Kansei Visualization System using Fuzzy Correspondence Analysis

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Abstract
In this paper, a visualization system using the individual fluctuation on evaluation is introduced. The system equips the fuzzy correspondence analysis which quantifies relationships between evaluation objects and items. The system can visualize these locations and circles expressing the relative fuzziness of the fluctuation. We define a similarity measure between the evaluation objects and items, taking into account both of the distance and the relative fuzziness. The obtained similarity measure is utilized to determine ranking of objects under a given linguistic quantifier and several evaluation items such as adjectives. The effectiveness of the system is shown through analysis of the subjective evaluation about traditional craft of Ishikawa Prefecture.

Keywords: Subjection Evaluation, Fuzziness, Correspondence Analysis, Traditional Craft, OWA operator

1 Introduction
The concern with Kansei engineering [1] is rapidly increasing since the importance of the subjective evaluation is focused on products design and customer-oriented manufacturing [2][3][4][5][6]. They can be powerful strength in marketing strategy since people might stress preferences more in consuming behaviour. Needless to say, the subjective evaluation on the various affairs such as the circumstance around the residence in the daily life has a role to support the activity [7]. Kansei engineering may be helpful to consider especially the peoples’ high-involvement affairs.

Traditional craft [8] is one of the high-involvement products. Although the brands in Japan have been protected by the Minister of Economy, Trade and Industry as the national product [9], the number of the customer is decreasing. Therefore, the explorer of the customers’ preferences has been required as one of the countermeasure. This is effective to investigate the brand image simultaneously from viewpoints of customers. The result of the analysis is also effective to consider both of the suppliers and customers [10].

Questionnaire survey is a popular way to collect the subjective evaluation data. A familiar scale is the semantic differential (SD) method [11]. There are several investigation style such as laddering method, the conjoint analysis. Comparing with those methods which require a lot of process, the SD method is well known approachable, traditional method although the defect has been discussed in especially psychological field. But it is rather easy to develop the questionnaire sheets. Originally the subjective evaluation leads an arduous undertaking feature since it has the fluctuation and changing. How can we change it advantage? To measure the subjective opinion is basically un-easy task, when the construction from the obtained result must be driven. There are, recently, researches to interpret the meaning of the obtained results [12][13] even though the analyst have to give them by himself. At the same time, grouping of the adjectives is carried out to consider a linguistic side in order to use the analysis result more [14].

The utilization of fuzzy concept is also promising. Recently, a correspondence analysis method with fuzzy data has been proposed [15]. In addition, there are several traditional methods to treat the fuzzy data. However, the identification of the fuzzy sets is required in advance. In this paper, we would like to focus on the fluctuation of the evaluation. The fuzzy correspondence analysis we employ can preserve the fluctuation which is observed in the original axes in the obtained eigenvectors.

Fuzzy correspondence analysis is proposed to analyze relationships between evaluation objects (samples) and evaluation items. The visualization of the obtained result is quite important in
order to construct broad gauged location. The visualization system is strongly required to discuss the preferences. So far, there are lots systems corresponding to each application such as [16] The developed system here can treat various applications when the fuzzy correspondence analysis with the evaluation fluctuation is utilized.

In this paper, the visualization system equipping analysis method and sorting function is introduced, after describing fuzzy correspondence analysis and the method to sort evaluated samples based on the quantitative result.

2 Subjective Evaluation Data

Let us introduce a set of evaluators \( E = \{1, 2, \cdots, K\} \), a set of evaluation items \( O = \{1, 2, \cdots, M\} \). Assume that \( E_m \) is a set of the evaluators who have evaluated Object \( m \), \( O_k \) is a set of the objects which are evaluated by evaluator \( k \). Sets of \( E \) and \( O \) can be described as follows:

\[
E = \bigcup_{m=1}^{M} E_m, \quad E_m \neq \phi, \forall m
\]

\[
O = \bigcup_{k=1}^{K} O_k \neq \phi, \forall k
\]

Data structure is highly depending on the condition of the questionnaire survey.

