^{MVP,\alpha'}$) of DMU_k as follows:

$$\begin{aligned}\theta_k^{MVP,\alpha'} &= \alpha'\theta_k^{E} - (1-\alpha')\theta_k^{IE} \\ &= \alpha'(1-d_k^{+*}) - (1-\alpha')\left(\frac{1}{1+d_k^{-*}}\right)\end{aligned} \tag{14}$$

Where superscript "*" indicate the optimal solution of formula (13).

The first term of formula (14) indicates the evaluation value by the aspect of the strong points and the second term indicates it by the aspect of the weak points. Therefore, the evaluation value ($\theta_k^{MVP,\alpha'}$) is measured on the range between -1 (-100%: inefficiency) and 1 (100%: efficiency).

4. Case Study

4.1. A data set

A data set used in this paper is demonstrated in Table 1. There are twelve DMU whose performance is measured by two inputs and three outputs.

Table 1. A data set

No.	DMU	Input		Output		
		Input1	Input2	Output1	Output2	Output3
1	A	10	8	23	32	21
2	B	26	10	37	47	32
3	C	40	15	80	148	68
4	D	35	28	76	104	60
5	E	30	21	23	40	20
6	F	33	10	38	89	41
7	G	37	12	78	175	65
8	H	50	22	68	200	77
9	I	31	15	48	86	33
10	J	12	10	16	35	16
11	K	20	12	64	74	23
12	L	45	26	72	58	35

Table 2. Evaluation value ($\theta^{MVP,1}$) and optimal solutions $(v^*, u^*, d^{+*}, d^{-*})$

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0.100	0	0.017	0.006	0.019	0	0	1.055	0	0.383	0	0.389	0	1.968	0	1.293	0
2	B	0.033	0.015	0	0	0.021	0	0	0.316	0	0.080	0	0.288	0	0.872	0	0.352	0
3	C	0.021	0.010	0	0	0.014	0	0	0.206	0	0.052	0	0.188	0	0.569	0	0.230	0
4	D	0.029	0	0.005	0.002	0.006	0	0	0.301	0	0.109	0	0.111	0	0.562	0	0.369	0
5	E	0.033	0	0.006	0.002	0.006	0	0	0.352	0	0.128	0	0.130	0	0.656	0	0.431	0
6	F	0	0.100	0	0	0.018	0.412	0	0.409	0	0.245	0	1.692	0	1.731	0	0.243	0
7	G	0.024	0.008	0.006	0	0.008	0	0	0.232	0	0.062	0	0.133	0	0.601	0	0.327	0
8	H	0.020	0	0	0.002	0.007	0	0	0.218	0	0.082	0	0.109	0	0.394	0	0.227	0
9	I	0.032	0	0.005	0.002	0.006	0	0	0.340	0	0.123	0	0.125	0	0.635	0	0.417	0
10	J	0.083	0	0	0.007	0.030	0	0	0.909	0	0.340	0	0.452	0	1.643	0	0.947	0
11	K	0.042	0.014	0.011	0	0.013	0	0	0.396	0	0.105	0	0.228	0	1.027	0	0.558	0
12	L	0.019	0.006	0.005	0	0.006	0	0	0.177	0	0.047	0	0.102	0	0.459	0	0.250	0

No.	DMU	Slack Variable											
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-
1	A	0	0	1.075	0	1.098	0	0.395	0	0	0	2.236	0
2	B	0	0	0.321	0	0.535	0	0.203	0	0.344	0	1.117	0
3	C	0	0	0.209	0	0.349	0	0.132	0	0.225	0	0.729	0
4	D	0	0	0.307	0	0.314	0	0.113	0	0	0	0.639	0
5	E	0	0	0.358	0	0.366	0	0.132	0	0	0	0.745	0
6	F	0	0	0.778	0	0.891	0	0.705	0	0.775	0	1.954	0
7	G	0	0	0.369	0	0.317	0	0.148	0	0	0	0.580	0
8	H	0	0	0.135	0	0.249	0	0.071	0	0.119	0	0.559	0
9	I	0	0	0.347	0	0.354	0	0.127	0	0	0	0.721	0
10	J	0	0	0.563	0	1.037	0	0.295	0	0.496	0	2.330	0
11	K	0	0	0.631	0	0.541	0	0.253	0	0	0	0.990	0
12	L	0	0	0.282	0	0.242	0	0.113	0	0	0	0.443	0

