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# Profit and Risk Sharing in Virtual Enterprises 

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#### Abstract

Member enterprises engaged in a virtual enterprise shirk both productive effort and risk because their efforts are unobservable to any other member enterprise. How to design the profit and risk sharing mechanism to prevent the member enterprises from free-riding is the key to cut down risks that virtual enterprises may meet. Based on the work of profit sharing in partnerships and the research on revenue sharing in supply chain and joint ventures, we propose a model of the profit and risk sharing contract aimed at coordinating a virtual enterprise composed of $n$ risk-averse member enterprises. We characterize optimal productive efforts, profit sharing and incentive intensity where every member enterprise performs different tasks and contributes different core resources, and demonstrates that improving the productive effort evaluation technique and keeping VEs in medium size are very important to the success of VEs.


Keywords: virtual enterprise, profit sharing, productive effort, risk aversion, incentive intensity

## 1. INTRODUCTION

A virtual enterprise (VE) is a temporary consortium of autonomous, diverse, and possibly geographically dispersed organizations that pool their resources to meet short-term objectives and exploit fast-changing market trends [1]. In the 1990s, the rapid rate of technological change, shortened product life cycles, diversified and individual customer requirement, and the globalization of markets have increased the pressure to improve new product development processes by business alliances. Many organizations are entering virtual enterprises (VEs) to catch the new opportunities quickly and overcome the risks associated with the opportunities.

The idea of VEs, that form themselves according to the needs and opportunities of the market, as well as remaining operational as long as these opportunities persist, put forward a number of benefits, such as flexibility, adaptability, complementary, and so on. In spite of substantial advantages of VEs, there are a number of problems associated with them, such as having to set up legally profit sharing contracts under
enormous time pressure and with incomplete information. This incompleteness arises from member enterprises not having sufficient background information about the other member enterprises or about the environment in which the VEs have to operate. Since VEs are profit driven, whether VEs could construct reasonable and efficient profit sharing mechanism in the operation process of VEs to prevent some members from gaining profit by harming others' profit, is the key on the successful running of VEs.

Das and Teng [2] have pointed out that the main two kinds of risk in alliances are performance risk and relational risk. Performance risk is the probability that a VE may fail even when member enterprises commit themselves fully to the VE. Relational risk is concerned with probability that member enterprises lack commitment to the VE and that their possible opportunistic behavior could undermine the prospects of a VE. Enterprises forms VE to reduce performance risk but induce relationship risk. Not surprisingly, member enterprises tend to be interested more in pursuing their self-interest than the common interest of the VE. Member enterprises are primarily motivated in enhancing their self-interest at the cost of other member enterprises and even the VE. Such opportunistic behaviors include shirking, appropriating the member enterprise's resources, distorting information, harboring hidden agendas, and delivering unsatisfactory products and services. Because these activities seriously jeopardize the viability of a VE, relational risk is an important component of the overall risk in VEs. This moral hazard problem exists in all the organizations which are formed by multiple enterprises, the researchers are pursuing to solve this problem through profit and risk sharing contract design.

In VEs, generally there are three kinds of methods to mitigate the high risk in the cooperation process, partner selection, cooperation contract design, and coordination mechanism design. From the literature review, we found that most of the corresponding researches are focused on partner selection [3-8]. After the partners are selected, however, how to design the cooperation contract, and how to coordinate the relationships between member enterprises in the process of cooperation? Unfortunately, there are few researchers who go deep into these problems, especially the profit and risk sharing in VEs,
though VE arrests many researchers' attention. Karjalainen et al. [9] have used a case study approach to explore the implementation of profit and risk sharing mechanisms in a real-life VE. It shows that lack of trust and a shared vision may have been the most important cause for the early decomposition of the VE. From the questionnaire of 10 profit sharing mechanisms in the 8 member enterprises, the authors got the result that the enterprises seemed to favor simple hierarchical rewarding mechanisms and fair straightforward incentive method. Feng and Chen [10] discussed the basic principles of the proportion making for profit/risk allocation within VEs, and proposed an algorithm to make the profits proportion for a type of VEs developing products by integrating the investment and risk factors, as well as using fuzzy evaluation method.