\( \diamond \) case1: (complete 3-way data)

\[
E_m = E, \forall m; \quad |E_m| = |E| = K
\]

\( \diamond \) case2: (one person evaluates only one object)

\[
|O_k| = 1, \forall k, \quad |E_m \cap E_{m'}| = 0, m \neq m', \quad \sum_{m=1}^{M} |E_m| = K
\]

\( \diamond \) case3: (some evaluate several objects)

\[
|O_k| \geq 1, \forall k, \quad |E| = K
\]

where, all evaluators have the original ID number.

Let be raw data set on the subjective evaluation \( \{z_{mnk}\} \). In this paper, the evaluation value \( z_{mnk} \) of evaluator \( k \) regarding evaluation object \( m \) from the standpoint of evaluation adjective \( n \) is given as a 7-level value.

\[
z_{mnk} \in \{1, 2, 3, 4, 5, 6, 7\}
\]

In addition, we assume that there is no lack of the evaluation data about the evaluation items.

\[
z_{mk} = (z_{m1k}, z_{m2k}, \cdots, z_{mNk})^t, \quad k \in E_m
\]

For case1, the averaged data is determined by

\[
z_{mn} = \frac{1}{K} \sum_{k=1}^{K} z_{m nk}, \quad m = 1, 2, \cdots, M, \quad n = 1, 2, \cdots, N.
\]

For case2 and 3, the averaged data is obtained as follows.

\[
z_{mn} = \frac{1}{|E_m|} \sum_{k=1}^{K} z_{m nk}, \quad m = 1, 2, \cdots, M, \quad n = 1, 2, \cdots, N.
\]

To analyze the data in case3, we have to re-assign the data structure.

3 Fuzzy Correspondence Analysis

3.1 Analysis based on the averaged data

Let the data matrix \( P \) based on the averaged data on the evaluators. The element is determined by

\[
p_{mn} = \frac{z_{mn}}{\sum_{m=1}^{M} \sum_{n=1}^{N} z_{mn}}.
\]

Where, the following conditions on \( p_{m\bullet}, p_{\bullet\bullet} \) are satisfied.

\[
p_{m\bullet} = \sum_{n=1}^{N} p_{mn}, \quad p_{\bullet\bullet} = \sum_{m=1}^{M} p_{mn}, \quad \sum_{m=1}^{M} p_{m\bullet} = \sum_{n=1}^{N} p_{\bullet\bullet} = 1
\]

In correspondence analysis, a quantity \( x_m \) is associated with evaluation object \( m \), and a quantity \( y_n \) is associated with evaluation item \( n \), and we find the following vectors by maximizing the correlation coefficient \( p_{xy} \) defined below:

\[
x = (x_1, x_2, \cdots, x_m)^t
\]

\[
y = (y_1, y_2, \cdots, y_n)^t
\]
Here, the correlation coefficient is defined by the following equation:

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \rightarrow \max.$$  \hspace{1cm} (17)

where $\sigma_{xy}, \sigma_x, \sigma_y$ are determined as follows:

$$\sigma_{xy} = \sum_{m=1}^{M} \sum_{n=1}^{N} p_{mn} x_m y_n - \sum_{m=1}^{M} \sum_{n=1}^{N} p_{m*} x_m \sum_{n=1}^{N} p_{*n} y_n$$  \hspace{1cm} (18)

$$\sigma_x = \sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{m=1}^{M} p_{mn} x_m^2 - \left( \sum_{m=1}^{M} \sum_{n=1}^{N} p_{m*} x_m \right)^2}$$  \hspace{1cm} (19)

$$\sigma_y = \sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{n=1}^{N} p_{*n} y_n^2 - \left( \sum_{m=1}^{M} \sum_{n=1}^{N} p_{*n} y_n \right)^2}$$  \hspace{1cm} (20)

The solution of the above maximization problem can be derived as the eigen value problem. The scores regarding to the evaluation target $m$ and the evaluation item $n$, on a two-dimensional plane using the second and third eigenvectors are given as follows:

$$(x_{2m}, x_{3m}), \quad m = 1, 2, \cdots, M$$

$$(y_{2n}, y_{3n}), \quad n = 1, 2, \cdots, N$$  \hspace{1cm} (21)

### 3.2 Modeling evaluation fluctuation

#### 3.2.1 Fuzzification of eigen vector on evaluation objects

In the previous section, the location, i.e, Equation (21), on the eigen vector space is determined by maximizing the correlation coefficient. In order to express the fluctuation on evaluation, each evaluator’s eigen vector is expressed using the distance from the location caused by averaged data.

