Performance Limits of Coded Multilevel DPSK in Cellular Mobile Radio

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Abstract— **Performance limits of coded multilevel differential PSK (MDPSK) in multipath Rayleigh fading channels are described. The simple "Gaussian metric" is assumed for reasons for practicality even though it is not the maximum likelihood. Channel cutoff rate R_0 of MDPSK is analyzed based on the metric. Account is taken of additive white Gaussian noise (AWGN), cochannel interference, and multipath channel delay spread. For the analysis of the spectrum efficiency of a cellular mobile radio system employing coded MDPSK, its service area is defined as the area in which, with a bit rate of R information bit/symbol ($R \leq R_0$), reliable communications are possible. Three optimal information bit rates are determined from the channel cutoff rate to minimize the required average signal energy per information bit-to-noise power spectral density ratio (E_b/N_0), to maximize the tolerable rms delay spread τ_{rms} and to maximize the spectrum efficiency.**

I. Introduction

SIGNALS received by mobile radio systems are subject to multipath fading which severely degrades transmission performance. One cause of error is additive white Gaussian noise (AWGN). Another is cochannel interference since cellular systems reuse the same frequency in spatially separated cells in order to use the available frequency spectra more effectively. If the symbol rate is higher than the channel coherence bandwidth, the fading channel tends to be frequency-selective. Intersymbol interference (ISI) is the main cause of errors in this case. The most efficient techniques in reducing the fading effects are diversity and coding [1]–[9].

Since Ungerboeck proposed trellis coded modulation (TCM) in 1982 [10], mobile radio applications of TCM with Viterbi decoding have attracted much attention. This is because TCM can greatly reduce the bit error rate (BER) without causing bandwidth expansion. Thus it is possible to improve the spectrum efficiency of cellular mobile radio systems.

The optimal code design procedure for fading channels is quite different from that for AWGN channels. In 1988, TCM code design criteria to achieve maximum performance in Rayleigh fading channels were presented for coherent PSK (CPSK) by Divsalar and Simon [2]. The design criteria consist of *maximizing the length of the shortest error event path and the product of the squared branch distances.* The BER reduces in inverse proportion to E_b/N_0 to the power of the shortest error event path length, where E_b/N_0 is the average signal energy per information bit-to-noise power spectral density ratio.

Using these criteria, Schlegel and Costello constructed several codes with rate $2/3$ for 8PSK [5], and Du, Vucetic, and Zhang constructed other rate $2/3$ codes for 8PSK [6] and rate $3/4$ codes for 16PSK [7]. Recently, Matsumoto and Adachi analyzed the BER performance of coded quaternary differential PSK (4DPSK) under multipath Rayleigh fading and showed that the above criteria are also useful in cochannel interference limited channels [8], and in frequency selective fading channels [9].

Decoding complexity of these codes is reasonable (this depends mainly on the number of states in the trellis diagram) given present technology. However, it is useful to investigate the performance limits of cellular mobile radio systems, assuming coding where the decoding complexity approaches the computationally achievable limit. This limit is determined by the channel cutoff rate (R_0). R_0 represents one of the upper bounds of the information bit rate [11], [12]. The error probability is, for R information bits/channel symbol transmission, less than or equal to $e^{-N(R_0-R)}$ where N is the code length. If $R \leq R_0$, the input information can be transmitted with an arbitrarily low BER (communications via coding channels satisfying this condition are referred to as "practical, reliable communications"). R_0 characterizes the coding channel only, and is independent of the specific code employed. Therefore, the performance limits of a signaling channel can be analyzed through R_0 independently of the code being used.

This paper analyzes the channel cutoff rate of Nyquist filtered multilevel differential PSK (MDPSK) in Rayleigh fading channels taking into account the AWGN, cochannel interference, and multipath delay spread. We assume the simple "Gaussian metric" [20] which is not the maximum likelihood, but because it is the most practical choice. Several performance limits of the cellular mobile radio system employing coded MDPSK are investigated based on the metric. Section II presents the system model used in the analysis. In Section III, the channel cutoff rate R_0 in a Rayleigh fading channel is theoretically analyzed through the Chernoff bound approach, taking into account AWGN, cochannel interference, and multipath delay spread. Section IV determines the optimal information bit rates that minimize the required average E_b/N_0 and maximize the tolerable delay spread.

Section V investigates the spectrum efficiency of cellular mobile radio systems employing coded MDPSK, where the service area is defined as the area in which practical, reli-

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able communications are possible. The spectrum efficiency is evaluated from the channel cutoff rate R_0 under a cochannel interference limited environment.

II. System Model

The block diagram of the system under investigation is shown in Fig. 1. The input bit stream is encoded by a trellis code encoder. The coded symbol sequence $\boldsymbol{x} = (x_1, x_2, \cdots, x_N)$ is block-interleaved symbol-by-symbol, and then mapped to a signal point of the MDPSK signal space, where M is the number of the MDPSK signal points. It is assumed that the interleaving degree is large enough to randomize burst errors produced by Rayleigh fading.