Evaluation Value		
$1 \times \theta^{E}$	$0 \times \theta^{IE}$	$\theta^{MVP,1}$
1	0	1
0.684	0	0.684
0.948	0	0.948
0.889	0	0.889
0.344	0	0.344
0.757	0	0.757
1	0	1
0.865	0	0.865
0.646	0	0.646
0.705	0	0.705
1	0	1
0.557	0	0.557

4.2. Multi-Viewpoint DEA's result

This paper calculates five patterns ($\alpha = 1,\ 0.75,\ 0.5,\ 0.25,\ 0)$. Table 2 shows the evaluation value ($\theta^{MVP,1}$) and optimal solutions $(v^*, u^*, d^{+*}, d^{-*})$ when the viewpoint parameter $\alpha = 1$. And Table 3 shows the evaluation value ($\theta^{MVP,0.75}$) and optimal solutions $(v^*, u^*, d^{+*}, d^{-*})$ when the viewpoint parameter $\alpha = 0.75$. The other pattern's results are put on Table A-1, A-2, and A-3 in Appendix.

Finding 1:

As shown in Table 1, when the viewpoint parameter $\alpha = 1$, the optimal solutions of formula (2) and formula (13) were the same about inefficiency DMU ($\theta^E < 1$). However in this case study, the optimal solutions of formula (2) was different from it of formula (13) about efficiency DMU ($\theta^E = 1$). The same result applied to inefficiency model (formula (9) and formula (13) with parameter $\alpha = 0$ in Appendix Table A-3). The cause of this result is the multiple solutions problem [3]. Thus, the proposed model is considered to integration DEA model of the efficiency and inefficiency model.

Finding 2:

Comparing the results of Table 1 to ones of Table 2, the combination of the optimal solution was change. Especially, focusing the ranking of evaluation value about DMU_2 and DMU_6, we can find that the order of DMU_2 and DMU_6 was reversed. As shown in Table 1, because DMU_2 has the multiplicity of strong points such as output 2 and output3, it is understood that DMU_2 has high rank roughly. In contrast, because DMU_6 has the only one strong point such as output 2, it is understood that DMU_6 has low rank roughly. Thus, proposed model can allow us to know a change in strong points and weak points for each DMU. The important point of this result is that proposed model can allow us to know the robustness for each DMU's evaluation value.

Table 3. Evaluation value ($\theta^{MVP,0.75}$) and optimal solutions $(v^*, u^*, d^{+*}, d^{-*})$

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0.069	0.039	0.019	0	0.026	0	0	0.622	0	0	0	0.444	0	1.911	0	0.849	0
2	B	0.038	0	0.007	0	0.013	0	0.060	0.308	0	0.057	0	0	0	0.722	0	0.451	0
3	C	0.025	0	0.005	0	0.009	0	0.039	0.200	0	0.037	0	0	0	0.469	0	0.293	0
4	D	0.020	0.011	0.006	0	0.008	0	0	0.178	0	0	0	0.127	0	0.546	0	0.243	0
5	E	0.031	0.003	0.006	0	0.012	0	0.054	0.237	0	0	0	0	0	0.622	0	0.343	0
6	F	0.030	0	0.006	0	0.010	0	0.048	0.242	0	0.045	0	0	0	0.569	0	0.355	0
7	G	0.027	0	0.005	0	0.009	0	0.042	0.216	0	0.040	0	0	0	0.507	0	0.317	0
8	H	0.020	0	0.004	0	0.007	0	0.031	0.160	0	0.030	0	0	0	0.375	0	0.234	0
9	I	0.031	0.003	0.006	0	0.012	0	0.053	0.234	0	0	0	0	0	0.613	0	0.338	0
10	J	0.057	0.032	0.016	0	0.022	0	0	0.512	0	0	0	0.365	0	1.572	0	0.699	0
11	K	0.047	0.004	0.009	0	0.018	0	0.082	0.359	0	0	0	0	0	0.941	0	0.519	0
12	L	0.021	0.002	0.004	0	0.008	0	0.036	0.160	0	0	0	0	0	0.419	0	0.231	0