Let us define $b_{mnk}, b_m$ as follows:

$$b_{mnk} = \frac{1}{N} \sum_{n=1}^{N} (z_{mnk} - z_{mn}),$$  \hspace{1cm} (22)

where $z_{mn} = \frac{1}{|E_m|} \sum_{k \in E_m} z_{mnk}$

$$b_m = \frac{1}{K} \sum_{k=1}^{K} b_{mnk}, \quad m = 1, 2, \cdots, M$$  \hspace{1cm} (24)

Then the eigen vector expressing each evaluator is defined in the following.

Using these vectors, the individual eigen vector is defined by the following equation.

$$\tilde{x}_{ik} = \tilde{x}_i + \lambda_i (b_k - b)$$  \hspace{1cm} (25)

where $\lambda_i$ is design parameters, and

$$b_k = (b_{1k}, b_{2k}, \cdots, b_{Mk})^t,$$

$$b = (b_1, b_2, \cdots, b_M)^t.$$  \hspace{1cm} (26)

The individual eigen vectors are featured by the following equation.

$$\frac{1}{K} \sum_{k=1}^{K} \tilde{x}_{ik} = \tilde{x}_i$$  \hspace{1cm} (27)

where

$$\tilde{x}_{ik} = (\tilde{x}_{i1k}, \tilde{x}_{i2k}, \cdots, \tilde{x}_{iMk})^t,$$

$$\tilde{x}_i = (\tilde{x}_{i1}, \tilde{x}_{i2}, \cdots, \tilde{x}_{iM})^t.$$  \hspace{1cm} (28)

When the individual vectors are averaged by the evaluators, it becomes the same with the original eigen vector.

#### 3.2.2 Fuzzy sets of evaluation objects

Let us define a fuzzy set which expresses the membership grade to the original eigen vector. According to the extension principle introduced by L.A. Zadeh, second and third elements of the eigen vector are mapped into a two-dimensional fuzzy set.

Let us define a fuzzy vector

$$\tilde{X}_i = (\tilde{X}_{i1}, \tilde{X}_{i2}, \cdots, \tilde{X}_{iM})^t,$$  \hspace{1cm} (29)

using the following membership function.

$$\mu_{\tilde{X}_i}(x) = \exp \{ -(\tilde{x} - \tilde{x}_i)^t D_X^{-1} (\tilde{x} - \tilde{x}_i) \}$$  \hspace{1cm} (30)

Where, $D_X$ is independent of $i$, and determined as follows:

$$D_X = \frac{1}{K} \sum_{k=1}^{K} (\tilde{x} - \tilde{x}_i)(\tilde{x} - \tilde{x}_i)^t = \frac{1}{K} \sum_{k=1}^{K} b_k b_k^t$$  \hspace{1cm} (31)

To make a mapping on the 2-dimensional plane consisting of second and third element of the eigen vector, a vector $a_m$ is defined.

$$a_m = (a_{m1}, a_{m2}, \cdots, a_{MM})^t,$$  \hspace{1cm} (32)

where

$$a_{mm'} = \begin{cases} 1, & m = m' \\ 0, & m \neq m' \end{cases}$$  \hspace{1cm} (33)
Now, the following mapping is considered.

$$X_{im} = a_m^t \tilde{X}_i$$  \hspace{1cm} (34)

According to the extension principle, the membership function of $X_{im}$ is identified as follows:

$$\mu_{X_{im}} = \max_{\{ \tilde{x} \mid \tilde{x} = a_m^t \tilde{x} \}} \mu_{\tilde{x}}(x)$$  \hspace{1cm} (35)

$$= \exp \left\{ - \frac{(x - a_m^t \tilde{x})^2}{(a_m^t D_X a_m)} \right\}$$  \hspace{1cm} (36)

The membership function on the 2 dimensional plane is defined by

$$\mu_{X_{im}} = \max_{\{ \tilde{x} \mid \tilde{x} = a_m^t \tilde{x} \}} \mu_{\tilde{x}}(x)$$  \hspace{1cm} (37)