To avoid ISI and to achieve a narrow-band spectrum, we use a Nyquist raised cosine transfer function for overall filter response. The function is shared equally between transmitter and receiver. The overall impulse response is

$$h(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\alpha \pi t/T)}{1-(2\alpha t/T)^2} \tag{1}$$

where T is the symbol duration, and α is the roll-off factor $(0 \leqq \alpha \leqq 1)$.

Signal transmission between mobile and base station takes place over multipath channels. We assume that fading is much slower than the symbol rate so that the multipath channel transfer function remains almost constant over several symbols. The received signal suffers from frequency selective fading. Cochannel interference and AWGN with single-sided spectral density of N_0 are added to the desired received signal. The resulting signal is then fed to the root Nyquist raised cosine filter for differential detection. The input to the differential detector can be written using the complex envelope representation as

$$\begin{aligned} z(t) &= z_s(t) + z_i(t) + z_n(t) \\ &= \int_{-\infty}^{\infty} s(t-\tau) g(\tau, t)\, d\tau + z_i(t) + z_n(t) \end{aligned} \tag{2}$$

where $s(t)$ is the desired signal component without fading, $g(\tau, t)$ is the baseband equivalent multipath channel impulse response with $\int_{-\infty}^{\infty} \langle |g(\tau,t)|^2 \rangle \, d\tau = 1$, $z_i(t)$ is cochannel interference, $z_n(t)$ is the filter output component due to the AWGN at the input, and $< \cdot >$ denotes the time average operation. The power spectrum of $z_n(t)$ has the raised cosine shape, and therefore, the samples $\{z_n(mT), m = \cdots, -2, -1, 0, 1, 2, \cdots\}$ form an independent, identically distributed sequence of random variables.

The overall response of the transmitter and receiver filters to the MDPSK signal is $s(t)$ and is expressed, with signal energy per symbol of E_s, as $s(t) = \sqrt{(2E_s/T)}\, d_s(t)$, where

$$d_s(t) = \sum_{m=-\infty}^{\infty} e^{j\phi_m} h(t - mT) \tag{3}$$

and $\phi_m \in \{2i\pi/M; i = 0 \sim M-1\}$ is the mth phase. With MDPSK modulation, the coded symbol to be transmitted is assigned to the phase difference $\Delta\phi_m = \phi_m - \phi_{m-1} \bmod 2\pi$.

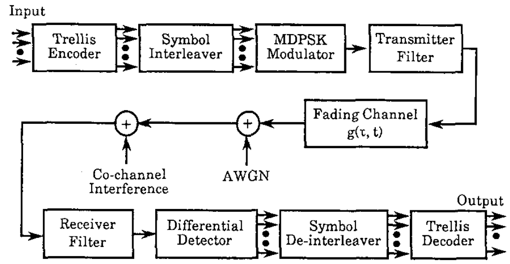

Fig. 1. Complex representation of system model.

The cochannel interference component without fading, $i(t)$, is also expressed as $i(t) = \sqrt{(2E_i/T)}\, d_i(t)$, where

$$d_i(t) = \sum_{m=-\infty}^{\infty} e^{j\psi_m} h(t + \Delta T - mT) \tag{4}$$

ψ_m is the mth phase of the cochannel interference and E_i is the cochannel interference signal energy per symbol. ΔT is the modulation timing offset between the desired and cochannel interference signals. The average received signal-to-noise power ratio (SNR) Γ is then defined as E_s/N_0, and the average received signal-to-interference power ratio (SIR)Λ is defined as E_s/E_i.

Since the impulse response at τ is due to the sum of many independent impulses caused by reflections from buildings and other obstacles, $g_d(\tau, t)$ can be assumed to be a zero-mean complex Gaussian process of t, and the delay-time correlation function is given by

$$\langle g(\tau, t) g^*(\tau - v, t - \mu) \rangle = \xi_s(\tau, \mu) \delta(v) \tag{5}$$

where the asterisk denotes complex conjugate and $\delta(\cdot)$ is the delta function. $\xi_s(\tau, 0)$ is called the multipath channel delay profile. The rms delay spread τ_{rms} is the important factor that determines the average BER under frequency selective fading. τ_{rms} is defined as

$$\tau_{\text{rms}} = \sqrt{\int_{-\infty}^{\infty} (\tau - \tau_{\text{mean}})^2 \xi_s(\tau, 0)\, d\tau} \tag{6}$$

where τ_{mean} is the mean time delay given by

$$\tau_{\text{mean}} = \int_{-\infty}^{\infty} \tau \xi_s(\tau, 0)\, d\tau \tag{7}$$

and $\int_{-\infty}^{\infty} \xi_s(\tau, 0)\, d\tau = 1$.

The MDPSK signal is detected by a differential detector. The sample sequence of the detector output $y_m = y(t_m)$ at the sampling timing $t = t_m$ is expressed in the complex form as

$$y_m = z_m z_{m-1}^* \tag{8}$$

where $z_m = z(t_m)$ and $z_{m-1} = z(t_m - T)$.

