Soft Decision Decoding of Block Codes Using Received Signal Envelope in Digital Mobile Radio

TADASHI MATSUMOTO, MEMBER, IEEE

Abstract—**The codeword error rate (WER) performance of noncoherent FSK with soft decision decoding of block codes using Chase's second algorithm is investigated in a Rayleigh fading channel. The received signal envelope is sampled and used as channel measurement information. The theoretical upper and lower bounds of the WER are derived assuming independent Rayleigh envelope samples in a received block. When the Golay (23, 12, 7) code is used, soft decision decoding with 6-bit error correction capability (3-bit error and 3-bit erasure) requires an average signal-to-noise power ratio about 5 dB lower than that for minimum distance decoding with 3-bit error correction to obtain a WER = 10^{-3}. The effects of bit interleaving on the WER performance when fading envelope variations is slow compared to the bit rate are investigated through computer simulations. When 23 × M_i bit interleaving is used for transmitting M_i Golay codewords, the simulation results show that $M_i \approx 0.2$ × (bit rate/fading maximum Doppler frequency) is sufficient to obtain the full advantages of soft decision decoding. The theoretical analysis was supported by laboratory experiments.**

I. Introduction

Multipath fading [1] severely degrades the digital signal transmission performance in mobile radios. Diversity reception [2], [3, ch. 10] is an effective technique for combating fading. Many diversity techniques have been proposed and studied for mobile radio applications [4, chs. 5, 6]. It is easy to implement a time diversity system in which the transmitter sends the same bit of information several times at time intervals well separated in relation to fading rate. The receiver then selects a decision result with the highest received signal envelope [3, pp. 425–426]. However, the major drawback of time diversity is its very low transmission rate which becomes $1/(\text{diversity order } N)$.

Another attractive technique is forward error correction (FEC) [5]. If a random error correcting (N, M) block code, such as the BCH code, is used, the transmission rate is equal to M/N and is much higher than that of the time diversity system. Minimum distance decoding has been widely utilized because of its simplicity in implementation. However, performance improvement by FEC with a minimum distance decoding is unsatisfactory for high-quality digital mobile radio. If FEC uses the received signal envelope, as does the time diversity system, it can further improve signal transmission performance. This is achieved by soft decision decoding using channel measurement information (CMI), which has recently attracted much attention because its performance is better than that of minimum distance decoding [6]–[8]. However, most soft decision algorithms are complex, and are applicable only to restricted classes of codes.

One approach to reducing decoding complexity is to generate a small set of candidate codewords and to select the most likely codeword as the output of the decoder. Chase [7] proposed three simplified algorithms based on this idea and investigated the codeword error rate (WER) performance in a Rayleigh fading environment through computer simulations. It has been shown that the second algorithm results in WER performance only slightly inferior to that of maximum likelihood decoding, while decoding complexity is greatly reduced. Experimental evaluations of bit error rate (BER) performance of a coherent GMSK system with soft decision decoding using the second algorithm were presented by Stjernvall *et al.* [9] and Ekemark *et al.* [10]. Great improvement in BER performance was obtained. However, there has been no theoretical analysis of improvement with soft decision decoding using the second algorithm.

This paper theoretically investigates the WER performance of a noncoherent FSK system with soft decision decoding using Chase's second algorithm in a Rayleigh fading environment. The received signal envelope is sampled and used as CMI to select unreliable bits and weaken the contribution of these bits to decoding. This process is similar to that in the time diversity system. The decoding algorithm is described in Section II. In Section III, assuming statistically independent signal envelope samples, the upper and lower bound estimates of the WER are theoretically analyzed. A performance comparison to time diversity is also given in Section III. In Section IV, the effects of bit interleaving on the WER performance when fading envelope variation is slow compared to the bit rate are investigated through computer simulations. Laboratory experimental results are presented in Section V.

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The author is with the NTT Radio Communication Systems Laboratories, Yokosuka 238, Japan.