No.	DMU	Slack Variable												Evaluation Value		
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-	$0.75\times\theta^E$	$0.25\times\theta^{IE}$	$\theta^{MVP,0.75}$
1	A	0	0.205	0.956	0	0.922	0	0.483	0	0	0	1.797	0	0.750	0.250	0.5
2	B	0	0.004	0.410	0	0.407	0	0.134	0	0	0	0.745	0	0.519	0.250	0.269
3	C	0	0.002	0.266	0	0.264	0	0.087	0	0	0	0.484	0	0.722	0.250	0.472
4	D	0	0.059	0.273	0	0.263	0	0.138	0	0	0	0.513	0	0.655	0.250	0.405
5	E	0	0.055	0.298	0	0.330	0	0.116	0	0	0	0.631	0	0.284	0.250	0.034
6	F	0	0.003	0.323	0	0.321	0	0.105	0	0	0	0.587	0	0.484	0.250	0.234
7	G	0	0.003	0.288	0	0.286	0	0.094	0	0	0	0.523	0	0.750	0.249	0.501
8	H	0	0.002	0.213	0	0.212	0	0.069	0	0	0	0.387	0	0.590	0.250	0.340
9	I	0	0.054	0.294	0	0.325	0	0.115	0	0	0	0.623	0	0.506	0.250	0.256
10	J	0	0.169	0.786	0	0.758	0	0.397	0	0	0	1.478	0	0.452	0.250	0.202
11	K	0	0.083	0.450	0	0.499	0	0.176	0	0	0	0.955	0	0.750	0.250	0.5
12	L	0	0.037	0.201	0	0.222	0	0.078	0	0	0	0.425	0	0.431	0.250	0.181

5. Conclusion

This paper has proposed a new decision support technique referred to as Multi-Viewpoint DEA model which integrated the DEA-efficiency model and the DEA-inefficiency model into one mathematical formulation. The proposed model allows us to analyze the performance of DMU by changing the viewpoint's parameter α between the strong points (especially, if $\alpha = 1$ then it becomes DEA-efficiency model) and weak points (if $\alpha = 0$ then it becomes DEA-inefficiency model). A case study has shown that the proposed model has two desirable features: (1) robustness of the evaluation value, and (2) unification between DEA-efficiency model and DEA-inefficiency model.

For the future study, we will apply the proposed method to practical problems which include volumes of data. We will also analytically compare our method to the traditional approaches [15, 16] and explore how to set the viewpoint's parameter.

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Appendix

Table A-1. Evaluation value ($\theta^{MVP,0.50}$) and optimal solutions $(v^*, u^*, d^{+*}, d^{-*})$

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0.038	0.078	0.030	0	0.020	0	0.116	0	0	0	1.104	0	0	1.670	0	0.053	0
2	B	0.038	0.0	0.004	0	0.021	0	0.160	0.162	0	0	0.249	0	0.255	0.631	0	0.237	0
3	C	0.025	0.0	0.003	0	0.014	0	0.104	0.105	0	0	0.162	0	0.166	0.410	0	0.154	0
4	D	0.011	0.022	0.009	0	0.006	0	0.033	0	0	0	0.315	0	0	0.477	0	0.015	0
5	E	0.021	0.017	0.005	0	0.015	0	0.062	0.085	0	0	0.257	0	0	0.601	0	0.096	0
6	F	0.030	0	0.004	0	0.017	0	0.126	0.127	0	0	0.196	0	0.201	0.497	0	0.187	0
7	G	0.027	0	0.003	0	0.015	0	0.112	0.114	0	0	0.175	0	0.179	0.443	0	0.167	0
8	H	0.020	0	0.002	0	0.011	0	0.083	0.084	0	0	0.129	0	0.133	0.328	0	0.123	0
9	I	0.032	0	0.004	0	0.018	0	0.134	0.136	0	0	0.209	0	0.214	0.529	0	0.199	0
10	J	0.031	0.063	0.025	0	0.016	0	0.094	0	0	0	0.897	0	0	1.357	0	0.043	0
11	K	0.033	0.028	0.007	0	0.023	0	0.099	0.134	0	0	0.407	0	0	0.952	0	0.152	0
12	L	0.015	0.012	0.003	0	0.010	0	0.044	0.060	0	0	0.183	0	0	0.428	0	0.069	0