After deinterleaving, the sample sequence of $y_k = z_k z_{k-1}^*$ is obtained. The deinterleaved sequence yk is then fed to the trellis code decoder.

$$\rho_k = \frac{\Gamma \int_{-\infty}^{\infty} \xi_s(\tau, T)\, d_s(kT-\tau)\, d_s^*((k-1)T-\tau)\, d\tau}{\left[\sqrt{\Gamma \int_{-\infty}^{\infty} \xi_s(\tau,0) |d_s(kT-\tau)|^2 d\tau + \frac{\Gamma}{\Lambda}\int_{-\infty}^{\infty} \xi_i(\tau,0) |d_i(kT+\Delta T-\tau)|^2 d\tau + 1}\right]} * \frac{+\frac{\Gamma}{\Lambda}\int_{-\infty}^{\infty} \xi_i(\tau,T)\, d_i(kT+\Delta T-\tau)\, d_i((k-1)T+\Delta T-\tau)\, d\tau}{\times\left[\sqrt{\Gamma \int_{-\infty}^{\infty} \xi_s(\tau,0) |d_s((k-1)T-\tau)|^2 d\tau + \frac{\Gamma}{\Lambda}\int_{-\infty}^{\infty} \xi_i(\tau,0) |d_i((k-1)T+\Delta T-\tau)|^2 d\tau + 1}\right]} \tag{12}$$

III. Channel Cutoff Rate

The channel cutoff rate R_0 is defined as the achievable information transmission rate (= *information bits/MDPSK symbol).* With length N code, the error probability is, for $R(\leq R_0)$ information transmission rate, less than or equal to $e^{-N(R_0-R)}$. The Chernoff bound approach is used to evaluate R_0 in Rayleigh fading channels. Account is taken of AWGN, cochannel interference, and multipath channel delay spread.

The channel cutoff rate R_0 is expressed as

$$R_0 = \log_2 M - \log_2 (1 + (M-1)D) \tag{9}$$

where D is given by

$$D = \frac{1}{M(M-1)} \min_{\lambda \geq 0} \sum_{\Delta\phi_k} \sum_{\Delta\phi_k' \neq \Delta\phi_k} D(\Delta\phi_k, \Delta\phi_k', \lambda) \tag{10}$$

and $D(\Delta\phi_k, \Delta\phi_k', \lambda)$ is where λ is the Chernoff parameter, the Chernoff bound of the average probability $\Pr(\Delta\phi_k \to \Delta\phi_k')$ that the transmitted phase difference $\Delta\phi_k$ is incorrectly received as $\Delta\phi_k'$. Here, we have assumed that the probability of choosing each signal point at the transmitter is $1/M$. To evaluate the Chernoff bound of the average probability $\Pr(\Delta\phi_k \to \Delta\phi_k')$, we first calculate the conditional probability density function (pdf) of the differential detector output.

A. Conditional pdf of Detector Output

The conditional pdf, $p\{z_{k-1}^*|z_k\}$, of z_{k-1}^* with z_k being given is given by [13],

$$p\{z_{k-1}^*|z_k\} = \frac{1}{2\pi\sigma_2^2\left(1-|\rho_k|^2\right)} \cdot \exp\left\{\frac{-\left|z_{k-1}^* - (\sigma_2/\sigma_1)\rho_k^* z_k\right|^2}{2\sigma_2^2\left(1-|\rho_k|^2\right)}\right\} \tag{11}$$

where $\rho_k = \langle z_k \cdot z_{k-1}^*\rangle / 2\sigma_1\sigma_2$ with $2\sigma_1^2 = \langle |z_k|^2 \rangle$ and $2\sigma_2^2 = \langle |z_{k-1}|^2 \rangle$. Since we are assuming statistically independent noise samples, ρ_k is given by (12) at the top of the page [14] where Γ and Λ are the average received SNR and SIR, respectively, defined in Section II. $\xi_i(\tau, 0)$ is the delay profile of the cochannel interference.

B. R_0 Expression

The decoder calculates the kth branch metric from the deinterleaver output y_k. For differential detection, the most practical choice for branch metric $m_b(y_k, x_k)$, the "Gaussian metric,"[1] is [15], [20]

$$m_b(y_k, x_k) = \mathrm{Re}(y_k x_k^*) = \mathrm{Re}\{(z_k z_{k-1}^*) x_k^*\}. \tag{13}$$

The transmitted phase difference $\Delta\phi_k$ is incorrectly received as $\Delta\phi_k'$ when $m_b(y_k, x_k) < m_b(y_k, x_k')$, where $x_k = \exp(j\Delta\phi_k)$ and $x_k' = \exp(j\Delta\phi_k')$. Therefore, $\Pr(\Delta\phi_k \to \Delta\phi_k')$ can be represented as

$$\Pr(\Delta\phi_k \to \Delta\phi_k') = \mathrm{Prob}\{\mathrm{Re}[z_k z_{k-1}^* e^{-j\Delta\phi_k}(1 - e^{j\delta\phi_k})] < 0\} \tag{14}$$

where $\delta\phi_k = \Delta\phi_k - \Delta\phi_k'$. $\mathrm{Re}\{z_k z_{k-1}^* e^{-j\Delta\phi_k}(1-e^{j\delta\phi_k})\}$ is, with z_k being given, a complex Gaussian variable whose mean and variance are

$$\mathrm{mean} = \frac{\sigma_2}{\sigma_1}\{\rho_c(1-\cos\delta\phi_k) + \rho_s \sin\delta\phi_k\}|z_k|^2 \tag{15}$$

and

$$\mathrm{variance} = 2\sigma_2^2\left(1-|\rho_k|^2\right)(1-\cos\delta\phi_n)|z_k|^2 \tag{16}$$

where $\rho_k e^{-j\Delta\phi_k} = \rho_c + j\rho_s$.