As we know, VE has many common characteristics with supply chain, partnerships and joint venture. Accordingly we could get some useful suggestion from the researches in these fields. The revenue sharing contract in supply chain proposed by Cachon and Lariviere [11] is a coordination mechanism offered by the supplier to the retailer, which modifies the retailer's profit (and also the supplier's) so as to incentive her to make decisions coherent with the supply chain total optimization. Under a revenue-sharing contract, a retailer pays a supplier a wholesale price for each unit purchased, plus a percentage of the revenue the retailer generates. Huddart \& Liang [12, 13] have studied the profit sharing and monitoring in partnerships. They considered partnerships among risk-averse professionals endowed with (i) a risky and personally-costly production technology and (ii) a personally-costly monitoring technology providing contractible noisy signals about partners' productive efforts. Every partner performs the same tasks and has the same characteristics. The authors illustrated partners' productive and monitoring efforts under different monitoring mechanisms.

Another kind of research we could use for reference is the revenue sharing and control rights in joint venture. Bai et al. [14] have investigated revenue sharing in joint venture with 2 enterprises. They presented a model of team production motivated by the stylized facts found from a sample of 200 joint-venture contracts. It shows that joint control can be optimal as well as unilateral control. Wang and Zhu [15] developed a two-period double moral hazard model with incomplete contracting to explore the implication of the possible adverse effects of unilateral control on the optimal allocation of control rights and revenue in a joint venture with two risk neutral enterprises. They identified conditions under which joint ownership and control become optimal
when unilateral control gives the controlling party opportunities to inefficiently extract private benefits at the expense of the joint revenue.

The researches on profit and risk sharing of supply chain are typically concentrated on the optimal price and quantity, and that of partnerships or joint ventures mostly on the control rights allocation. And as we know, there are many differences between VEs and supply chain, partnerships or joint venture. At first, the inter-firm relationships in VEs are not similar to the supplier-retailer relationship within supply chain, especially in the VE where the ownership and control right are diffused among all member enterprises. In partnerships, all the partners have the same characteristics, perform the same task, on the contrary every member in VEs has different core resources and competence, and VEs will decompose if lack any of the core resources or core competence. Joint ventures are just like traditional enterprises, the relationships between partners are stable in long term. However, the most distinguishing feature of VEs is that VEs will disband soon after the objectives of VEs are realized. Consequently, we cannot deal with the risk and profit sharing problem in VEs as we do in risk management of enterprises, supply chain, joint ventures and partnerships. It is more feasible and realistic for us to control the moral hazard problem in VEs by focusing on productive effort and risk taken by member enterprises.

From the literature review, we found that researchers had carried out some valuable investigations on the profit and risk sharing in VE. However, relatively little has been published in terms of quantitative research. This paper aims to make a contribution towards filling this gap. Based on the research of profit sharing in supply chain, partnerships and joint ventures, the profit and risk sharing contract in a VE composed of $n$ risk-averse member enterprises is proposed in this paper. We characterize optimal productive efforts, profit sharing and incentive intensity where every member enterprise performs different tasks and contributes different core resources. The remaining of this paper is organized as follows. Section 2 addresses the assumption and describes our model of the profit and risk sharing mechanism. Section 3 and 4 analyze this model and take some numerical analysis to get the direction results of the sharing mechanism. Section 5 addresses the needs for further advances and concludes the paper.

## 2. ASSUMPTIONS AND MODEL

In usual setting with a risk-neutral principal of earlier
research, the principal costlessly bears the risk associated with any risky compensation payment schedule. We consider a setting with $n$ risk-averse members, and every member enterprise is simultaneously an owner who shares in the net output of the VE and an agent who produces output, that's to say the ownership and control right are diffused among all member enterprises.

Because member enterprises are subject to moral hazard, awarding a fixed share of VE output invites undersupply of effort. This is unavoidable if the VE output is the only contractible variable. Holmström [16] shows that free riding cannot be eliminated if every partner lacks information about the actions of the other partners, the budget is balanced and output varies continuously with agents' action choices. In our model, member enterprises commit to contract parameters that induce unobservable productive effort choices and risk-taken by each member. Like earlier work, we assume that the productive effort of each member enterprise is unobservable during production process. Unlike earlier work, we assume the productive effort is contractible and publicly verifiable after the project finished. In our paper, we use process evaluation to produces informative signals of members' productive efforts, and process evaluation is represented as an activity that is publicly costly to every member enterprise. The benefits of process evaluation are twofold. First, the output will increase since the improved incentives and peer monitoring created by process evaluation. Second, the process evaluation will achieve better risk sharing or other synergies. We assume that every member is endowed with the same risk attitudes, personal cost of process evaluation (hence we ignore the evaluation cost in our model), and the personal cost of productive effort is direct proportional to personal core resource value. Our model also assumes that every member select its own productive effort and risk-taken by profit maximization principle and there is no side-payment between any member enterprises.