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II. Algorithm

In Rayleigh fading, the signal-to-noise power ratio (SNR) varies over a wide range and the bit error probability of each bit in the received block is widely distributed. The bits having a low SNR are unreliable. The WER can be improved by weakening the contribution of these unreliable bits to decoding. This can be achieved by erasure correction. Erasure bits are selected by finding K bits, from the received block, having the lowest envelope samples. The resulting word with $N - K$ remainder bits is decoded into a codeword after minimum distance decoding with C-bit error correction carried out 2^K times where N is the word length.

Chase's second algorithm, using received signal envelope samples, consists of the following three steps.

Step 1: Find the K bits having the lowest signal envelope samples.

Step 2: These K bits are assumed to have been erased. Applying 2^K patterns to the erasure bits, C-bit error correction is performed for each of the 2^K patterns by minimum distance decoding where $0 \leqq C \leqq d$ and $2d + 1$ is the minimum distance of the code. Then, the set Ω of the candidate codewords is obtained.

Step 3: Using the bit error probability, with the SNR's derived from the signal envelope samples, calculate the *a posteriori* probability that $Y = (y_1, \cdots, y_N)$ is received when each candidate codeword is assumed to have been transmitted where Y is the received word. Then, the codeword with the maximum *a posteriori* probability is selected as the output of the decoder.

The geometric sketch for decoding with $2d + 1 = 7$ is shown in Fig. 1. When $C \leqq \lfloor (2d - K)/2 \rfloor$, the set Ω contains a single codeword [see Fig. 1(a)] where $\lfloor X \rfloor$ denotes the greatest integer less than or equal to X. Thus, Step 3 is not needed. When $\lfloor (2d - K)/2 \rfloor < C \leqq (2d - K)$, Ω contains several codewords, and Step 3 is required to select the most likely among the candidates [see Fig. 1(b)].

The above algorithm is applied to a noncoherent FSK system. The bit error probability is given by

$$p_E(\gamma) = \tfrac{1}{2} \exp(-\gamma/2) \tag{1}$$

where $\gamma = R^2/2N_0$ is the SNR, with R and N_0 being the received signal envelope and average noise power, respectively. Therefore, Step 3 can be approximated as finding a codeword which minimizes the sum of SNR's associated with the bit positions where the received word and candidate codeword have different symbols (see part A of the Appendix). Thus, the algorithm can be summarized as follows.

Find the codeword X_j

such that

$$\sum_{i=1}^{N} \gamma_i (y_i \oplus x_{ji}) \rightarrow \text{Min} \tag{2}$$

subject to $X_j \in \Omega$

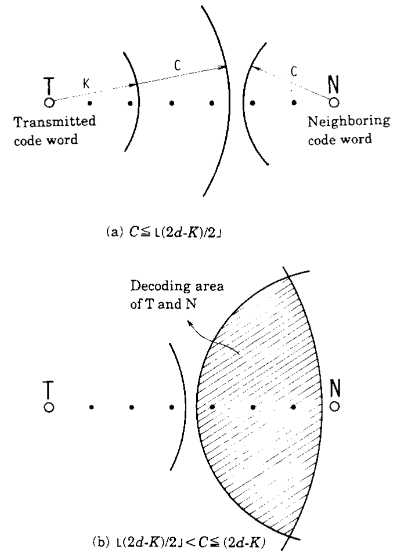

Fig. 1. Geometric sketch for decoding.

where γ_i is the SNR associated with the ith bit y_i of the received word, $X_j = (x_{j1}, \cdots, x_{jN})$ is the jth codeword of Ω, and $\oplus$ denotes the modulo two sum.