Using the characteristic function of the Gaussian pdf, the conditional Chernoff bound $d(\Delta\phi_k, \Delta\phi_k', \lambda \mid |z_k|)$ is given by

$$\begin{aligned} d(\Delta\phi_k, \Delta\phi_k', \lambda \mid |z_k|) &= E\exp\Bigg\{\Bigg[-\lambda\frac{\sigma_2}{\sigma_1}\left(\rho_c(1-\cos\delta\phi_k) - \lambda\rho_s\sin\delta\phi_k\right) \\ &\quad + \sigma_2^2\lambda^2\left(1-|\rho_k|^2\right)\cdot(1-\cos\delta\phi_k)\Bigg]|z_k|\Bigg\}. \end{aligned} \tag{17}$$

The ensemble average in (17) is taken over all the possible patterns of adjacent and cochannel interference symbol sequences and the distribution of the modulation timing offset ΔT. Thus the Chernoff bound $D(\Delta\phi_k, \Delta\phi_k', \lambda)$ in (10) can be calculated by averaging $d(\Delta\phi_k, \Delta\phi_k', \lambda \mid |z_k|)$ over the pdf of $|z_k|^2$, and over possible interference and adjacent symbol patterns and δT as

$$D(\Delta\phi_k, \Delta\phi_k', \lambda) = E\int_0^{\infty} d(\Delta\phi_k, \Delta\phi_k', \lambda \mid |z_k|)\, p\left(|z_k|^2\right) d|z_k|^2. \tag{18}$$

[1]The maximum likelihood metric for differential PSK was derived in [20], however, it is quite complicated to implement. The "Gaussian metric" given by (13) is most useful from the implementation point of view. R_0 derived from this "Gaussian metric" is the "mismatched" cutoff rate.

Using the pdf $p\left(|z_k|^2\right)$ of $|z_k|^2$ given by

$$p\left(|z_k|^2\right) = \frac{1}{2\sigma_1^2}\exp\left(-|z_k|^2/2\sigma_1^2\right) \tag{19}$$

$D(\Delta\phi_k, \Delta\phi'_k, \lambda)$ becomes that which is shown by (20) at the bottom of the page where $\varepsilon = \lambda\sigma_1\sigma_2$.

C. Numerical Calculations

If the transmission symbol rate is smaller than the coherence bandwidth of the fading channel and no cochannel interference arrives at the receiver, only AWGN and random FM noise cause errors. For nonfrequency selective and nontime selective fading (very slow flat fading), the error cause is cochannel interference when the received SNR is sufficiently large. If both the received average SNR and SIR are sufficiently large and fading is nontime selective, the error cause is the ISI from the adjacent symbols due to the fading frequency selectivity. Therefore, R_0 characteristics can be better understood by independently considering the effects of each error cause on R_0.

1) AWGN and Random FM Noise: The effects of only AWGN and random FM noise on R_0 are considered, and thus the averaging processes with respect to the adjacent and cochannel interference symbol patterns are not necessary. The autocorrelation ρ_k is obtained by letting $\Lambda \to \infty$ and $\xi_s(\tau, \mu) = \xi_s(\mu)\delta(\tau)$, and becomes

$$\rho_k e^{-j\Delta\Phi_k} = \rho_c + j\rho_s = \xi_s(T)\frac{\Gamma}{\Gamma+1}. \tag{21}$$

If the fading complex envelope has a symmetric power spectrum, the autocorrelation $\xi_s(T)$ has a real value, and $\Pr(\Delta\phi_k \to \Delta\phi'_k) = \Pr(\Delta\phi'_k \to \Delta\phi_k)$ (this channel is referred to as a "symmetric channel"). In this case, D is given by

$$\begin{aligned} D &= \frac{1}{M(M-1)}\sum_{\Delta\phi_k}\sum_{\Delta\phi'_k \neq \Delta\phi_k}\min_{\lambda \geq 0} D(\Delta\phi_k, \Delta\phi'_k, \lambda) \\ &= \frac{1}{M(M-1)}\sum_{\Delta\phi_k}\sum_{\Delta\phi'_k \neq \Delta\phi_k}\frac{1}{1 + \frac{\rho_c^2(1-\cos\delta\phi_k)}{2\left(1-|\rho_k|^2\right)}}. \end{aligned} \tag{22}$$

Assuming that equal amplitude multipath waves arrive from all directions with equal probability, $\xi_s(\mu) = J_0(2\pi f_D \mu)$ [16], where $J_0(\cdot)$ is the Bessel function and f_D is the maximum Doppler frequency given by vehicle speed/carrier wavelength. Fig. 2 shows the channel cutoff rate R_0 of coded MDPSK (M = 2, 4, 8, 16, and 32) versus average SNR Γ for $f_D T \to 0$ (very slow fading). It is found from Fig. 2 that the cutoff rate approaches $\log_2 M$ bits/symbol as Γ increases. If the required information bit rate is less than, but very close to $\log_2 M$ bits/symbol, coded $(2^{1+\log_2^M})$DPSK is preferable. This is because the reduction in Γ possible with coded $(2^{i+\log_2^M})$DPSK, where $i \geqq 2$, is very small compared to the reduction realized by using $(2^{1+\log_2^M})$DPSK. For example, if the required information bit rate is 0.9 bits/symbol, coded DPSK is preferable. This is because the SNR value for 0.9 bits/symbol information bit rate is about 8 dB for coded MDPSK's with $M \geqq 4$, while it is 15 dB for 2DPSK.