The notation used in our model are shown as below:
$x$, the output of VE;
$e_{i}$, the productive effort of member enterprise i;
$r_{i}$, the risk taken by member enterprise i ;
$\alpha_{i}$, the resource contributed by member enterprise i ;
$\beta_{i}$, the profit sharing factor of member enterprise i;
$t_{i}$, the incentive intensity to member enterprise i;
$t_{i j}$, the transfer payment factor from member enterprise j to member enterprise i ;
$s_{i}$, the productive effort of member enterprise i evaluated by process evaluation;
$\theta$, the risk aversion parameter of all member enterprises;

$$
\begin{array}{ll}
c\left(e_{i}, r_{i}\right) & \text { the cost of member enterprise } \mathrm{i} \\
p\left(e_{i}, r_{i}\right) & \text { the net profit of member enterprise i. }
\end{array}
$$

Now, we will present the basic model. Like Huddart \& Liang [12, 13], we also assumes linear sharing rules, exponential utility, and normally distributed random variables. Consider a set $N=\{1,2, \ldots, n\}$ of member enterprises in a VE. Each member enterprise $i \in \mathrm{~N}$ contributes core resource $\alpha_{i}$, chooses a level of productive effort, $e_{i} \geq 0$ and a level of risk-taken. $r_{i} \geq 0$. Both productive effort and risk are personally costly to the member enterprise. Denote the cost and the net profit to member enterprise $i$ as $c\left(e_{i}, r_{i}\right)$ and $p\left(e_{i}, r_{i}\right)$ respectively. When member enterprise $i$ exerts effort $e_{i}$ and take the risk $r_{i}$, he incurs cost

$$
\begin{equation*}
c\left(e_{i}, r_{i}\right)=\frac{1}{2} \alpha_{i} e_{i}^{2}+r_{i}, \tag{1}
\end{equation*}
$$

and the total output of the VE is

$$
\begin{align*}
& x=\sum_{i \in N}\left(\alpha_{i} e_{i}+\varepsilon_{i}\right)  \tag{2}\\
& \text { s. t. } \quad \varepsilon_{i} \sim N\left(0, g\left(r_{i}\right)\right)
\end{align*}
$$

The $\varepsilon_{i}$ are mutually independent and normally distributed random variables. In this formulation, member enterprise's contribution to VE output does not depend on the effort choices of other members, which considerably simplifies the analysis by making each member enterprise's productive effort choice separable from the effort choice of the other member enterprises.

We assume VE output is observable, so that a member enterprise's profit may depend on $x$. The process evaluation provides $s_{i}$, the productive effort of member enterprise $i$ evaluated by the process evaluation after VE objective finished, defined as $s_{i}=e_{i}+\zeta_{i}$, where $\zeta_{i} \sim N\left(0, \sigma^{2}\right)$ means the evaluation variance of the process evaluation technique.

Member enterprise's revenue is assumed to be a linear function of the observables, thus member enterprise $i$ receives fraction $\beta_{i}$ of VE output, $x$; a transfer payment that is a multiple $t_{i}$ of the evaluated productive effort, $s_{i}$. Call $t_{i}$ the incentive intensity to member enterprise $i$. Also, member enterprise $i$ bears share $t_{j i}$ of the payment to member enterprise $j$ based on the evaluated productive effort $s_{j}$. Therefore, member enterprise $i$ 's net profit is

$$
\begin{align*}
p\left(e_{i}, r_{i}\right)= & \beta_{i} x+t_{i} s_{i}-\sum_{j \in N i i} t_{j i} s_{j}-c\left(e_{i}, r_{i}\right)  \tag{3}\\
& \text { s. t. } \quad \sum_{i \in N} \beta_{i}=1,  \tag{4}\\
& \text { and } \quad t_{i}=\sum_{j \in N i} t_{i j} \text { for all } i \in N, \tag{5}
\end{align*}
$$

where $M i$ denotes all elements in $N$ except the $i$ th
elements, equation (4) and (5) are the budget-balancing constraints. Budget balancing requires that the total output of VE should be divided among the member enterprises, hence (4). Further, since the transfer payment to one member enterprise based on the evaluation result $s_{i}$, necessarily is funded by other member enterprises, we have (5).