III. Bound Estimation

A. Expression for Lower Bound

The SNR variations in the received block are assumed to be statistically independent. When the number of errors occurring in the remainder bits is $C + 1$ or more, the candidate set does not always contain the transmitted codeword. On the other hand, if the number is C or less, the received word is decoded either into the transmitted codeword (correct decoding) or into a neighboring codeword (incorrect decoding). Therefore, the WER P_w is represented as[1]

$$P_w = P_w' + P_w'' \tag{3}$$

where P_w' is the probability that $C + 1$ or more errors occur in the remainder bits, and P_w'' is the probability that the number of errors in the remainder bits is C or less, but the codeword selected from the candidates is incorrect. When $C \leqq \lfloor (2d - K)/2 \rfloor$, obviously $P_w'' = 0$ and the WER is given exactly by P_w'. Otherwise, $P_w'' > 0$. Therefore, the lower bound of the WER is represented by P_w' and is given by

lower bound of WER

$$= P_w' = 1 - \sum_{i=0}^{C} \binom{N-K}{i} \cdot P_{b1}^i (1 - P_{b1})^{N-K-i} \tag{4}$$

where P_{b1} is the average bit error probability of the $N -$

[1]Word error is defined as two possible outcomes: erroneous decoding, in which the decoder delivers incorrect information, and detecting error, in which a received word has errors detected but not corrected.

K remainder bits and $\binom{A}{B}$ is the binomial coefficient. Equation (4) gives the exact WER when $C \leqq \lfloor (2d - K)/2 \rfloor$. The P_{b1} value can be calculated by averaging $p_E(\gamma)$ with the probability density function (pdf) $p_c(\gamma)$ of the SNR associated with the $N - K$ remainder bits. From part B of the Appendix, $p_c(\gamma)$ is obtained as

$$p_c(\gamma) = \frac{N}{N-K} \cdot p(\gamma) \sum_{i=1}^{N-K} \binom{N-1}{K+i-1} \cdot P(\gamma)^{K+i-1} \cdot \{1 - P(\gamma)\}^{N-K-i} \quad (5)$$

where $p(\gamma)$ is the pdf and $P(\gamma)$ is the cumulative distribution function of γ for any of N bits in the received block. In the Rayleigh fading environment, $p(\gamma) = 1/\Gamma \cdot \exp(-\gamma/\Gamma)$ and $P(\gamma) = 1 - \exp(-\gamma/\Gamma)$ where Γ is the average SNR. Fig. 2 shows $p_c(\gamma)$ for $N = 23$ with K as a parameter. It is shown that as the number K of the erasure bits increases, the SNR's of the $N - K$ remainder bits increase. Particularly, when $K = N - 1$, $p_c(\gamma)$ is identical to the pdf of γ for N-branch time diversity. It can be anticipated from Fig. 2 that the bit error probability P_{b1} of $N - K$ remainder bits is reduced as K increases. The P_{b1} value is given by

$$P_{b1} = \int_0^\infty p_E(\gamma)\, p_c(\gamma)\, d\gamma = \frac{N}{N-K} \sum_{i=1}^{N-K} \cdot \sum_{r=0}^{K+i-1} (-1)^r \cdot \frac{\binom{N-1}{K+i-1} \cdot \binom{K+i-1}{r}}{2(N-K-i+r+1)+\Gamma}. \quad (6)$$

The P_w's are calculated using the P_{b1} values and are plotted in Fig. 3, versus K, with C as a parameter for $\Gamma = 13$ dB. The greater K becomes, the smaller the P'_w obtained.

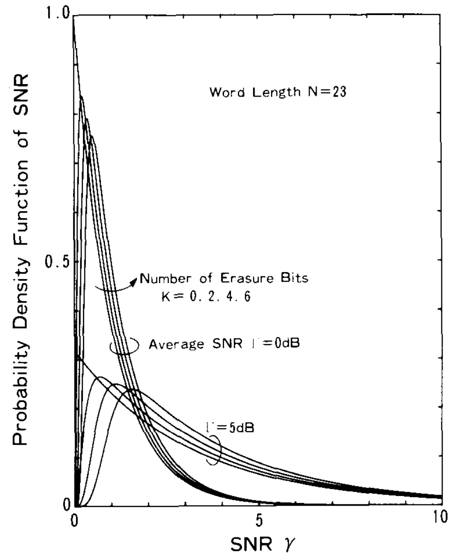

Fig. 2. Probability density functions of SNR for remainder bits.