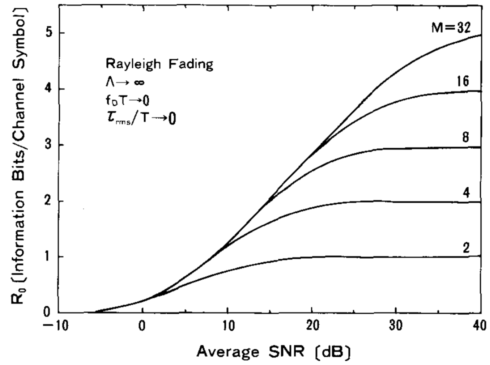

Fig. 2. Channel cutoff rate due to AWGN.

For a given average SNR, the most preferable value of M, that which yields the maximum increase in information bit rate without excessively increasing complexity, can be determined from Fig. 2. For example, coded 8DPSK is the best choice if Γ = 15 dB. This is because the information bit rate achieved by coded 4DPSK is 1.6 bits/symbol while that by 8DPSK is 2.0 bits/symbol. The other MDPSK schemes yield relatively insignificant increases in information bit rate.

If $f_D T$ has a nonzero value, fading is time selective and the channel cutoff rate R_0 for $\Gamma \to \infty$ is less than $\log_2 M$ because of random FM noise. Fig. 3 shows the channel cutoff rate R_0 versus $f_D T$. As $f_D T$ increases, R_0 decreases. The $f_D T$ value at which R_0 begins to deviate from $\log_2 M$ decreases as M increases. This is because the noise margin is decreased at large M values. Therefore, MDPSK transmission with large M values becomes more sensitive to random FM noise.

2) Cochannel Interference: To analyze the cochannel interference effect on R_0, we assume $f_D T \to 0$, $\Gamma \to \infty$, and that fading is not frequency selective. If the cochannel interference symbol pattern is fixed, the autocorrelation ρ_k is obtained by letting $\Gamma \to \infty$ and $\xi_i(\tau, \mu) = \delta(\tau)$. We note that $d_i(t)$ is identical to $d_s(t)$ except that the transmitted phase sequence is replaced by the interference phase sequence. Unfortunately, the imaginary part of ρ_k is not zero and $\Pr(\Delta\phi_k \to \Delta\phi'_k)$ is not always identical to $\Pr(\Delta\phi'_k \to \Delta\phi_k)$, i.e., the channel is unsymmetric.

For such unsymmetric channels, derivation of the optimal Λ in (10) is analytically difficult. Here, we assume that for each transmitted symbol $\Delta\phi_k$, the worst pattern of cochannel

$$D(\Delta\phi_k, \Delta\phi'_k, \lambda) = E\,\frac{1}{1 + 2\varepsilon\left\{\rho_c(1-\cos\delta\phi_k) + \rho_s\sin\delta\phi_k\right\} - 2\varepsilon^2\left(1-|\rho_k|^2\right)(1-\cos\delta\phi_k)} \tag{20}$$

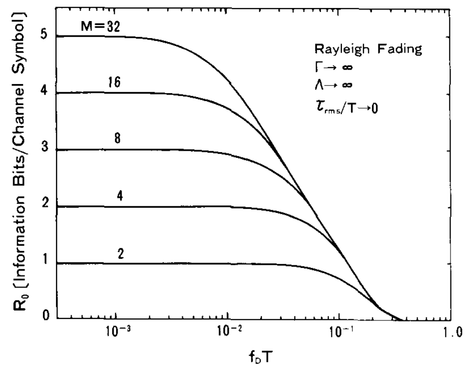

Fig. 3. Channel cutoff rate due to random FM noise.

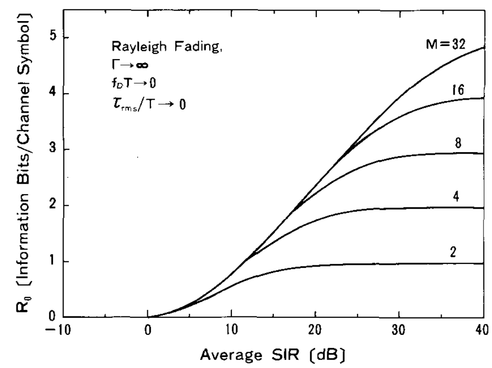

Fig. 4. Channel cutoff rate due to cochannel interference.