The circle which illustrates the $\alpha$-level set is denoted by

$$(x_2 - a_m^t \tilde{x}_2)^2 + (x_3 - a_m^t \tilde{x}_3)^2 = (a_m^t D_X a_m)(-\log(\alpha)) \equiv s_m^2.$$  \hspace{1cm} (38)

For the evaluation items, the similar idea is applied. The circle on the evaluation items is described as follows:

$$(y_2 - a_m^t \tilde{y}_2)^2 + (y_3 - a_m^t \tilde{y}_3)^2 = (a_m^t D_Y a_m)(-\log(\alpha)) \equiv i_n^2$$  \hspace{1cm} (39)

Then the relationships between the evaluation targets and the evaluation items are illustrated.

### 4 Similarity Measure

Let us define a similarity measure. The similarity measure is determined by the obtained scores and the radiuses by the fuzzy correspondence analysis.

For evaluation target $m$ and evaluation adjective $n$, the similarity measure $s_{mn}$ here is defined by

$$s_{mn} = \frac{\exp \{s_m + t_n\}}{\exp \{2 \max \{s_m, t_n\}\} \exp \{d_{mn}\}}$$  \hspace{1cm} (40)

where, $(x_{2mn}, x_{3mn})$ indicates the score of evaluation target $m$, and $s_m$ denotes the radius, $(y_{2n}, y_{3n})$ indicates the score of evaluation item $n$ and $t_n$ denotes the radius, and $d_{mn}$ is the Euclidean distance between evaluation target $m$ and evaluation item $n$.

The measure is defined so that the value of $s_{mn}$ is equal to 1.0 when two circles have completely the same locations and the radiuses. In addition, if there are two different circles in the plane, the following condition is satisfied.

$$s_{mn} = \exp \{s_m + t_n\} \exp \{2r\} < 1$$  \hspace{1cm} (41)

This formulation provides the smaller value of the similarity when the considered circles are located far.

### 5 Preference Ranking

This section introduces the OWA operator proposed by Yager [17] in order to determine the recommendation order from the products to customers who mention the plural adjectives as their preferences.

The definition of OWA operator proposed by Yager is described as follows: OWA operator $F$ of $n$ dimension is a mapping

$$Q : [0, 1]^n \rightarrow [0, 1]$$  \hspace{1cm} (42)

which is expressed as

$$Q(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i,$$  \hspace{1cm} (43)

by using the weighted vector

$$W = (w_1, w_2, \ldots, w_n), w_i \in [0, 1], \sum_{i=1}^{n} w_i = 1$$  \hspace{1cm} (44)

Where, $b_i$ is the $i$-th biggest value in the sequence from $a_1$ to $a_n$ when $n$ is the number of the selected adjectives.

Now that we would like to propose samples which satisfy the preferences of a customer. Let us introduce the following notation. Let $O$ a set of evaluation target and $W$ a set of the desired adjectives.

$$O = \{O_1, O_2, \cdots, O_M\}$$  \hspace{1cm} (45)

$$W = \{W_1, W_2, \cdots, W_N\}$$  \hspace{1cm} (46)

Assume that an index of a customer is $A$. The vector of the selected evaluation items can be indicated as follows:

$$W_A = \{W_{A1}, W_{A2}, \cdots, W_{AK}\} \subset W, K \leq N$$  \hspace{1cm} (47)
In order to assign the meaning of the weight, the customer is required to select Linguistic Quantifier (LQ) from “all”, “almost all”, “as many as possible”, “most”, “at least half” and “there exists”.

The natures of membership functions of fuzzy quantifiers are required to be satisfied as follows:

\[
Q : [0, 1]^n \rightarrow [0, 1],
\]

\[
Q(0) = 0,
\]

\[
Q(r) = 1 \quad \forall r \in [0, 1],
\]

\[
Q(r) \text{ is not a decreasing function.}
\]

Membership function examples proposed by Zadeh in Reference [18] are indicated in Table 1.