No.	DMU	Slack Variable											
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-
1	A	0	1.330	0	0	0.226	0	0.427	0	0	0.706	0.845	0
2	B	0	0.292	0	0	0.284	0	0.054	0	0	0	0.673	0
3	C	0	0.190	0	0	0.185	0	0.035	0	0	0	0.438	0
4	D	0	0.380	0	0	0.064	0	0.122	0	0	0.202	0.241	0
5	E	0	0.320	0	0	0.212	0	0.120	0	0	0	0.560	0
6	F	0	0.230	0	0	0.224	0	0.042	0	0	0	0.531	0
7	G	0	0.205	0	0	0.200	0	0.038	0	0	0	0.473	0
8	H	0	0.152	0	0	0.148	0	0.028	0	0	0	0.350	0
9	I	0	0.245	0	0	0.238	0	0.045	0	0	0	0.565	0
10	J	0	1.080	0	0	0.183	0	0.347	0	0	0.573	0.687	0
11	K	0	0.506	0	0	0.336	0	0.190	0	0	0	0.886	0
12	L	0	0.228	0	0	0.151	0	0.086	0	0	0	0.399	0

Evaluation Value		
$0.50\times\theta^{E}$	$0.50\times\theta^{IE}$	$\theta^{MVP,0.50}$
0.500	0.448	0.052
0.419	0.500	-0.081
0.500	0.430	0.070
0.500	0.500	0.000
0.199	0.500	-0.301
0.407	0.500	-0.093
0.500	0.415	0.085
0.500	0.500	0.000
0.381	0.500	-0.119
0.327	0.500	-0.173
0.500	0.500	0.000
0.301	0.500	-0.199

Table A-2. Evaluation value ($\theta^{MVP,0.25}$) and optimal solutions $(v^*, u^*, d^{+*}, d^{-*})$

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0.036	0.080	0.051	0	0.001	0	0.195	0	0.187	0	1.515	0	0.436	1.565	0	0	0
2	B	0.038	0	0.020	0	0.008	0	0.246	0	0	0	0.614	0	0.663	0.531	0	0.178	0
3	C	0.025	0	0.013	0	0.005	0	0.160	0	0	0	0.399	0	0.431	0.345	0	0.116	0
4	D	0.010	0.023	0.014	0	0.000	0	0.056	0	0.053	0	0.433	0	0.125	0.447	0	0	0
5	E	0.013	0.029	0.018	0	0.001	0	0.071	0	0.068	0	0.549	0	0.158	0.567	0	0	0
6	F	0.030	0	0.016	0	0.006	0	0.194	0	0	0	0.484	0	0.522	0.419	0	0.140	0
7	G	0.027	0	0.014	0	0.006	0	0.173	0	0	0	0.432	0	0.466	0.373	0	0.125	0
8	H	0.020	0	0.011	0	0.004	0	0.128	0	0	0	0.319	0	0.345	0.276	0	0.093	0
9	I	0.032	0	0.017	0	0.007	0	0.206	0	0	0	0.515	0	0.556	0.446	0	0.149	0
10	J	0.029	0.065	0.041	0	0.001	0	0.159	0	0.152	0	1.229	0	0.354	1.270	0	0	0
11	K	0.050	0	0.026	0	0.010	0	0.319	0	0	0	0.799	0	0.861	0.691	0	0.231	0
12	L	0.022	0	0.012	0	0.005	0	0.142	0	0	0	0.355	0	0.383	0.307	0	0.103	0

No.	DMU	Slack Variable											
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-
1	A	0	1.757	0	0	0	0.165	0.399	0	0	1.596	0	0
2	B	0	0.666	0	0.058	0	0.038	0.012	0	0	0.705	0	0
3	C	0	0.433	0	0.038	0	0.025	0.008	0	0	0.458	0	0
4	D	0	0.502	0	0	0	0.047	0.114	0	0	0.456	0	0
5	E	0	0.637	0	0	0	0.060	0.144	0	0	0.578	0	0
6	F	0	0.524	0	0.046	0	0.030	0.009	0	0	0.556	0	0
7	G	0	0.468	0	0.041	0	0.027	0.008	0	0	0.496	0	0
8	H	0	0.346	0	0.030	0	0.020	0.006	0	0	0.367	0	0
9	I	0	0.558	0	0.049	0	0.032	0.010	0	0	0.591	0	0
10	J	0	1.426	0	0	0	0.134	0.323	0	0	1.295	0	0
11	K	0	0.865	0	0.076	0	0.049	0.016	0	0	0.917	0	0
12	L	0	0.385	0	0.034	0	0.022	0.007	0	0	0.407	0	0

