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Description	

A Matched Filter Approximation for SC/MMSE Iterative Equalizers

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***Abstract*—This letter proposes a new iterative ISI equalization algorithm that offers low computational complexity: order L^2 with channel memory length L. The proposed algorithm is an extension of Reynolds and Wang's SC/MMSE (Soft Canceler followed by MMSE filter) equalizer: approximations are used properly to reduce the computational complexity. It is shown that the approximations used in the proposed algorithm do not cause any serious performance degradations from the trellis-based iterative equalization algorithms.**

***Index Terms*—Approximation, broadband mobile communication, iterative equalization.**

I. Introduction

A KEY issue toward mobile multimedia communications is to create technologies for broadband signal transmission that can support high quality services. Reducing the effects of the severe Intersymbol interference (ISI) inherent within broadband mobile communications requires a technological breakthrough. Iterative equalization [1], which is based on the Turbo decoding concept, is known as an excellent technique for reducing ISI effects. The maximum *a posteriori* probability (MAP) algorithm and its derivatives, such as Log-MAP and Max-Log-MAP as well as soft output Viterbi algorithm (SOVA) [2], can be used as the soft-input/soft-output (SISO) algorithm needed for iterative equalization. However, their computational complexities increase exponentially with channel memory length L since they use a trellis diagram of the channel. Reynolds and Wang recently proposed a computationally efficient iterative equalization algorithm for severe ISI channels [3], which was derived from the iterative multiuser detector in [4] for CDMA systems. The iterative equalizer in [3] consists of a soft canceler (SC) followed by a linear adaptive filter whose taps are determined adaptively based on the minimum mean square error (MMSE) criterion, and hence is referred to as SC/MMSE in this letter for convenience. SC/MMSE's computational complexity is of the order of L^3 since it requires matrix inversion.

This letter proposes a new version of SC/MMSE that offers further reduced computational complexity by eliminating the need for matrix inversion. For the first iteration, the filter taps are determined adaptively using the training sequence transmitted for channel estimation. For the 2nd and later iterations, the MMSE filter is replaced by a matched filter matched to the channel. This approximation significantly reduces the complexity without causing any serious performance degradation. Computational complexity of the proposed SC/MMSE algorithm is of the order of L^2.

II. Principle and Complexity of SC/MMSE

Fig. 1 shows a block diagram of the SC/MMSE iterative equalizer. Its conceptual basis is to replicate the ISI components by using the log-likelihood ratio (LLR) of the coded bits, fed back from the channel decoder, and to subtract the soft replica of the ISI components from the received composite signal vector. Adaptive linear filtering then takes place to remove the interference residuals; taps of the linear filter are determined adaptively so as to minimize the mean square error (MSE) between the filter output and the signal point corresponding to the coded symbol. The LLR of the filter output is then calculated. After de-interleaving, the LLR values of the filter output are brought to the channel decoder as extrinsic information. SISO decoding is performed by the channel decoder. The process described above is repeated in an iterative manner. The key point of this scheme is that it offers much lower computational complexity than iterative equalizers using a trellis diagram of the channel.

Assuming that there are M antenna diversity branches, the adaptive MMSE filter has ML taps. Vector $\boldsymbol{m}(n)$ corresponding to the filter taps is given by

$$\boldsymbol{m}(n) = [\boldsymbol{H}(n)\boldsymbol{\Lambda}(n)\boldsymbol{H}^H(n) + \sigma^2\boldsymbol{I}]^{-1}\boldsymbol{h}(n) \tag{1}$$

which is an MMSE solution to the minimization problem

$$\boldsymbol{m}(n) = \arg\min_{\boldsymbol{m}(n)} E\{\|b(n) - \boldsymbol{m}^H(n)\hat{\boldsymbol{r}}(n)\|\}^2. \tag{2}$$

$\boldsymbol{H}(n)$ is the space-time channel matrix [3], [6] whose dimensionality is $ML \times (2L-1)$. $\boldsymbol{\Lambda}(n)$ is an $(2L-1)\times(2L-1)$ diagonal matrix

$$\begin{aligned}\boldsymbol{\Lambda}(n) = \mathrm{diag}\{&1-\tilde{b}^2(n+(L-1)), \ldots,\\ &1-\tilde{b}^2(n+1),\, 1,\, 1-\tilde{b}^2(n-1), \ldots,\\ &1-\tilde{b}^2(n-(L-1))\}\end{aligned} \tag{3}$$

where $\tilde{b}(k)$, $n-(L-1) < k < n+(L+1)$, is a soft estimate of the coded bit $b(n)$, given by

$$\tilde{b}(k) = \tanh\left[\frac{\lambda_2[b(k)]}{2}\right]. \tag{4}$$

n denotes the symbol timing index. $\lambda_2[b(k)]$ is the extrinsic information provided by the channel decoder. $\hat{\boldsymbol{r}}(n)$ is the soft canceler output given by

$$\hat{\boldsymbol{r}}(n) = \boldsymbol{r}(n) - \boldsymbol{H}(n)\cdot\tilde{\boldsymbol{b}}(n) \tag{5}$$