We consider a two-period model involving $n$ risk-averse member enterprises in a VE. At the beginning of period 1 , the member enterprises cooperatively sign a contract that allocates the public revenue from the project including choosing contract parameters $\left\langle\beta_{i}, t_{i}, t_{i j}\right\rangle$, as well as the process evaluation method that collect correlative information during period 1 and evaluate all member enterprises' productive effort in period 2. In period 1 (ex ante), each member enterprise can make an unobservable productive effort $e_{i}$, and take an unobservable risk $r_{i}$ to develop a joint project of the VE, privately and non-cooperatively. In period 2 (ex post), the productive efforts are evaluated, after the objective $x$ realized but before profit sharing. After the profit is allocated, the VE is dismissed and every member enterprises have no any relation about this finished project. The timing of events is illustrated in Figure 1.


Figure 1 The Timing of Events

## 3. MODEL ANALYSIS

### 3.1 The optimization problem of VE

Let $E=\left\{e_{i}\right\}_{i \in N}, \quad R=\left\{r_{i}\right\}_{i \in N}, \quad A=\left\{\alpha_{i}\right\}_{i \in N}, \quad B=\left\{\beta_{i}\right\}_{i \in N}$, $T=\left\{t_{i}\right\}_{i \in N}$, and $H=\left\{t_{i j} \mid i, j \in N\right.$ and $\left.i \neq j\right\}$. Optimal VEs should produce the largest profit and therefore are solutions to the following program:

$$
\begin{equation*}
\max _{\{B, T, H, E, R\}} \sum_{i \in N} E\left[U\left(p\left(e_{i}, r_{i}\right)\right)\right] \tag{6}
\end{equation*}
$$

s. t. $E\left[U\left(p\left(e_{i}, r_{i}\right)\right)\right] \geq E\left[U\left(p\left(\hat{e}_{i}, \hat{r}_{i}\right)\right)\right]$

$$
\begin{equation*}
\text { for all } \hat{e}_{i} \text { and } \hat{r}_{i}, \text { for all } i \in N \text {, and } \tag{7}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{E}\left[\mathrm{U}\left(p\left(e_{i}, r_{i}\right)\right)\right] \geq K_{i} \quad \text { for all } i \in N \tag{8}
\end{equation*}
$$

The incentive compatibility (IC) constraint (7) requires that each member enterprise prefers the equilibrium choices of $e_{i}$ and $r_{i}$, to any other pairs of effort and risk choices $\left(\hat{e}_{i}, \widehat{r}_{i}\right)$, holding fixed the choices of the other member enterprises. The participation constraint (8) always holds if one assumes that a member enterprise's alternative to membership in one VE is membership in
some other VE or a sole enterprise.
To any member enterprise, his objective is to maximize the utility of his net profit, i.e.:

$$
\begin{array}{lc} 
& \max _{\left\{e_{i}, r_{i}\right\}} E\left[U\left(p\left(e_{i}, r_{i}\right)\right)\right] \\
\text { s. t. } & E\left[U\left(p\left(e_{i}, r_{i}\right)\right)\right] \geq K_{i} \tag{10}
\end{array}
$$

Given exponential utility function with risk aversion parameter $\theta$, we describe the member enterprise's preferences over the net profit by Von Neumann-Morgenstern utility function with constant absolute risk aversion,

$$
\begin{equation*}
U\left(p\left(e_{i}, r_{i}\right)\right)=-\exp \left(-\theta p\left(e_{i}, r_{i}\right)\right) \tag{11}
\end{equation*}
$$

