<table>
<thead>
<tr>
<th>Title</th>
<th>Soft Decision Decoding of Block Codes Using Received Signal Envelope in Digital Mobile Radio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Matsumoto, Tadashi</td>
</tr>
<tr>
<td>Citation</td>
<td>IEEE Journal on Selected Areas in Communications, 7(1): 107-113</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1989-01</td>
</tr>
<tr>
<td>Type</td>
<td>Journal Article</td>
</tr>
<tr>
<td>Text version</td>
<td>publisher</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/10119/4635">http://hdl.handle.net/10119/4635</a></td>
</tr>
<tr>
<td>Rights</td>
<td>Copyright (c)1989 IEEE. Reprinted from IEEE Journal on Selected Areas in Communications, 7(1), 1989, 107-113. This material is posted here with permission of the IEEE. Such permission of the IEEE does not in any way imply IEEE endorsement of any of JAIST’s products or services. Internal or personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution must be obtained from the IEEE by writing to <a href="mailto:pubs-permissions@ieee.org">pubs-permissions@ieee.org</a>. By choosing to view this document, you agree to all provisions of the copyright laws protecting it.</td>
</tr>
</tbody>
</table>
Abstract—The codeword error rate (WER) performance of noncoherent FSK with soft decision decoding of block codes using Chase's second algorithm is investigated in a Rayleigh fading channel. The received signal envelope is sampled and used as channel measurement information. The theoretical upper and lower bounds of the WER are derived assuming independent Rayleigh envelope samples in a received block. When the Golay (23, 12, 7) code is used, soft decision decoding with 6-bit error correction capability (3-bit error and 3-bit erasure) requires an average signal-to-noise power ratio about 5 dB lower than that for minimum distance decoding with 3-bit error correction to obtain a WER = 10^{-3}. The effects of bit interleaving on the WER performance when fading envelope variations is slow compared to the bit rate are investigated through computer simulations. When 23 \times M, bit interleaving is used for transmitting M, Golay codewords, the simulation results show that M_i = 0.2 \times (\text{bit rate/fading maximum Doppler frequency}) is sufficient to obtain the full advantages of soft decision decoding. The theoretical analysis was supported by laboratory experiments.

I. INTRODUCTION

MULTIPATH fading [1] severely degrades the digital signal transmission performance in mobile radios. Diversity reception [2], [3, ch. 10] is an effective technique for combating fading. Many diversity techniques have been proposed and studied for mobile radio applications [4, chs. 5, 6]. It is easy to implement a time diversity system in which the transmitter sends the same bit of information several times at time intervals well separated in relation to fading rate. The receiver then selects a decision result with the highest received signal envelope [3, pp. 425–426]. However, the major drawback of time diversity is its very low transmission rate which becomes 1/(diversity order N).

Another attractive technique is forward error correction (FEC) [5]. If a random error correcting (N, M) block code, such as the BCH code, is used, the transmission rate is equal to M/N and is much higher than that of the time diversity system. Minimum distance decoding has been widely utilized because of its simplicity in implementa-
II. ALGORITHM

In Rayleigh fading, the signal-to-noise power ratio (SNR) varies over a wide range and the bit error probability of each bit in the received block is widely distributed. The bits having a low SNR are unreliable. The WER can be improved by weakening the contribution of these unreliable bits to decoding. This can be achieved by erasure correction. Erasure bits are selected by finding \( K \) bits, from the received block, having the lowest envelope samples. The resulting word with \( N - K \) remainder bits is decoded into a codeword after minimum distance decoding with \( C \)-bit error correction carried out \( 2^K \) times where \( N \) is the word length.

Chase’s second algorithm, using received signal envelope samples, consists of the following three steps.

Step 1: Find the \( K \) bits having the lowest signal envelope samples.

Step 2: These \( K \) bits are assumed to have been erased. Applying \( 2^K \) patterns to the erasure bits, \( C \)-bit error correction is performed for each of the \( 2^K \) patterns by minimum distance decoding where \( 0 \leq C \leq d \) and \( 2d + 1 \) is the minimum distance of the code. Then, the set \( \Omega \) of the candidate codewords is obtained.