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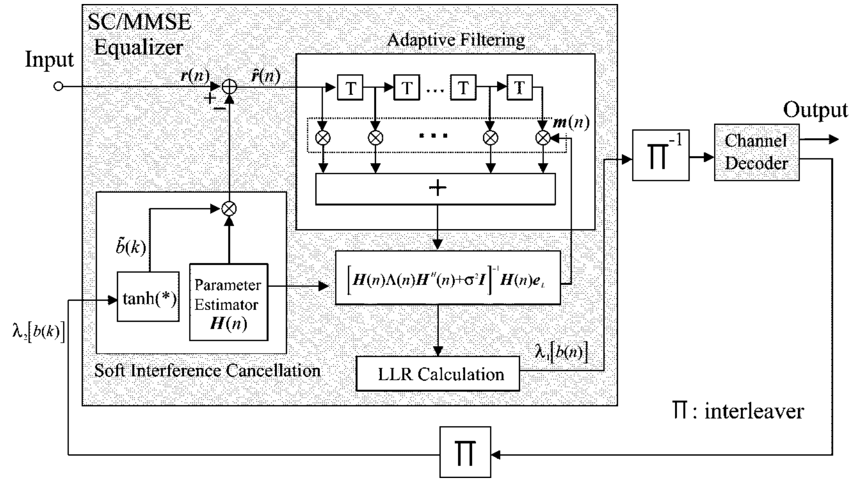

Fig. 1. Block diagram of SC/MMSE iterative equalizer.

where $\boldsymbol{r}(n)$ is the received signal vector, and

$$\tilde{\boldsymbol{b}}(n) = [\tilde{b}(n+(L-1)) \cdots \tilde{b}(n+1)\ 0\ \tilde{b}(n-1) \cdots \tilde{b}(n-(L-1))]^T. \tag{6}$$

Obviously, calculation of $\boldsymbol{m}(n)$ given by (1) requires matrix inversion, which incurs order L^3 complexity.

III. Approximation

The algorithm proposed in this paper aims to eliminate the matrix inversion needed to calculate the MMSE filter taps. It is obvious that $\boldsymbol{\Lambda}(n) = \boldsymbol{I}$ for the first iteration since no extrinsic information is provided by the channel decoder. Thus, the MMSE filter taps can be determined adaptively by using the training sequence transmitted to estimate the channel matrix $\boldsymbol{H}(n)$. In the 2nd and later iterations, $\boldsymbol{\Lambda}(n) \neq \boldsymbol{I}$, and hence, this technique cannot be used. However, if the soft estimates of the coded bits, obtained by using the LLR, are perfect, which is more likely to happen at higher iteration numbers, diagonal matrix $\boldsymbol{\Lambda}(n)$ becomes

$$\boldsymbol{\Lambda}(n) = \tilde{\boldsymbol{\Lambda}} = \mathrm{diag}(0, \ldots, 0, 1, 0, \ldots 0). \tag{7}$$

$\tilde{\boldsymbol{\Lambda}}$ is no longer a function of symbol timing index n, and $\boldsymbol{H}(n)\tilde{\boldsymbol{\Lambda}}\boldsymbol{H}^H(n)$ becomes a rank-one matrix. Hence, the MMSE filter taps can be obtained as

$$\boldsymbol{m}(n) = [\boldsymbol{h}(n)\boldsymbol{h}^H(n) + \sigma^2\boldsymbol{I}]^{-1}\boldsymbol{h}(n) \tag{8}$$

where $\boldsymbol{h}(n)$ is the Lth column vector of the space-time channel matrix $\boldsymbol{H}(n)$. By using the matrix inversion lemma [5], $\boldsymbol{m}(n)$ becomes

$$\begin{aligned}\boldsymbol{m}(n) &= [\boldsymbol{h}(n)\boldsymbol{h}^H(n)+\sigma^2\boldsymbol{I}]^{-1}\boldsymbol{h}(n)\\ &= \left[\frac{1}{\sigma^2}\boldsymbol{I} - \frac{1}{\sigma^4}\left(1+\frac{1}{\sigma^2}\boldsymbol{h}^H(n)\boldsymbol{h}(n)\right)^{-1}\right]\boldsymbol{h}(n)\boldsymbol{h}^H(n)\boldsymbol{h}(n)\\ &= \alpha\boldsymbol{h}(n)\end{aligned} \tag{9}$$