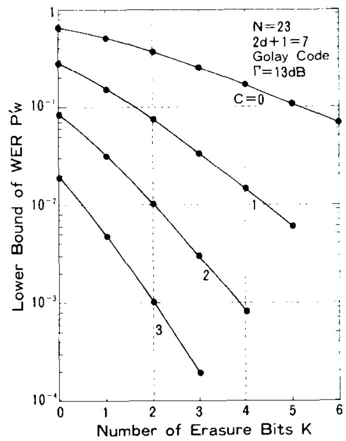

Fig. 3. Lower bound of word error rate for SNR = 13 dB.

B. Expression for Upper Bound

Since P''_w depends on the algebraic structure of the code, an exact derivation of P''_w is difficult. However, it is possible to derive an upper bound of P''_w. The upper bound of the WER is then expressed as P'_w plus the upper bound of P''_w. To derive the upper bound of P''_w, only the neighboring codewords are considered for the incorrect decoding. In the soft decision decoding described in Section II, the decoder selects a codeword, as output, which minimizes the sum of SNR's associated with the bit positions where the received word and candidate codeword have different symbols. To derive the expression of the upper bound of P''_w, we assume that m errors in the K erasure bits and n errors in the $N - K$ remainder bits are produced in the received word. The Hamming distance between the received word and the transmitted codeword is $m + n$. Thus, the Hamming distance e between the received word and the neighboring codewords satisfies the following triangle inequality [11, p. 16]:

$$e + (m + n) \geqq 2d + 1. \quad (7)$$

We assume that the e bits consist of s bits in erasures and t bits in the remainders. Let the sum of the SNR's associated with the $m + n$ error bits be denoted by γ_e and that associated with the $e(= s + t)$ bits by γ_c. The transmitted codeword is correctly selected if $\gamma_e < \gamma_c$. It is obvious that the greater the probability of γ_c being small, the larger the probability of erroneous decoding. The condition for e and s yielding the smallest γ_c is $e = 2d + 1 - m - n$, $s = K - m$, and these e bits are found from the $N - (m + n)$ bits which are correctly received for the transmitted codeword. Therefore, the upper bound of the probability that the received word containing m errors in the erasure bits and n errors in the remainder bits is erroneously decoded can be calculated as the probability of $\gamma_e \geqq \gamma_c$ when the above condition is assumed to be always satisfied.

Thus, the upper bound of P''_w is obtained by upper bound of P_w''

$$= \sum_{m=0}^{K} \sum_{n=2d+1-K-C}^{C} \binom{K}{m} \cdot \binom{N-K}{n} \cdot P_w^{(mn)} \tag{8}$$

where $P_w^{(mn)}$ is the probability that $\gamma_e \geqq \gamma_c$ when m errors are produced in the erasures and n errors in the remainders. This upper bound is obviously independent of the algebraic structure of the code.

Thus, the upper bound of the WER is given by

upper bound of WER

$$= 1 - \sum_{i=0}^{C} \binom{N-K}{i} \cdot P_{b1}^{i} \cdot (1 - P_{b1})^{N-K-i} + \sum_{m=0}^{K} \sum_{n=2d+1-K-C}^{C} \binom{K}{m} \cdot \binom{N-K}{n} \cdot P_w^{(mn)}. \tag{9}$$

The equation for $P_w^{(mn)}$ is

$$P_w^{(mn)} = P_0^{(mn)} \cdot \text{Prob}\,(\gamma_e \geqq \gamma_c) \tag{10}$$

where $P(mn)_0$ is the probability that m errors are produced in the erasures and n errors in the remainders. The derivation of $P_0^{(mn)}$ is presented in part C of the Appendix. Prob $(\gamma_e \geqq \gamma_c)$ can be calculated by numerical double integration with respect to the pdf's of γ_e and γ_c. The pdf's of γ_e and γ_c are derived using the characteristic function approach, as shown in part D of the Appendix.