Step 3: Using the bit error probability, with the SNR’s derived from the signal envelope samples, calculate the a posteriori probability that \( Y = (y_1, \ldots, y_N) \) is received when each candidate codeword is assumed to have been transmitted where \( Y \) the received word. Then, the code word with the maximum a posteriori probability is selected as the output of the decoder.

The geometric sketch for decoding with \( 2d + 1 = 7 \) is shown in Fig. 1. When \( \lfloor (2d - K)/2 \rfloor \), the set \( \Omega \) contains a single codeword [see Fig. 1(a)] where \( \lfloor X \rfloor \) denotes the greatest integer less than or equal to \( X \). Thus, Step 3 is not needed. When \( \lfloor (2d - K)/2 \rfloor < C \leq (2d - K) \), \( \Omega \) contains several codewords, and Step 3 is required to select the most likely among the candidates [see Fig. 1(b)].

The above algorithm is applied to a noncoherent FSK system. The bit error probability is given by

\[
p_e(\gamma) = \frac{1}{2} \exp(-\gamma/2)
\]

where \( \gamma = R^2/2N_0 \) is the SNR, with \( R \) and \( N_0 \) being the received signal envelope and average noise power, respectively. Therefore, Step 3 can be approximated as finding a codeword which minimizes the sum of SNR’s associated with the bit positions where the received word and candidate codeword have different symbols (see part A of the Appendix). Thus, the algorithm can be summarized as follows.

**Find** the codeword \( X_f \)

such that

\[
\sum_{i=1}^{N} \gamma_i(y_i \oplus x_{fi}) \rightarrow \text{Min}
\]

subject to \( X_f \in \Omega \)

where \( \gamma_i \) is the SNR associated with the \( i \)th bit \( y_i \) of the received word, \( X_f = (x_{f1}, \ldots, x_{fN}) \) is the \( f \)th codeword of \( \Omega \), and \( \oplus \) denotes the modulo two sum.

III. BOUND ESTIMATION

A. Expression for Lower Bound

The SNR variations in the received block are assumed to be statistically independent. When the number of errors occurring in the remainder bits is \( C + 1 \) or more, the candidate set does not always contain the transmitted codeword. On the other hand, if the number is \( C \) or less, the received word is decoded either into the transmitted codeword (correct decoding) or into a neighboring codeword (incorrect decoding). Therefore, the WER \( P_w \) is represented as

\[
P_w = P'_w + P'_w
\]

where \( P'_w \) is the probability that \( C + 1 \) or more errors occur in the remainder bits, and \( P'_w \) is the probability that the number of errors in the remainder bits is \( C \) or less, but the codeword selected from the candidates is incorrect. When \( C \leq \lfloor (2d - K)/2 \rfloor \), obviously \( P'_w = 0 \) and the WER is given exactly by \( P'_w \). Otherwise, \( P'_w > 0 \). Therefore, the lower bound of the WER is represented by \( P'_w \) and is given by

lower bound of WER

\[
P'_w = 1 - \sum_{i=0}^{C} \binom{N-K}{i} \cdot P_{b_i} \cdot (1 - P_{b_i})^{N-K-i}
\]

where \( P_{b_i} \) is the average bit error probability of the \( N - 1 \) word error is defined as two possible outcomes: erroneous decoding, in which the decoder delivers incorrect information, and detecting error, in which a received word has errors detected but not corrected.
\( K \) remainder bits and \( (\frac{d}{2}) \) is the binomial coefficient. Equation (4) gives the exact WER when \( C \leq \lfloor (2d - K)/2 \rfloor \). The \( P_{\text{bi}} \) value can be calculated by averaging \( p_e(\gamma) \) with the probability density function (pdf) \( p_e(\gamma) \) of the SNR associated with the \( N - K \) remainder bits. From part B of the Appendix, \( p_e(\gamma) \) is obtained as