TABLE I
Simulation Parameters

Modulation	BPSK
Information symbol	256
Training symbol	64
Channel coding	Convolutional coding (R=1/2,K=3)
Interleaver	Random
Diversity	2-branch antenna
Channel mode	10-path Rayleigh fading ($f_D T_S$=1/12000)

where $\alpha = 1/(\boldsymbol{h}(n)\boldsymbol{h}^H(n) + \sigma^2)$. If the channel variation due to fading is slow enough compared with the frame length, the symbol timing index n is no longer needed, and thus $\boldsymbol{m}(n) \equiv \boldsymbol{m}$ and $\boldsymbol{h}(n) \equiv \boldsymbol{h}$ henceforth. It is obvious from (9) that the MMSE filter $\boldsymbol{m}^H\hat{\boldsymbol{r}}(n)$ is equivalent to the matched filter matched to the channel. This is quite reasonable given a sufficient number of iterations, since almost all ISI components can be eliminated by the soft canceler, and the role of the MMSE filter at this iteration stage is merely to maximize the signal energy which can be done by the matched filter.

Now, we reach the key idea that the matched filter $\alpha\boldsymbol{h}^H$ be used instead of (1) even from the 2nd iteration. This approximation significantly reduces the computational complexity since matrix inversion is no longer needed. Furthermore, the tap vector $\boldsymbol{m}$ does not have to be updated at each iteration. The computational complexity of SC/MMSE is of the order of L^2 with this approximation. Surprisingly, this approximation does not cause any serious performance degradation, as is shown in Section IV.

IV. Simulation Results

The performance of the proposed algorithm was evaluated through a series of computer simulations. Table I summarizes the simulation parameters. Fig. 2 shows BER performances of the SOVA and original SC/MMSE iterative equalizers. BER performance with the maximum likelihood sequence estimator (MLSE) followed by a hard decision Viterbi decoder is also shown in Fig. 2 (indicated by MLSE-VA). It is found that both

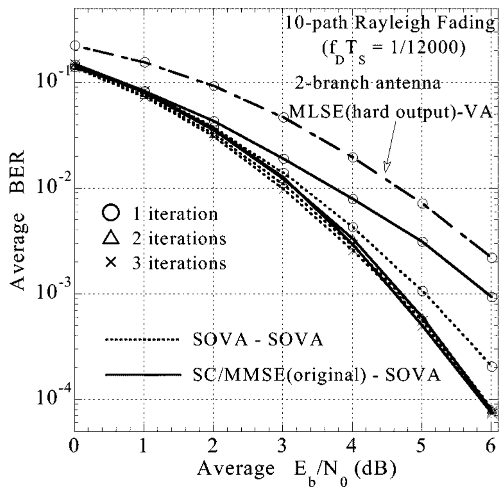

Fig. 2. Comparison of SOVA and SC/MMSE.

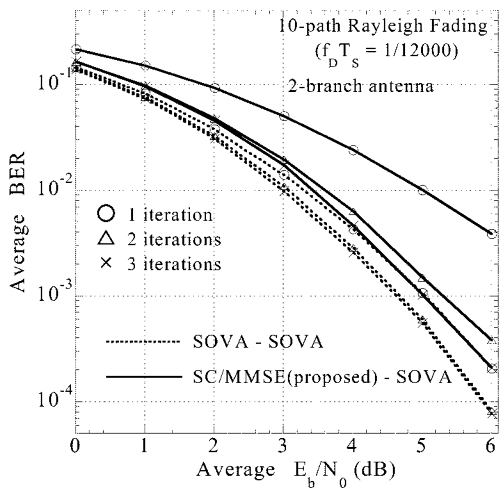

Fig. 3. BER performances of proposal SC/MMSE.

SOVA and SC/MMSE offer improved BER performance over MLSE-VA. With SOVA the performance difference between 1st and 2nd iterations is very small. This means that most of the performance improvement over MLSE-VA is achieved by the first iteration; this is offset by SOVA's relatively heavy computational burden. To the contrary, reduced-complexity algorithms such as the original SC/MMSE and the proposed SC/MMSE algorithm may not achieve the same performance at low iteration numbers, but the performance difference becomes insignificant as the iteration process proceeds. As shown in Fig. 2, SC/MMSE's BER is worse at the first iteration than that of SOVA, but the performance difference becomes very small after two-to-three iterations. Fig. 3 shows the BER performances of the SOVA and the proposed SC/MMSE algorithms. At the first iteration, the proposed SC/MMSE offers worse performance than with the original SC/MMSE. However, after three iterations, the proposed SC/MMSE almost matches the performance of the original SC/MMSE, and thus that of SOVA.

V. Conclusion

This letter has proposed a new iterative ISI equalization algorithm that offers low computational complexity: order L^2 with channel memory length L. BER performance of the proposed algorithm was evaluated through computer simulations. Results show that the proposed algorithm basically matches the performances of SOVA. Therefore, the proposed algorithm can solve the complexity problem inherent in trellis-based iterative equalizers.

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