Then, we make the usual transformation of expected utility into mean-variance terms. Member enterprise $i$ 's expected utility is

$$
\begin{align*}
& E\left[U\left(p\left(e_{i}, r_{i}\right)\right)\right] \\
& =E\left[p\left(e_{i}, r_{i}\right)\right]-\frac{\theta}{2} \operatorname{Var}\left[p\left(e_{i}, r_{i}\right)\right] \\
& =\left(\beta_{i} \alpha_{i}+t_{i}\right) e_{i}+\sum_{j \in N \backslash i}\left(\beta_{i} \alpha_{j}-t_{j i}\right) e_{j}-c\left(e_{i}, r_{i}\right) \\
& \quad-\frac{\theta}{2}\left[\beta_{i}^{2}\left(\sum_{j \in N} g\left(r_{j}\right)\right)+t_{i}^{2} \sigma^{2}+\sum_{j \in N \backslash i} t_{j i}{ }^{2} \sigma^{2}\right] \tag{12}
\end{align*}
$$

### 3.2 First-order conditions

In our model, contract parameters $<\mathrm{B}, \mathrm{T}, \mathrm{H}>$ are chosen to maximize the expected utility of a VE. Next, each member enterprise $i$ chooses the levels of productive effort and risk-taken that maximizes his utility of net profit given these contract parameters. Note that the second-order conditions on $e_{i}$ and $r_{i}$ are satisfied provided functions $c$ and $g$ are convex. From (12), the first-order conditions (FOC) on the productive effort of member enterprise $i$ is

$$
\begin{equation*}
0=\alpha_{i} \beta_{i}+t_{i}-\frac{\partial c\left(e_{i}, r_{i}\right)}{\partial e_{i}} \tag{13}
\end{equation*}
$$

Also from (12), the first-order condition on the risk taken by the member enterprise $i$ is

$$
\begin{equation*}
0=-\frac{\partial c\left(e_{i}, r_{i}\right)}{\partial r_{i}}-\frac{\theta}{2} \beta_{i}^{2}\left(\sum_{j \in N} \frac{\partial g\left(r_{j}\right)}{\partial r_{i}}\right) \tag{14}
\end{equation*}
$$

The optimization problem is solved by substituting the optimal productive effort and risk-taken for each member enterprise $i, e_{i}{ }^{*}$ and $r_{i}{ }^{*}$, into (6) and then solving for optimal T, and H. Since the productive effort affects the mean of a VE's output, but not the variance, the
optimal productive effort $e_{i}^{*}$ given by (13), does not depend directly on the member enterprises' aversion to risk, $\theta$. On the contrary, changes in the risk taken by member enterprise do affect the variance of payment to member enterprises, so the risk taken by member enterprise $r_{i}{ }^{*}$, given by (14) depends directly on the member enterprises' risk aversion to risk. However, risk taken by member enterprise is governed by choice of B, which also affects productive effort, so risk aversion does have an indirect effect on productive effort.

Similar with Huddart \& Liang's assumption on monitoring effort [13], we assume that

$$
\begin{equation*}
g\left(r_{i}\right)=\frac{1}{r_{i}} \tag{15}
\end{equation*}
$$

This assumption means that more risk the member enterprise $i$ taken, more precise of the VE's output. It's obvious that the variance of a VE's output is near to zero if the all the member enterprises in the VE take the risk as high as possible. For this assumption and under moral hazard with respect to productive efforts and risk-taken, optimal productive efforts, risk-taken are as follows by substituting (1) and (15) into (13) and (14).

$$
\begin{align*}
e_{i} & =\beta_{i}+\frac{t_{i}}{\alpha_{i}}  \tag{16}\\
r_{i} & =\sqrt{\frac{\theta}{2}} \beta_{i} \tag{17}
\end{align*}
$$

FOC with respect to $r_{i}$ demonstrates that more risk averse the member enterprise $i$ is or more profit sharing factor he gets, more risk he will take. About the productive effort $e_{i}$, we will discuss it in next subsection.

### 3.2 Optimality of Sharing Contract

We assume the share of the transfer payment to member enterprise j born by member enterprise $i$ is the same across member enterprises $i \in N \backslash j$. Then, from equation (5) we have

$$
\begin{equation*}
t_{j i}=\frac{t_{j}}{n-1} \quad \text { for all } i \text { and } j \text { and } j \neq i \tag{18}
\end{equation*}
$$