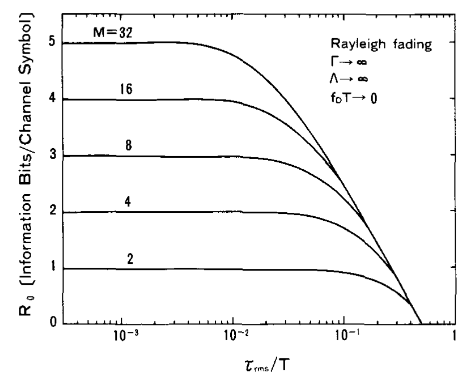

Fig. 5. Channel cutoff rate due to delay spread.

interference is always received. This assumption gives a lower bound of the channel cutoff rate R_0, and is reasonable since in a fading environment, errors are mainly caused when the desired signal fades and the modulating symbol difference is not important. Hence, we assume for calculation simplicity that the cochannel interference modulation timing is synchronized to the desire signal modulation timing, so that the ensemble average over the distribution of ΔT is not necessary.

For the cochannel interference with worst symbol pattern, ρ_k becomes

$$\rho_k e^{-j\Delta\Phi_k} = \frac{\Lambda - 1}{\Lambda + 1}. \tag{23}$$

Apparently, $\rho_s = 0$ and the channel is symmetric if the fading complex envelope has a symmetric power spectrum. Thus for the worst cochannel interference, the D value is obtained by substituting (23) into (22).

Fig. 4 shows the lower bound of the channel cutoff rate R_0 versus average SIR Λ. The cutoff rate approaches $\log_2 M$ bits/symbol as average SIR increases, as in Fig. 2. It is found by comparing Fig. 4 to Fig. 2 that for a certain value of R_0, the required Λ value is 3 dB larger than the required Γ value. This is because we assume the worst case cochannel interference.

3) Delay Spread: If $\Gamma \to \infty$, $\Lambda \to \infty$, and $f_D T \to 0$, the main error cause is the ISI from the adjacent symbols on each side due to the fading frequency selectivity. As in the cochannel interference environment, the channel is not symmetric. Thus assuming that for each symbol $\Delta\phi_k$ considered, the worst patter adjacent symbols on both sides are always transmitted, we can derive the lower bound of R_0. With this assumption, $\Pr(\Delta\phi_k \to \Delta\phi'_k) = \Pr(\Delta\phi'_k \to \Delta\phi_k)$ and the D value is given by

$$D = \frac{1}{M(M-1)} \cdot \sum_{\Delta\phi_k} \sum_{\Delta\phi'_k \neq \Delta\phi_k} \frac{1}{1 + \frac{\{\rho_c(1-\cos\delta\phi_k)+\rho_s \sin\delta\phi_k\}^2}{2(1-|\rho_k|^2)(1-\cos\delta\phi_k)}}. \tag{24}$$

Substituting (24) into (9) yields a lower bound of the channel cutoff rate R_0.

For the transmitted phase difference $\Delta\phi_k = \phi_k - \phi_{k-1}$, the worst adjacent symbols on each side are $\phi_{k-u} = \pi + \phi_{k-1}$ $(u = 2, 3, \cdots)$ and $\phi_{k+v} = \pi + \phi_k$ $(v = 1, 2, \cdots)$. Assuming that the rms delay spread is much smaller than $2T$, the ISI from two adjacent symbols on each side are considered. Fig. 5 shows the lower bound of the channel cutoff rate R_0 versus rms delay spread $\tau_{\rm rms}/T$ normalized by the symbol duration T. A double spike delay profile and roll-off factor of $\alpha = 0.5$ were assumed. As $\tau_{\rm rms}/T$ increases, R_0 decreases. The $\tau_{\rm rms}/T$ values at which the R_0 begins to deviate from $\log_2 M$ becomes smaller as M becomes larger. This is because of the margin decrease due to the increased M value.

To achieve a required information bit rate, the most preferable value of M in frequency selective fading can be determined from Fig. 5. For example, if 1.5 information bits/symbol transmission is required, coded 8DPSK is most reasonable. This is because for coded 4DPSK, $\tau_{\rm rms}/T = 0.15$ is tolerable, and the value is increased to 0.2 by 8DPSK (increases with other MDPSK schemes with $M \geqq 16$ are very slight).

If other values of α are used for the roll-off factor, different ISI effects appear. Fig. 6 shows R_0 versus α with $\tau_{\rm rms}/T$ as a parameter. It is found from this figure that R_0 is insensitive to α when $\tau_{\rm rms}/T = 0.01$. For $\tau_{\rm rms}/T = 0.1$, a comparatively larger R_0 is achieved by larger α.

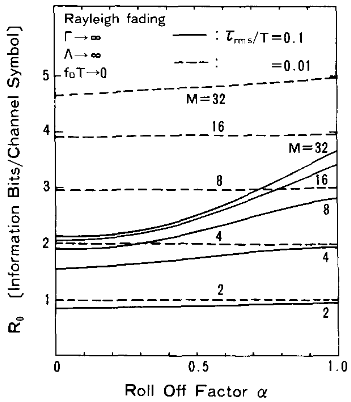

Fig. 6. Channel cutoff rate versus roll off factor.