\[
p_e(\gamma) = \frac{N}{N - K} \cdot p(\gamma) \sum_{i=1}^{N-K} \left( \begin{array}{c} N - 1 \\ K + i - 1 \end{array} \right) \\
\cdot \left( 1 - P(\gamma) \right)^{N-K-i} \tag{5}
\]

where \( p(\gamma) \) is the pdf and \( P(\gamma) \) is the cumulative distribution function of \( \gamma \) for any of \( N \) bits in the received block. In the Rayleigh fading environment, \( p(\gamma) = 1/\Gamma \cdot \exp (-\gamma/\Gamma) \) and \( P(\gamma) = 1 - \exp (-\gamma/\Gamma) \) where \( \Gamma \) is the average SNR. Fig. 2 shows \( p_e(\gamma) \) for \( N = 23 \) with \( K \) as a parameter. It is shown that as the number \( K \) of the erasure bits increases, the SNR's of the \( N - K \) remainder bits increase. Particularly, when \( K = N - 1 \), \( p_e(\gamma) \) is identical to the pdf of \( \gamma \) for \( N \)-branch time diversity. It can be anticipated from Fig. 2 that the bit error probability \( P_{\text{bi}} \) of \( N - K \) remainder bits is reduced as \( K \) increases. The \( P_{\text{bi}} \) value is given by

\[
P_{\text{bi}} = \int_0^\infty p_e(\gamma) p_e(\gamma) \, d\gamma
\]

\[
= \frac{N}{N - K} \sum_{i=1}^{N-K} \sum_{r=0}^{K+i-1} (-1)^r \left( \begin{array}{c} N - 1 \\ K + i - 1 \end{array} \right) \\
\cdot \left( \begin{array}{c} r \\ r \end{array} \right) \\
\cdot 2^{N-K-i+r+1} + \Gamma. \tag{6}
\]

The \( P_{\text{bi}} \)'s are calculated using the \( P_{\text{bi}} \) values and are plotted in Fig. 3, versus \( K \), with \( C \) as a parameter for \( \Gamma = 13 \) dB. The greater \( K \) becomes, the smaller the \( P_{\text{bi}} \) obtained.

**B. Expression for Upper Bound**

Since \( P_{\text{bi}}^* \) depends on the algebraic structure of the code, an exact derivation of \( P_{\text{bi}}^* \) is difficult. However, it is possible to derive an upper bound of \( P_{\text{bi}}^* \). The upper bound of the WER is then expressed as \( P_{\text{bi}}^* \) plus the upper bound of \( P_{\text{er}}^* \). To derive the upper bound of \( P_{\text{bi}}^* \), only the neighboring codewords are considered for the incorrect decoding. In the soft decision decoding described in Section II, the decoder selects a codeword, as output, which minimizes the sum of SNR's associated with the bit positions where the received word and candidate codeword have different symbols. To derive the expression of the upper bound of \( P_{\text{bi}}^* \), we assume that \( m \) errors in the \( K \) erasure bits and \( n \) errors in the \( N - K \) remainder bits are produced in the received word. The Hamming distance between the received word and the transmitted codeword is \( m + n \). Thus, the Hamming distance \( e \) between the received word and the neighboring codewords satisfies the following triangle inequality [11, p. 16]:

\[
e + (m + n) \geq 2d + 1. \tag{7}
\]

We assume that the \( e \) bits consist of \( s \) bits in erasures and \( t \) bits in the remainders. Let the sum of the SNR's associated with the \( m + n \) error bits be denoted by \( \gamma_e \), and that associated with the \( e(=s+t) \) bits by \( \gamma_e \). The transmitted codeword is correctly selected if \( \gamma_e < \gamma_c \). It is obvious that the greater the probability of \( \gamma_e \) being small, the larger the probability of erroneous decoding. The condition for \( e \) and \( s \) yielding the smallest \( \gamma_e \), is \( e = 2d + 1 - m - n, s = K - m, \) and these \( e \) bits are found from the \( N - (m + n) \) bits which are correctly received for the transmitted codeword. Therefore, the upper bound of the probability that the received word containing \( m \) errors in the erasure bits and \( n \) errors in the remainder bits is erroneously decoded can be calculated as the probability of \( \gamma_e \geq \gamma_c \) when the above condition is assumed to be always satisfied.