<table>
<thead>
<tr>
<th>Fuzzy Q.</th>
<th>Membership Function</th>
</tr>
</thead>
</table>
| there exists | \( Q(r) = \begin{cases} 
0 & \text{if } r < 1/K \\
1 & \text{if } r \geq 1/K 
\end{cases} \) |
| for all | \( Q(r) = \begin{cases} 
1 & \text{if } r = 1 \\
0 & \text{if } r \neq 1 
\end{cases} \) |
| almost all | \( Q(r) = r \) |
| at least half | \( Q(r) = \begin{cases} 
2r & \text{if } 0 \leq r \leq 0.5 \\
1 & \text{if } 0.5 \leq r \leq 1 
\end{cases} \) |
| as many as possible | \( Q(r) = \begin{cases} 
0 & \text{if } 0 \leq r \leq 0.5 \\
2r - 1 & \text{if } 0.5 \leq r \leq 1 
\end{cases} \) |
| most | \( Q(r) = \begin{cases} 
1 & \text{if } 0 \leq r \leq 0.3 \\
2r - 0.6 & \text{if } 0.3 \leq r \leq 0.8 \\
0 & \text{if } 0.8 \leq r \leq 1 
\end{cases} \) |

6 Analysis of Subjective Evaluation Data about Traditional Craft

6.1 Preparation of evaluation items

Pairs of adjectives as evaluation items are considered here. We assumed that the given words by customers are adjectives. The function of the product is, of course, required, however, the preferences on the product are focused on this study, because the technique to deal with the subjective evaluation data is discussed. The demand on the pairs of adjectives can be summarized as follows: (i) Frequent adjectives used in the traditional craft shop, (ii) Impressive adjectives came from evaluation samples (which are mentioned as below) by designers, (iii) Because the prepared evaluation samples are part of products this time, adjectives related to the feeling of a substance are included positively. Therefore, the pre-investigation was carried out in a small number of evaluators. The determined pairs of adjectives are shown in Table 2.

6.2 Evaluation objects

The prepared evaluation objects are made in each manufacture. Some objects are shown in from Figure 1 to Figure 8. In this case, the pictures of evaluation objects are prepared for questionnaire survey, instead of the real one, since they are so expensive. All evaluators are also required to come to the room to perform their evaluation. Therefore, the complete 3-way data is obtained in this case.

6.3 System instruction

The developed system description is shown in Figure 9. It consists of two parts, which identify “relationships between objects and adjectives”, “preference expression”. The necessary input for each part is also described in Figure 9.

The system overview is displayed in Figure 10. Let us focus on each function. In Figure 11, the result on the traditional craft is shown. Each color is corresponding to evaluation object (red), left side adjective of a pair of
evaluation item (yellow) and right side adjective (blue). The semi-transparent circle is utilized in order to see other circles at the same time, even if they are so close or overlapped. When the mouse across any circle, the name of the corresponding items or objects will be appeared such as “yamanaka32” in the graph. In addition, when an evaluation object is selected to see the circumstance, the corresponding circles is stressed by elastic movement.

The next phase is object sorting using linguistic quantifiers. As you can see at left bottom of the overview of the system in Figure 10, there is the search tool menu.

In Figure 12, the obtained result is shown when adjectives such as “un-conventional”, “fun” and “casual”, and “there exists” linguistic quantifier are selected as the user’s preference. The blue circles in the graph are corresponding to the selected adjectives. In the left hand side, the sorted objects are displayed.

The obtained recommendation of samples are shown in Table 3. The relationships between these adjectives and samples are illustrated in Figure 12.
Table 3. Obtained Objects Ranking

<table>
<thead>
<tr>
<th>Fuzzy Quantifier</th>
<th>Sample Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>LQ=all</td>
<td>37 27 42 15 14</td>
</tr>
<tr>
<td>LQ=almost all</td>
<td>37 27 42 15 14</td>
</tr>
<tr>
<td>LQ=as many as possible</td>
<td>27 37 20 42 19</td>
</tr>
<tr>
<td>LQ=most</td>
<td>37 27 42 19 36</td>
</tr>
<tr>
<td>LQ=at least half</td>
<td>20 27 19 37 36</td>
</tr>
<tr>
<td>LQ=there exists</td>
<td>20 27 19 36 37</td>
</tr>
</tbody>
</table>

7 Concluding Remarks

A fuzzy correspondence analysis was introduced. The preference ranking method based on the defined similarity and calculation by the OWA operator was also introduced.

References


[9] Trade the Minister of Economy and Industry. The law for the promotion of traditional craft industries and traditional crafts designated.