C. Discussion

From the lower and upper bounds estimation, it is obvious that the algorithm has the capability of up to $(K + C)$-bit error correction. The calculated upper and lower bounds of the WER for the Golay code ($N = 23$ and $2d + 1 = 7$) are shown in Fig. 4. The parameters of K and C are set at $K = 3$, $C = 3$ and $K = 3$, $C = 2$. The WER's for minimum distance decoding with 2- and 3-bit error correction ($K = 0$, $C = 2$ and $K = 0$, $C = 3$) are also shown for comparison. Bound estimation is very tight: when $K = 3$ and $C = 2$, the lower bound is almost equal to the upper bound; when $K = 3$ and $C = 3$, the difference between the upper and lower bounds is only about 1.5 dB. Soft decision decoding with $K = 3$ and $C = 2$ requires an average SNR of 14 dB for a WER of 10^{-3}, which is about 4 dB lower than that for minimum distance decoding with 3-bit error correction ($K = 0$, $C = 3$). When $K = 3$ and $C = 3$, it requires an average SNR of 13 dB. If errors less than or equal to 6 bits can be corrected for Golay code, a WER of 10^{-3} is achieved at an average SNR of about 11 dB, which is the lower bound given by Chase [7]. The WER estimation for $K = 3$ and $C = 3$ more closely approximates actual WER performance than that given by Chase's lower bound.

Here, we compare the BER performance after soft decision decoding of the Golay (23, 12, 7) code to that of 2-branch time diversity. They have almost the same transmission rate. The BER of an (N, M) block code after decoding is evaluated from the word error rate P_w by $P_w \cdot 2^{M-1}/(2^M - 1)$ [12], the upper bound of the BER is found to be about 1×10^{-5} for an average SNR of 15 dB when $K = 3$ and $C = 3$. The BER of time diversity can be calculated from $4/\{(2 + \Gamma)(4 + \Gamma)\}$ [3, p. 462], and is about 3×10^{-3} for an average SNR of 15 dB. Thus, it is seen that the BER performance with soft decision decoding is better than that with the time diversity system.

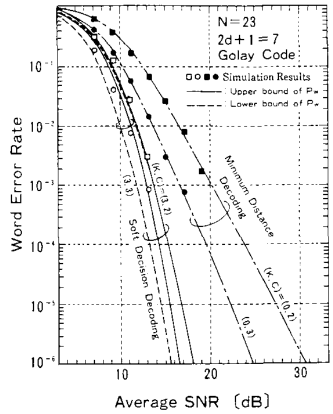

Fig. 4. Word error rate.

IV. Effect of Bit Interleaving

In Section III, the envelope variations were assumed to be statistically independent. However, the envelope variation is in many cases so slow that the signal envelope samples in the received block are statistically correlated. It is anticipated that envelope correlation degrades the WER performance of the soft decision decoding. The bit interleaving technique can be introduced to randomize the envelope variations. This section investigates the effect of the bit interleaving through computer simulation for the Golay (23, 12, 7) code. In the simulation, the bit interleaving technique with degree M_i was employed: write M_i codewords as rows of an $N \times M_i$ bit array in a memory and transmit the bits by reading the columns sequentially.

A time-varying Rayleigh envelope is generated based on a model in which a mobile station moves with constant speed and many multipath waves with identical amplitude and uniformly distributed phase come from all directions. The envelope is sampled with a normalized sampling period $f_D T$ where f_D is the maximum Doppler frequency given by (vehicle speed/carrier wavelength) and $T = M_i T_b$ with bit rate T_b^{-1}.

Simulation results for the WER are plotted in Fig. 4 for $f_D T = 1$ where the envelope variations are considered statistically independent. These results agree well with the theoretical ones. The values of average SNR required for a WER of 10^{-2} are plotted in Fig. 5 for both soft decision decoding and minimum distance decoding with K and C as parameters. The WER performance degrades as $f_D T$ decreases, e.g., the envelope correlation between any two

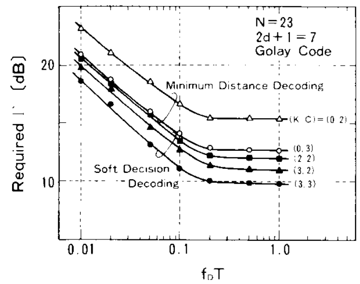

Fig. 5. Required SNR for obtaining WER = 10^{-2} versus normalized Doppler frequency $f_D T$.