Thus, the upper bound of \( P_{\text{bi}}^* \) is obtained by
upper bound of $P_w$

$$P_w = \sum_{m=0}^{K} \sum_{n=2d+1}^{C} \binom{K}{m} \cdot \binom{N-K}{n} \cdot P_w(mn)$$

where $P_w(mn)$ is the probability that $\gamma_e \geq \gamma_c$ when $m$ errors are produced in the erasures and $n$ errors in the remainders. This upper bound is obviously independent of the algebraic structure of the code.

Thus, the upper bound of the WER is given by

$$1 - \sum_{i=0}^{C} \binom{N-K}{i} \cdot P_{bl}^i \cdot (1 - P_{bl})^{N-K-i}$$

$$+ \sum_{m=0}^{K} \sum_{n=2d+1}^{C} \binom{K}{m} \cdot \binom{N-K}{n} \cdot P_w(mn).$$

The equation for $P_w(mn)$ is

$$P_w(mn) = P_{0}(mn) \cdot \text{Prob}(\gamma_e \geq \gamma_c)$$

where $P_{0}(mn)$ is the probability that $m$ errors are produced in the erasures and $n$ errors in the remainders. The derivation of $P_{0}(mn)$ is presented in part C of the Appendix. Prob.$(\gamma_e \geq \gamma_c)$ can be calculated by numerical double integration with respect to the pdf’s of $\gamma_e$ and $\gamma_c$. The pdf’s of $\gamma_e$ and $\gamma_c$ are derived using the characteristic function approach, as shown in part D of the Appendix.

C. Discussion

From the lower and upper bounds estimation, it is obvious that the algorithm has the capability of up to $(K + C)$-bit error correction. The calculated upper and lower bounds of the WER for the Golay code ($N = 23$ and $2d + 1 = 7$) are shown in Fig. 4. The parameters of $K$ and $C$ are set at $K = 3$, $C = 3$ and $K = 3$, $C = 2$. The WER’s for minimum distance decoding with 2- and 3-bit error correction ($K = 0$, $C = 2$ and $K = 0$, $C = 3$) are also shown for comparison. Bound estimation is very tight: when $K = 3$ and $C = 2$, the lower bound is almost equal to the upper bound; when $K = 3$ and $C = 3$, the difference between the upper and lower bounds is only about 1.5 dB. Soft decision decoding with $K = 3$ and $C = 2$ requires an average SNR of 14 dB for a WER of $10^{-5}$, which is about 4 dB lower than that for minimum distance decoding with 3-bit error correction ($K = 0$, $C = 3$). When $K = 3$ and $C = 3$, it requires an average SNR of 13 dB. If errors less than or equal to 6 bits can be corrected for Golay code, a WER of $10^{-5}$ is achieved at an average SNR of about 11 dB, which is the lower bound given by Chase [7]. The WER estimation for $K = 3$ and $C = 3$ more closely approximates actual WER performance than that given by Chase’s lower bound.

Here, we compare the BER performance after soft decision decoding of the Golay (23, 12, 7) code to that of 2-branch time diversity. They have almost the same transmission rate. The BER of an $(N, M)$ block code after decoding is evaluated from the word error rate $P_w$ by $P_w \cdot \frac{2^{M-1}}{(2^M - 1)}$ [12], the upper bound of the BER is found to be about $1 \times 10^{-5}$ for an average SNR of 15 dB when $K = 3$ and $C = 3$. The BER of time diversity can be calculated from $4/(\{2 + \Gamma\}(4 + \Gamma))$ [3, p. 462], and is about $3 \times 10^{-5}$ for an average SNR of 15 dB. Thus, it is seen that the BER performance with soft decision decoding is better than that with the time diversity system.