Substituting (16) and (18) into (12), and then substituting (12) into (6), we could get the expected utility of a VE:
$\sum_{i \in N} \mathrm{E}\left[U\left(p\left(e_{i}, r_{i}\right)\right)\right]$
$=\sum_{i \in N}\left[\left(\alpha_{i} e_{i}-\frac{1}{2} \alpha_{i} e_{i}{ }^{2}-r_{i}\right)-\frac{\theta}{2}\left(\beta_{i}{ }^{2}\left(\sum_{i \in N} \frac{1}{r_{i}}\right)+t_{i}^{2} \sigma^{2}+\sum_{j \in N i} \frac{t_{j}{ }^{2} \sigma^{2}}{(n-1)^{2}}\right)\right]$

$$
\begin{align*}
& =\sum_{i \in N}\left[\left(\alpha_{i} \beta_{i}+t_{i}-\frac{1}{2} \alpha_{i} \beta_{i}^{2}-\frac{1}{2} \frac{t_{i}^{2}}{\alpha_{i}}-\beta_{i} t_{i}-r_{i}\right)\right. \\
& \left.-\frac{\theta}{2}\left(\beta_{i}{ }^{2}\left(\sum_{j \in N} \frac{1}{r_{j}}\right)+t_{i}{ }^{2} \sigma^{2}+\sum_{j \in N \backslash i} \frac{t_{j}^{2} \sigma^{2}}{(n-1)^{2}}\right)\right] \tag{19}
\end{align*}
$$

Taking just the terms involving $t_{i}$ from expression (19), the FOC with respect to $t_{i}$ gives the optimal incentive intensity:

$$
\begin{equation*}
t_{i}=\frac{1-\beta_{i}}{\frac{1}{\alpha_{i}}+\frac{n \theta}{n-1} \sigma^{2}} \tag{20}
\end{equation*}
$$

Substituting (20) into (16) yields the optimal productive effort:

$$
\begin{equation*}
e_{i}=\beta_{i}+\frac{1-\beta_{i}}{1+\frac{n \theta \alpha_{i}}{n-1} \sigma^{2}} \tag{21}
\end{equation*}
$$

Usually, incentive mechanism is used to deal with the moral hazard problem due to information asymmetry. From (20), we found that the incentive intensity $t_{i}$ is increasing in VE size $n$, that is to say a VE need more incentive mechanism since free riding becomes severe in a large VE. From (21), however, we find an interesting result that the member enterprises prefer to choose high productive effort ( $e_{i} \uparrow$ ) in a large VE. That is to say the incentive mechanism in our model is very effective. Maybe it is better that a VE keeps in medium size since it is dangerous for a VE if only one member in one core competence when this member get into trouble. At the same time, we should give the member enterprise more incentive who contributes more resources $\left(\alpha_{i} \uparrow\right)$, or gets less profit from the VE's output $\left(\beta_{i \downarrow} \downarrow\right.$, or he is willing to bear more risk $(\theta \downarrow)$.

It is clear that $\beta_{i} \leq e_{i} \leq 1$, and when $\theta \rightarrow \infty$ or $\sigma \rightarrow \infty, e_{i} \rightarrow \beta_{i}$, and $e_{i} \rightarrow 1$ when $\theta \rightarrow 0$ or $\sigma \rightarrow 0$. This implies that more risk averse the member enterprises are $(\theta \uparrow)$, or more imprecise the process evaluation $(\sigma \downarrow)$, more productive efforts he will choose, otherwise, he prefer to choose low productive efforts. And when perfect signals of productive efforts are available, every member enterprise will contribute his largest effort. From (20) we could also see that more imprecise the process evaluation, more incentive he should get. Unfortunately, when the process evaluation has great variance, the member enterprise cannot get more profit even he gets high incentive intensity ( $t_{i} \uparrow$ ). This observation seems very important to us, it gives us the implication that we should improve the precision of process evaluation method to guarantee the VE objective completed.

## 4. NUMERICAL ANALYSIS

In this section, for having a better understanding of our model, we process some numerical analysis based on the equations (20) and (21) to make further investigation.

### 4.1 Productive Efforts and Incentive Intensity

At first, we consider the optimal productive efforts and incentive intensity in the following case: $n=3, \beta_{i}=\alpha_{i}$, $\alpha_{1}=1 / 2, \alpha_{2}=1 / 3$, and $\alpha_{3}=1 / 6$.