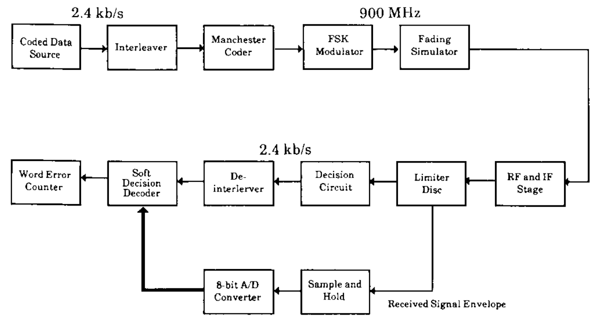

Fig. 6. Laboratory experiment system.

bits becomes large. When $f_D T \geqq 0.2$, the required SNR is almost the same as that when $f_D T = 1$. For example, $M_i T_b \geqq 5 \times 10^{-3}$ is necessary to obtain the full advantage of using soft decision decoding when $f_D = 40$ Hz.

V. Laboratory Experimental Results

Laboratory experiments were conducted for the Golay (23, 12, 7) code. The laboratory experimental system is outlined in Fig. 6. A 2.4 kbit/s bit stream was interleaved, Manchester-coded, and fed to the 900 MHz FSK modulator with a frequency deviation of 2.0 kHz. The fading FSK signal was generated by a Rayleigh fading simulator. The maximum Doppler frequency f_D of the fading simulator was set at 40 Hz, corresponding to a typical vehicle speed of 48 km/h for the 900 MHz band. From the computer simulation results in Section IV, bit interleaving degree M_i was set at 64 (i.e., $f_D T = 1.06$) so that the envelope variations could be regarded as statistically independent.

A limiter-discriminator type receiver was used.[2] An approximately Gaussian-shaped ceramic filter with a center frequency of 455 kHz and a 3 dB bandwidth of 6 kHz was adopted for the predetection bandpass filter. The frequency discriminator output was low-pass-filtered by a four-pole Butterworth filter with 3 dB bandwidth of 2.4 kHz for postdetection noise reduction. The filter output was fed to a decision circuit, and a data stream was regenerated.

The logarithmically compressed IF signal was envelope-detected. The envelope-detector output was low-pass-filtered by a two-pole Butterworth filter with a 3 dB bandwidth of 2 kHz. The filter output was sampled using an 8-bit A/D converter. The regenerated data stream and the envelope samples were delivered to an 8-bit microcomputer, which carried out the soft decision decoding. The envelope samples were anti-logarithmically expanded using a lookup table on a ROM. The experimental and theoretical WER's versus the channel BER's are shown in Fig. 7 for soft decision decoding with 6-bit ($K = 3$, $C = 3$) error correction and minimum distance decoding with 3-bit ($K = 0$, $C = 3$) error correction. The experimental results agree well with the theoretical ones.

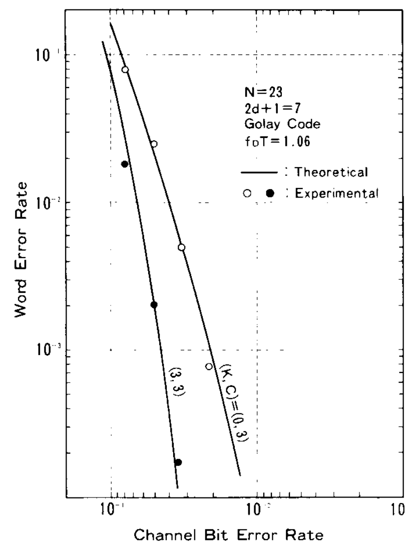

Fig. 7. Word error rate versus channel error rate.