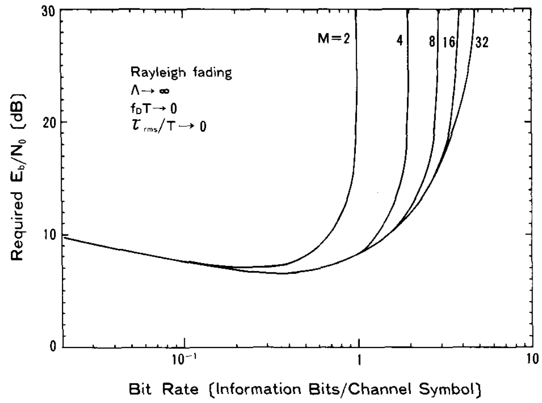

Fig. 7. Required E_b/N_0 versus information bit rate.

IV. Optimal Code Rates

This section determines, for practical, reliable communication, the optimal code rates from the derived cutoff rate R_0 to minimize the required average E_b/N_0 and to maximize the tolerable channel delay spread under frequency selective fading.

A. AWGN

For a given value of R information bits/channel symbol, the average E_b/N_0 is given by

$$\text{Average } E_b/N_0 = \frac{\Gamma}{R}. \tag{25}$$

The channel cutoff rate R_0 is a function of the average SNR Γ if fading is not frequency selective and no cochannel interference arrives at the receiver. Therefore, practical, reliable communication is possible if

$$\text{Average } E_b/N_0 \geqq \frac{R_0^{-1}(R)}{R}. \tag{26}$$

Fig. 7 shows the required average E_b/N_0 versus information bit rate R for very slow fading. It is found from this figure that there exists an optimum R which minimizes the required average E_b/N_0. This is because a higher R value requires larger average SNR for reliable communications. On the other hand, a lower R increases the bandwidth, which increases the required average E_b/N_0. The optimum R is found to be around 0.25 information bits/channel symbol for 2DPSK, and is around 0.4 for other MDPSK schemes. The minimum value of required E_b/N_0 is slightly larger with 2DPSK than with other MDPSK schemes.

B. Delay Spread

The lower the rate $r = R/\log_2 M$ of codes for error correction, the higher the channel symbol rate *for a given information bit rate* $1/T_b$, and the transmission performance becomes more sensitive to the fading frequency selectivity. On the contrary, as code rate r increases, the error correction capability decreases. Therefore, there exists an optimum code rate which maximizes the tolerable rms delay spread.

If $\Gamma \to \infty$, $\Lambda \to \infty$ and fading is very slow, the channel cutoff rate R_0 is a function of τ_{rms}/T. Since

$$R = T/T_b \tag{27}$$

practical, reliable communication is possible if

$$\frac{\tau_{\mathrm{rms}}}{T_b} \leqq R \times R_0^{-1}(R). \tag{28}$$

Fig. 8 shows τ_{rms}/T_b versus code rate r for frequency selective Rayleigh fading with a double spike delay profile. It is obvious that a larger τ_{rms}/T_b value can be tolerated with larger M values. The optimal code rate for the 32DPSK is around 0.3 (1.5 information bits/symbol), and the maximum τ_{rms}/T_b value is 1.5. For other MDPSK schemes with $M \leqq 16$, the optimum r value tends to be larger than that for 32DPSK, and its range tends to be broader.

V. Spectrum Efficiency

The spectrum efficiency η of a cellular system is defined as the product of three factors: time, frequency, and space [17]. *For a given channel bandwidth* (variable information bit rate), the spectrum efficiency η of coded MDPSK with R information bits/channel symbol can be represented as

$$\eta \propto \frac{R}{\left[1 + (\kappa\Lambda_{\mathrm{th}})^{1/A}\right]^2} \tag{29}$$

where A is the propagation constant (a typical value is $A = 3.5$) [18], κ is the fading margin for the allowable probability Q of geographical outage at the cell fringe due to shadow fading, and Λ_{th} is the average SIR necessary to achieve the specific BER after decoding when $E_s/N_0 \to \infty$. Here, we have assumed a hexagonal cell layout. Both the desired and cochannel interference signals suffer from independent log-normal shadowing with identical standard deviation δ (in decibels). Thus κ can be obtained from

$$Q = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{\kappa}{2\delta}\right\} \tag{30}$$

where erfc(·) is the complementary error function.

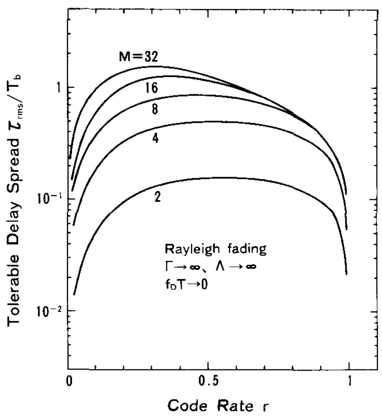

Fig. 8. Tolerable delay spread versus code rate.