Figure 2 presents the isoquants of the equilibrium productive efforts, $e_{i}$, as a function of member enterprise risk aversion, $\theta$, (horizontal axis) and evaluation variance of productive effort, $\sigma$, (vertical axis). Figure 3 presents the isoquants of the equilibrium incentive intensity, $t_{i}$, as a function of member enterprise risk aversion, $\theta$, (horizontal axis) and evaluation variance of productive effort, $\sigma$, (vertical axis). From the figures, we could draw out that, as the risk aversion parameter or the evaluation variance of productive effort increases, the productive effort of every member enterprise and the incentive intensity to them decreases. In turn, this suggests that evaluation technique is very important for VEs since it will affect the success of VEs severely.


Figure 2 Optimal Productive Efforts


Figure 3 Optimal Incentive Intensity

Now we consider the effect of core resources contributed by the member enterprises on the productive efforts.





Figure 4 The Effect of Contributed Resource on Optimal Productive Efforts

Figure 4 shows that the member enterprise who contributed more core resource will choose lower productive effort when the evaluation variance $\sigma$ is very small; however, it will choose higher productive effort when the evaluation variance $\sigma$ and risk aversion $\theta$ are very big. This finding is a litter conflicted with the suggestion from figure 2 and 3 . However it accords with the reality in our real life, those powerful member enterprises are often lazier than those with small power. It also gives us an implication that a VE is easier to success if it is combined with the member enterprises who have near important core competence.

### 4.2 VE Size

In this part, we consider the optimal productive efforts and incentive intensity of the member enterprise in the following four cases: $n=2,4,8$, and $16 ; \beta_{i}=\alpha_{i}, \alpha_{1}=1 / 2$.

Figure 5 presents four contour plots of equilibrium variable values as functions member enterprise risk aversion and evaluation variance. The top two panels relate to the same optimal productive efforts of member enterprise 1 under different VE size, 2, 4, 8, or 16 member enterprises; the bottom two panels relate to the same VE size under different productive efforts.

It shows that productive effort increases as the number of member enterprises increases when the risk aversion, evaluation variance and contributed resource are the same. It suggests that more member enterprises, more productive efforts the member enterprises will choose. As we know, it is dangerous for VE if only one member in one core competence when this member get into trouble. So in the forming of VE, more than one member in one core competence should be selected to introduce competition and reduce risk.

We should mention that free riding problem becomes severe in large VE and from previous analysis of (20) we know that the incentive intensity $t_{i}$ is bigger when $n$ is bigger. The risk imposed by our profit sharing contract can be reduced either by making the process evaluation more precise or by reducing the change in payment associated with a change in the process evaluation result. As a result, the incentive intensity $t_{i}$ is bigger when $n$ is bigger, and then the change in payment associated with a change in the process evaluation result is bigger and will bring more risk to VE. It is also a litter more difficult for the VE to be in agreement on the high transfer payment, so maybe it is better that VE keep in medium size.





Figure 5 The Effect of VE Size on Optimal Productive Efforts

## 5. CONCLUSIONS

This paper considers the profit and risk sharing mechanism in VE. Our model could get the Pareto
optimal sharing rules to maximize the production of VE, and could reduce the "free-riding" phenomena effectively by the incentive mechanism in the model. From the analysis of our model, we could draw out some useful and valuable conclusion as follows.

More risk averse the member enterprises are, or more imprecise the process evaluation, more productive efforts he will choose, otherwise, he prefer to choose low productive efforts. This observation seems important to us, it give us the implication that we should improve the precision of process evaluation method. Evaluating productive efforts is difficult because judgments about quality are necessary and because effort is multi-faceted, including lots of elements. Producing and interpreting quantitative information about such efforts so that it is contractible is critical to the success of VE combined of many members. Likely, this evaluating process is best accomplished by experts in the same professional field with VE's project.

Another observation important to the success of VE is that keeping in medium size is better for VE. Of course, selecting more than one member enterprise in one core competence and selecting near powerful member enterprises in VE is also very important for reducing the risk.

This work could be regarded as a first step of designing profit and risk sharing mechanism for VE. Future work includes extensions to insurance mechanism and synergistic effect in primal model. Insurance is a good way to transfer the risk to the insurance company. Synergistic effect is an important problem in VE, however, in the model of this paper, a member enterprise's contribution to VE output does not depend on the effort choices of other member enterprises. It may also be interesting to study profit and risk sharing mechanism and specific process evaluation technique in more general or practical environment, such as new product development VE, which could be undertaken on the basis of this work.

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