IV. EFFECT OF BIT INTERLEAVING

In Section III, the envelope variations were assumed to be statistically independent. However, the envelope variation is in many cases so slow that the signal envelope samples in the received block are statistically correlated. It is anticipated that envelope correlation degrades the BER performance of the soft decision decoding. The bit interleaving technique can be introduced to randomize the envelope variations. This section investigates the effect of the bit interleaving through computer simulation for the Golay (23, 12, 7) code. In the simulation, the bit interleaving technique with degree $M_i$ was employed: write $M_i$ codewords as rows of an $N \times M_i$ bit array in a memory, and transmit the bits by reading the columns sequentially.

A time-varying Rayleigh envelope is generated based on a model in which a mobile station moves with constant speed and many multipath waves with identical amplitude and uniformly distributed phase come from all directions. The envelope is sampled with a normalized sampling period $f_0T$ where $f_0$ is the maximum Doppler frequency given by (vehicle speed/crarrier wavelength) and $T = M_i T_b$ with bit rate $T_b^{-1}$.

Simulation results for the WER are plotted in Fig. 4 for $f_0T = 1$ where the envelope variations are considered statistically independent. These results agree well with the theoretical ones. The values of average SNR required for a WER of $10^{-5}$ are plotted in Fig. 5 for both soft decision decoding and minimum distance decoding with $K$ and $C$ as parameters. The BER performance degrades as $f_0T$ decreases, e.g., the envelope correlation between any two
bits becomes large. When \( f_d T \geq 0.2 \), the required SNR is almost the same as that when \( f_d T = 1 \). For example, \( M_j T_b \geq 5 \times 10^{-3} \) is necessary to obtain the full advantage of using soft decision decoding when \( f_d = 40 \) Hz.

V. LABORATORY EXPERIMENTAL RESULTS

Laboratory experiments were conducted for the Golay (23, 12, 7) code. The laboratory experimental system is outlined in Fig. 6. A 2.4 kbit/s bit stream was interleaved, Manchester-coded, and fed to the 900 MHz FSK modulator with a frequency deviation of 2.0 kHz. The fading FSK signal was generated by a Rayleigh fading simulator. The maximum Doppler frequency \( f_d \) of the fading simulator was set at 40 Hz, corresponding to a typical vehicle speed of 48 km/h for the 900 MHz band. From the computer simulation results in Section IV, bit interleaving degree \( M_i \) was set at 64 (i.e., \( f_d T = 1.06 \)) so that the envelope variations could be regarded as statistically independent.

A limiter-discriminator type receiver was used. An approximately Gaussian-shaped ceramic filter with a center frequency of 455 kHz and a 3 dB bandwidth of 6 kHz was adopted for the predetection bandpass filter. The frequency discriminator output was low-pass-filtered by a four-pole Butterworth filter with 3 dB bandwidth of 2.4 kHz for postdetection noise reduction. The filter output was fed to a decision circuit, and a data stream was regenerated.

The logarithmically compressed IF signal was envelope-detected. The envelope-detector output was low-pass-filtered by a two-pole Butterworth filter with a 3 dB bandwidth of 2 kHz. The filter output was sampled using an 8-bit A/D converter. The regenerated data stream and the envelope samples were delivered to an 8-bit microcomputer, which carried out the soft decision decoding. The envelope samples were anti-logarithmically expanded using a lookup table on a ROM. The experimental and theoretical WER's versus the channel BER's are shown in Fig. 7 for soft decision decoding with 6-bit \((K = 3, C = 3)\) error correction and minimum distance decoding with 3-bit \((K = 0, C = 3)\) error correction. The experimental results agree well with the theoretical ones.

IV. CONCLUSION

WER performance with Chase's second algorithm using a received signal envelope was theoretically investigated for noncoherent FSK in a Rayleigh fading channel. The theoretical upper and lower bounds of the WER were derived assuming independent signal envelope variations in a received block. When a Golay (23, 12, 7) code is used, soft decision decoding with 6-bit error correction capability (3-bit error and 3-bit erasure) required an average SNR about 5 dB lower than that for minimum distance decoding with 3-bit error correction for a WER of \( 10^{-3} \). The effects of bit interleaving on the WER performance when fading envelope variation is slow compared to the bit rate were investigated through computer simulations. When \( 23 \times M_i \) bit interleaving is used for transmitting \( M_i \) Golay codewords, the simulation results show that \( M_i \approx 0.2 \times \) (bit rate/fading maximum Doppler frequency) is sufficient. The theoretical analysis was supported by laboratory experiments.
APPENDIX