For $\Gamma \rightarrow \infty$ and nonfrequency selective and nontime selective fading, R_0 is a function of average SIR Λ, and thus practical, reliable communication is possible if

$$\Lambda \leqq R_0^{-1}(R). \tag{31}$$

If the service area of the cellular system is defined as the area in which, with R bits-per-symbol transmission, practical, reliable communication is possible, then η can be evaluated by using $R_0^{-1}(R)$ instead of Λ_{th} in (29). Fig. 9 shows the calculated relative spectrum efficiency η (nomalized by that for 1 bit-per-symbol coded 4DPSK) versus information bit rate R for $Q = 1\%$, $A = 3.5$, and $\delta = 6$ dB. It is found from this figure that the optimum information bit rates which maximize the spectrum efficiency are around 0.5 bits/symbol for 2DPSK, 1 bit/symbol for 4DPSK, and 1.4 bits/symbol for other MDPSK schemes with $M \geqq 8$. The maximum spectrum efficiency values with $M \geqq 8$ remain over broader ranges of the information bit rate than with 2 and 4DPSK. This implies that for a given channel bandwidth, a higher information bit rate is possible at the cost of a slight degradation in spectrum efficiency. On the contrary, *for a given information bit rate* (variable channel symbol rate), η becomes

$$\eta \propto \frac{R \times \log_2 M}{\left[1 + \left\{\kappa R_0^{-1}(R)\right\}^{1/K}\right]^2}. \tag{32}$$

Fig. 10 shows η (relative value) versus code rate $r = R/\log_2 M$. It is obvious that for a given information bit rate, a greater spectrum efficiency is achieved with larger values of M. The optimal code rate for 32DPSK is around 0.3 (1.5 information bits/symbol), and the maximal spectrum efficiency is 2.6 times as large as that for 1 bit-per-symbol coded 4DPSK. For other MDPSK schemes with $M \geqq 16$, the optimal r value tends to be larger than that for 32DPSK, and its range tends to be broader.

VI. Conclusion

Performance limits of coded MDPSK in frequency selective Rayleigh fading channels have been described. The channel cutoff rate of MDPSK was analyzed based on the "Gaussian metric"; AWGN, cochannel interference and multipath channel delay spread were taken into account. For practical, reliable communications over cellular mobile radio systems employing coded MDPSK, the three optimal information bit rates that achieve 1) minimum required average signal energy per information bit-to-noise power spectral density ratio (E_b/N_0), 2) maximum tolerable rms delay spread τ_{rms}, and 3) maximum spectrum efficiency were determined from the channel cutoff rate. It has been shown that the optimal information bit rate (= information bits /MDPSK symbol) which minimizes the required average E_b/N_0 is around 0.25 information bits/symbol for 2DPSK, and 0.4 bits/symbol for other MDPSK schemes with $M \geqq 4$. For a constant information bit rate ($= 1/T_b$), a larger τ_{rms}/T_b value can be tolerated with larger values of M. It has been shown that the optimal code rate for 32DPSK is around 0.3 (1.5 information bits/symbol), and the maximum τ_{rms}/T_b value is 1.5.

In the analysis of the spectrum efficiency, the service area was defined as the area in which practical, reliable communications are possible. It has been shown that for a given channel bandwidth, the spectrum efficiencies are maximized when the information bit rate is around 0.5 information bits/symbol for 2DPSK, 1 bit/symbol for 4DPSK, and 1.4 bits/symbol for other MDPSK schemes. For a given information bit rate,

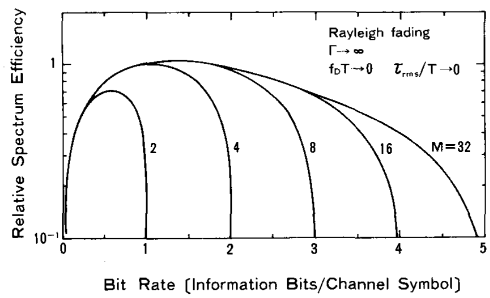

Fig. 9. Relative spectrum efficiency versus information bit rate (constant channel bandwidth).

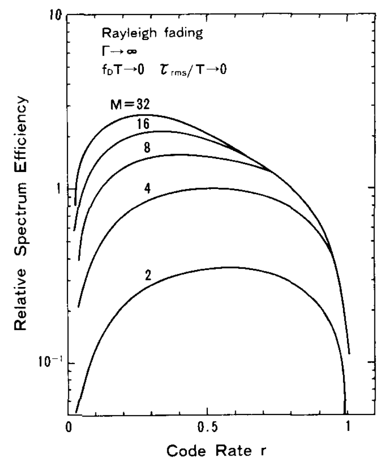

Fig. 10. Relative spectrum efficiency versus code rate (constant information bit rate).

spectrum efficiency is increased with larger M values. The optimal code rate for 32DPSK is around 0.3, and the maximal spectrum efficiency is 2.6 times as large as that for 1 bit-per-symbol coded 4DPSK.

It has been assumed throughout this paper that interleaving degree is large enough to randomize the burst errors produced by fading. However, this assumption is not always reasonable because the delay time caused by interleaving must be within an acceptable limit and, therefore, burst errors may not be fully randomized. Slow frequency-hopping with coded DPSK [19] may solve this problem. Another assumption used in this paper was computationally achievable decoder complexity. Sequential decoding may be used in the decoder of the trellis code if its Viterbi decoder complexity becomes unreasonable. These practical investigation topics are left for further study.

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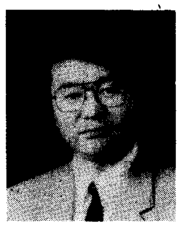

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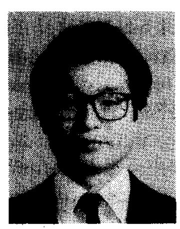

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