IV. Conclusion

WER performance with Chase's second algorithm using a received signal envelope was theoretically investigated for noncoherent FSK in a Rayleigh fading channel. The theoretical upper and lower bounds of the WER were derived assuming independent signal envelope variations in a received block. When a Golay (23, 12, 7) code is used, soft decision decoding with 6-bit error correction capability (3-bit error and 3-bit erasure) required an average SNR about 5 dB lower than that for minimum distance decoding with 3-bit error correction for a WER of 10^{-3}. The effects of bit interleaving on the WER performance when fading envelope variation is slow compared to the bit rate were investigated through computer simulations. When $23 \times M_i$ bit interleaving is used for transmitting M_i Golay codewords, the simulation results show that $M_i \approx 0.2 \times$ (bit rate/fading maximum Doppler frequency) is sufficient. The theoretical analysis was supported by laboratory experiments.

[2]Limiter-discriminator detection is widely used for reception of an FSK signal. The BER for FSK with limiter-discriminator can be approximated by $(1/2)\exp(-a\gamma)$, in which parameter a is experimentally obtained [3, p. 339]. Hence, this algorithm can also be applied.

Appendix

A. Practical Approach of Step 3

The *a posteriori* probability that $Y = (y_1, \cdots, y_N)$ is received when a candidate codeword $X_j = (x_{j1}, \cdots, x_{jN}) \in \Omega$ is assumed to have been transmitted is calculated by

$$a \text{ posteriori probability} = \prod_{y_i = x_{ji}} \{1 - P_E(\gamma_i)\} \prod_{y_i \neq x_{ji}} \cdot P_E(\gamma_i) \approx \prod_{y_i \neq x_{ji}} P_E(\gamma_i). \tag{A1}$$

Using (1), the *a posteriori* probability can be approximated by

$$\begin{aligned} &a \text{ posteriori probability} \\ &\approx (1/2)^k \exp\left\{-\left(\sum_{i=1}^{k} \gamma_i\right)\Big/2\right\}, \\ &= \exp\left\{-k \cdot \log 2 - \left(\sum_{i=1}^{k} \gamma_i\right)\Big/2\right\} \end{aligned} \tag{A2}$$

where k is the number of the bits whose symbol $y_i \neq x_{ji}$. In a Rayleigh fading environment, the variations in the received SNR γ are 30–40 dB (1000–10 000 in real value). The value of $k \cdot \log 2 \approx 0.693k$ can be regarded neglegible compared to $\Sigma\gamma_i$ when the comparison among the *a posteriori* probabilities for some codewords in Ω is made. Therefore, Step 3 can be approximated by (2).

B. Derivation for pdf's of γ for Erasures and Remainders

We assume that the $N - K$ remainder bits are rearranged in order of increasing SNR. The probability that SNR γ_i of the ith bit from the lowest is $\gamma \leqq \gamma_i < \gamma + \Delta\gamma$ is given by

$$\begin{aligned} &\text{Prob}\{\gamma \leqq \gamma_i < \gamma + \Delta\gamma\} \cdot \Delta\gamma \\ &= \binom{N-1}{K+i-1} \cdot P(\gamma)^{K+i-1} \cdot \{1 - P(\gamma)\}^{N-K-i} \\ &\quad \cdot p(\gamma)\Delta\gamma, \qquad 1 \leqq i \leqq N - K. \end{aligned} \tag{B1}$$

The ith bit may be any one of the N bits in the received block. By summing up the probability with respect to the subscript i, the pdf $p_c(\gamma)$ of SNR for the $N - K$ remainder bits can be obtained. The pdf of SNR for the K erasure bits can be obtained in the same way.