A. Practical Approach of Step 3

The a posteriori probability that \( Y = (y_1, \ldots, y_N) \) is received when a candidate codeword \( X_j = (x_{j1}, \ldots, x_{jN}) \in \Omega \) is assumed to have been transmitted is calculated by

\[
\text{a posteriori probability} = \prod_{y_i \neq x_{ji}} \left[ 1 - P_e(y_i) \right] \prod_{y_i = x_{ji}} P_e(y_i),
\]

where \( k \) is the number of the bits whose symbol \( y_i \neq x_{ji} \). In a Rayleigh fading environment, the variations in the received SNR is 30-40 dB (1000-10 000 in real value).

The value of \( k \cdot \log 2 = 0.693k \) can be regarded negligible compared to \( \gamma \) when the comparison among the a posteriori probabilities for some codewords in \( \Omega \) is made. Therefore, Step 3 can be approximated by (2).

B. Derivation for pdf's of \( y_e \) and \( y_r \)

Let the sum of the SNR's associated with the erroneously received \( n + m \) bits and that associated with the correctly received \( e \) bits be expressed as

\[
y_e = y_{1e} + \cdots + y_{me} + y_{1r} + \cdots + y_{nr}\]

and

\[
y_r = y_{1r} + \cdots + y_{er} + y_{1r} + \cdots + y_{nr}\]

where

\( y_{1e}, \ldots, y_{me} \): SNR's associated with the errors in the erasures, 
\( y_{1r}, \ldots, y_{nr} \): SNR's associated with the errors in the remainders, 
\( y_{1r}, \ldots, y_{nr} \): SNR's associated with the correctly received bits in the erasures, 
\( y_{1r}, \ldots, y_{nr} \): SNR's associated with the correctly received bits in the remainders.

The conditional pdf's of \( y_i(i = 1, 2, \cdots, m) \), associated with the erroneously received bits in erasures, and of \( y_j' (j = 1, 2, \cdots, n) \), associated with the erroneously received bits in remainders, are obtained from Bayes' theorem and are given by

\[
\text{pdf of SNR for erasures (or remainders)}
\]

Then, the pdf of \( \gamma \), is obtained by the inverse Fourier transformation of the product of characteristic functions.
of \( \gamma_1, \gamma_2, \ldots, \gamma_m \) and \( \gamma'_1, \gamma'_2, \ldots, \gamma'_t \) whose pdf's are given by (D3).

The condition pdf's of \( \gamma_k (k = 1, 2, \ldots, s) \), associated with the correctly received bits in erasures, and of \( \gamma'_l (l = 1, 2, \ldots, t) \), associated with the correctly received bit in remainders, can be also obtained using Bayes' theorem, and are given by

\[
\text{pdf of SNR for erasures (or remainders)} = \frac{1 - \text{BER when SNR is } \gamma_k}{1 - \text{average BER for erasures (or remainders)}}.
\]

The pdf of \( \gamma_c \) can also be obtained in the same way as the pdf of \( \gamma_e \).

ACKNOWLEDGMENT

The author wishes to thank Dr. M. Shinji, Executive Manager of NTT Radio Communication Systems Laboratories, for his encouragement throughout this study.

REFERENCES


Tadashi Matsumoto (M'84) was born in Tokyo, Japan, on July 19, 1955. He received the B.S. and M.S. degrees in electrical engineering from Keio University, Yokohama-shi, Japan, in 1978 and 1980, respectively.

Since joining NTT in 1980, he participated in the R&D project of NTT's high capacity mobile communication system until 1987. Since May 1987 he has been researching digital mobile radio channels. He is a Senior Research Engineer at NTT Radio Communication Systems Laboratories, Yokosuka-shi.

Mr. Matsumoto is a member of the Institute of Electronics, Information and Communication Engineers of Japan.