Evaluation Value		
$0.25\times\theta^{E}$	$0.75\times\theta^{IE}$	$\theta^{MVP,0.25}$
0.250	0.627	-0.377
0.250	0.750	-0.500
0.250	0.536	-0.286
0.250	0.667	-0.417
0.108	0.750	-0.642
0.215	0.750	-0.535
0.250	0.511	-0.261
0.250	0.728	-0.478
0.250	0.727	-0.477
0.169	0.750	-0.581
0.250	0.391	-0.141
0.250	0.750	-0.500

Table A-3. Evaluation value ($\theta^{MVP,0}$) and optimal solutions $(v^*, u^*, d^{+*}, d^{-*})$

No.	DMU	Weight					Slack Variable											
		v1	v2	u1	u2	u3	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d5+	d5-	d6+	d6-
1	A	0	0.125	0	0.066	0	0	1.100	0	1.834	0	7.838	0	3.325	0	0	0	4.591
2	B	0.038	0	0.001	0.028	0	0	0.545	0	0.371	0	2.723	0	1.678	0	0	0	1.278
3	C	0.025	0	0	0	0.038	0	0.538	0	0.550	0	1.550	0	1.375	0	0	0	0.713
4	D	0	0.036	0	0.019	0	0	0.314	0	0.524	0	2.239	0	0.950	0	0	0	1.312
5	E	0.033	0	0	0.021	0.008	0	0.508	0	0.379	0	2.317	0	1.500	0	0	0	1.096
6	F	0.030	0	0.040	0	0	0	0.606	0	0.675	0	1.950	0	1.943	0	0	0	0.502
7	G	0.027	0	0	0	0.041	0	0.581	0	0.595	0	1.676	0	1.486	0	0	0	0.770
8	H	0.020	0	0.026	0	0	0	0.400	0	0.445	0	1.287	0	1.283	0	0	0	0.331
9	I	0.032	0	0	0	0.048	0	0.694	0	0.710	0	2.000	0	1.774	0	0	0	0.919
10	J	0	0.100	0.091	0	0	0	1.300	0	2.378	0	5.804	0	4.139	0	0	0	2.470
11	K	0.050	0	0	0	0.075	0	1.075	0	1.100	0	3.100	0	2.750	0	0	0	1.425
12	L	0.022	0	0	0.014	0.006	0	0.339	0	0.253	0	1.544	0	1.000	0	0	0	0.731

No.	DMU	Slack Variable											
		d7+	d7-	d8+	d8-	d9+	d9-	d10+	d10-	d11+	d11-	d12+	d12-
1	A	0	9.984	0	10.375	0	3.769	0	1.047	0	3.356	0	0.556
2	B	0	3.591	0	3.776	0	1.286	0	0.542	0	1.397	0	0
3	C	0	1.513	0	1.638	0	0.463	0	0.300	0	0.363	0	0.188
4	D	0	2.853	0	2.964	0	1.077	0	0.299	0	0.959	0	0.159
5	E	0	2.954	0	3.142	0	1.033	0	0.463	0	1.067	0	0
6	F	0	1.962	0	1.173	0	0.958	0	0.269	0	1.924	0	1.482
7	G	0	1.635	0	1.770	0	0.500	0	0.324	0	0.392	0	0.203
8	H	0	1.295	0	0.774	0	0.632	0	0.177	0	1.270	0	0.978
9	I	0	1.952	0	2.113	0	0.597	0	0.387	0	0.468	0	0.242
10	J	0	5.922	0	4.009	0	2.883	0	0.461	0	4.643	0	3.974
11	K	0	3.025	0	3.275	0	0.925	0	0.600	0	0.725	0	0.375
12	L	0	1.969	0	2.094	0	0.689	0	0.308	0	0.711	0	0

Evaluation Value		
$0\times\theta^{E}$	$1\times\theta^{IE}$	$\theta^{MVP,0}$
0	0.476	-0.476
0	0.730	-0.730
0	0.392	-0.392
0	0.513	-0.513
0	1.000	-1.000
0	0.666	-0.666
0	0.379	-0.379
0	0.564	-0.564
0	0.626	-0.626
0	0.685	-0.685
0	0.580	-0.580
0	1.000	-1.000