C. Derivation of pdf's of $P_0^{(mn)}$

Letting the average bit error probability of the erasure bits be denoted by P_{b2}, the probability $P_0^{(mn)}$ that m errors are produced in the erasures and n errors in the remainders is obtained from

$$\begin{aligned} P_0^{(mn)} = \binom{K}{m} \cdot \binom{N-K}{n} \cdot P_{b2}^m \cdot (1 - P_{b2})^{K-m} \\ \cdot P_{b1}^n \cdot (1 - P_{b1})^{N-K-n}. \end{aligned} \tag{C1}$$

The value of P_{b2} can be calculated by averaging $p_E(\gamma)$ over the pdf of the SNR associated with the K erasure bits. In a way similar to that in which the pdf of the SNR for the $N - K$ remainders is derived, the pdf for the K erasure bits can be derived as

$$\begin{aligned} \frac{N}{K} p(\gamma) \sum_{i=1}^{K} \binom{N-1}{N-K+i-1} \\ \cdot \{1 - P(\gamma)\}^{N-K+i-1} \cdot P(\gamma)^{K-i}. \end{aligned} \tag{C2}$$

Hence, P_{b2} is given by

$$\begin{aligned} P_{b2} = \frac{N}{K} \sum_{i=1}^{K} \cdot \sum_{r=0}^{K-i} (-1)^r \\ \cdot \frac{\binom{N-1}{N-K+i-1} \cdot \binom{K-i}{r}}{2(N-K+i+r) + \Gamma}. \end{aligned} \tag{C3}$$

D. Derivation for pdf's of γ_e and γ_c

Let the sum of the SNR's associated with the erroneously received $n + m$ bits and that associated with the correctly received e bits be expressed as

$$\gamma_e = \gamma_1 + \cdots + \gamma_m + \gamma_1' + \cdots + \gamma_n' \tag{D1}$$

and

$$\gamma_c = \gamma_1'' + \cdots + \gamma_s'' + \gamma_1''' + \cdots + \gamma_t''' \tag{D2}$$

where

- $\gamma_1, \cdots, \gamma_m$: SNR's associated with the errors in the erasures,
- $\gamma_1', \cdots, \gamma_n'$: SNR's associated with the errors in the remainders,
- $\gamma_1'', \cdots, \gamma_s''$: SNR's associated with the correctly received bits in the erasures,
- $\gamma_1''', \cdots, \gamma_t'''$: SNR's associated with the correctly received bits in the remainders.

The conditional pdf's of $\gamma_i (i = 1, 2, \cdots, m)$, associated with the erroneously received bits in erasures, and of $\gamma_j' (j = 1, 2, \cdots, n)$, associated with the erroneously received bits in remainders, are obtained from Bayes' theorem and are given by

$$\begin{aligned} &\text{pdf of SNR for erasures (or remainders)} \\ &\cdot \frac{\text{BER when SNR is } \gamma_i (\text{or } \gamma_j')}{\text{average BER for erasures (or remainders)}}. \end{aligned} \tag{D3}$$

Then, the pdf of γ_e is obtained by the inverse Fourier transformation of the product of characteristic functions of $\gamma_1, \gamma_2, \cdots, \gamma_m$ and $\gamma_1', \gamma_2', \cdots, \gamma_n'$ whose pdf's are given by (D3).

The condition pdf's of γ_k'' ($k = 1, 2, \cdots, s$), associated with the correctly received bits in erasures, and of γ_l''' ($l = 1, 2, \cdots, t$), associated with the correctly received bit in remainders, can be also obtained using Bayes' theorem, and are given by

$$\text{pdf of SNR for erasures (or remainders)} \cdot \frac{[1 - \text{BER when SNR is } \gamma_k'' \text{ (or } \gamma_l''')]}{1 - \text{average BER for erasures (or remainders)}}. \tag{D4}$$

The pdf of γ_c can also be obtained in the same way as the pdf of γ_e.

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Tadashi Matsumoto (M'84) was born in Tokyo, Japan, on July 19, 1955. He received the B.S. and M.S. degrees in electrical engineering from Keio University, Yokohama-shi, Japan, in 1978 and 1980, respectively.

Since joining NTT in 1980, he participated in the R&D project of NTT's high capacity mobile communication system until 1987. Since May 1987 he has been researching digital mobile radio systems. His interest is in error control strategies such as FEC's and/or ARQ's in mobile radio channels. He is a Senior Research Engineer at NTT Radio Communication Systems Laboratories, Yokosuka-shi.

Mr. Matsumoto is a member of the Institute of Electronics, Information and Communication Engineers of Japan.