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Author(s)	瀧田，康晴
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A Proof-Theoretic Approach to Involutive Substructural Logics

Yasuharu Takida (510062)

School of Information Science,
Japan Advanced Institute of Science and Technology

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1 Introduction

Roughly speaking, substructural logics are obtained from Classical logic (LK) or Intuitionistic logic (LJ) by delating all or some of structural rules when they are formalized in sequent systems. Until now, there are many studies in such systems. For example, *what sort of logical properties hold or not*, or *clarifying relations between structural rules and logical properties*, and so on. In these studies, there are mainly two methods, i.e., syntactical and semantical ones. In this thesis, we apply a syntactical method for involutive substructural logics, which are substructural logics satisfying the double negation law.

The main purpose of this paper is to extend Kolmogorov's and Glivenko's theorems to those of substructrual logics. To do this, we need to introduce new translations. By using them, we can show that similar results of Kolmogorov and Glivenko hold on substructural logics.

2 Basic substructural logics

The most fundamental substructural logic FL is obtained from LJ by eliminating all of structrual rules. The CFL is defined from LK in the same way as above. The $InFL$ is the FL which has the double negation law (or involution) $\neg\neg\alpha \Rightarrow \alpha$ as an initial sequent. The FLe, $CFLe$ and $InFLe$ is obtained from FL, CFL and $InFL$, respectively, by adding the exchange rule. The $CFLe$ and FLe are called also MALL (multiplicative additive linear logic) and intuitionistic linear logic, respectively.

In this paper, we assume that the exchange rule always holds. So, we do not deal with non-commutative cases.

3 positive fragments

In this section, we discuss about positive fragments, which play an important role when we study a translation on substructrual logics.

Definition 1. *(positive formula)*
A formula ϕ is positive if it does not have any occurrences of $\neg$.

Definition 2. *(positive fragment)*
For each logic L, L^+ (called positive fragment of L) is the set of all positive formula provable in L.

Then, the following holds.

Theorem 1. $CFLew^+ \vdash \Gamma \Rightarrow \alpha$ *iff* $FLew^+ \vdash \Gamma \Rightarrow \alpha$

Theorem 2. $CFLe^+ \vdash \Gamma \Rightarrow \alpha$ *iff* $FLe^+ \vdash \Gamma \Rightarrow \alpha$

On the other hand, we can't show the similar result does not hold between $LK^+(CFLecw^+)$ and $LJ^+(FLecw^+)$ since the existence of right-contraction rule deteriorates the situation.

4 The $Kolmogorov$-style translation

In this section, we introduce new translation S. To understand our idea, we first explain Kolmogorov's theorem and its extension for substructural logic due to Matsuda.

Definition 3. *(The Kolmogorov-style translation)*
For any formula C, $T(C)$ is defined inductively as follow;

$T(\bot) := \neg\neg\bot$
$T(\top) := \top$
$T(p) := \neg\neg p$ *(p is a propositional variable)*
$T(A \wedge B) := \neg\neg(T(A) \wedge T(B))$
$T(A \rightarrow B) := \neg\neg(T(A) \rightarrow T(B))$
$T(A \vee B) := \neg(\neg T(A) \wedge \neg T(B))$
$T(\neg A) := \neg T(A)$

The following result is well-known as Kolmogorov's theorem.

Theorem 3. $LJ \vdash T(\Gamma) \Rightarrow T(D)$ *iff* $LK \vdash \Gamma \Rightarrow D$

Matsuda extended the Kolmogorov-style translation to substructrual logics, and showed Theorem 4.

Definition 4.
$T(0) := 0$
$T(1) := \neg\neg 1$
$T(A \bullet B) := \neg\neg(T(A) \bullet T(B))$

Theorem 4.

$$\begin{array}{rcl} FLe \vdash T(\Gamma) \Rightarrow T(D) & \textit{iff} & CFLe \vdash \Gamma \Rightarrow D \\ FLew \vdash T(\Gamma) \Rightarrow T(D) & \textit{iff} & CFLew \vdash \Gamma \Rightarrow D \\ FLec \vdash T(\Gamma) \Rightarrow T(D) & \textit{iff} & CFLec \vdash \Gamma \Rightarrow D \end{array}$$

We introduce an alternative translation S for substructrual logics as follows;

Definition 5. *(translation S)*
$S(p) := \neg\neg p$ *(p is a propositional variable or constant)*
$S(\neg A) := \neg S(A)$
$S(A \bullet B) := \neg\neg(S(A) \bullet S(B))$
$S(A \vee B) := \neg\neg(S(A) \vee S(B))$
$S(A \wedge B) := S(A) \wedge S(B)$
$S(A \rightarrow B) := S(A) \rightarrow S(B)$

Theorem 4 holds also for S, since

Lemma 1. $FLe \vdash S(D) \Leftrightarrow T(D)$. *is provable in FLe*

By Lemma 1, we can replace all of S in Theorem 3 and 4 by T.

5 *Glivenko*'s theorem

Theorem 5. *(Glivenko's theorem)*
For any formula A, A is provable in classical propositional logic LK if $\neg\neg A$ is provable in intuitionistic propositional logic LJ.

This theorem shows that we can embed LK into LJ by the double-negation translation.

Let L be an involutive substructural logic such that
$L \supseteq FLe$. Define K_0 by

$$\begin{array}{rcrl} K_0 & : & FLe & + \ \neg(\neg\neg\alpha\to\beta)\to\neg(\alpha\to\neg\neg\beta) \\ & & & + \ \neg(\alpha\wedge\beta)\to\neg(\neg\neg\alpha\wedge\neg\neg\beta) \end{array}$$

Then, we can show the following

Theorem 6. $CFLe \vdash \varphi \quad$ *iff* $\quad K_0 \vdash \neg\neg\varphi$

Theorem 7. $CFLew \vdash \varphi \quad$ *iff* $\quad K_0 + \neg\beta\to\neg(\alpha\bullet\beta) \vdash \neg\neg\varphi$

Theorem 8. $CFLec \vdash \varphi \quad$ *iff* $\quad K_0 + \neg(\alpha\bullet\alpha)\to\neg\alpha \vdash \neg\neg\varphi$

In fact, each logic in the right-hand side is the smallest logic for which Glivenko theorem holds with respect to the logic in the left-hand side.

6 Conclusion

In the present thesis, we extend both *Kolmogorov* translation and *Glivenko* translation to translation among substructural logics, and show translation results by using proof-theoretic methods.

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