A Satisfactory-Oriented Approach to Multiexpert Decision-Making With Linguistic Assessments

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***Abstract*—This paper proposes a multiexpert decision-making (MEDM) method with linguistic assessments, making use of the notion of random preferences and a so-called *satisfactory principle*. It is well known that decision-making problems that manage preferences from different experts follow a common resolution scheme composed of two phases: an *aggregation phase* that combines the individual preferences to obtain a collective preference value for each alternative; and an *exploitation phase* that orders the collective preferences according to a given criterion, to select the best alternative/s. For our method, instead of using an aggregation operator to obtain a collective preference value, a random preference is defined for each alternative in the aggregation phase. Then, based on a *satisfactory principle* defined in this paper, that says that it is perfectly satisfactory to select an alternative as the best if its performance is as at least "good" as all the others under the same evaluation scheme, we propose a linguistic choice function to establish a rank ordering among the alternatives. Moreover, we also discuss how this linguistic decision rule can be applied to the MEDM problem in multigranular linguistic contexts. Two application examples taken from the literature are used to illuminate the proposed techniques.**

***Index Terms*—Decision making, linguistic hierarchies, linguistic variables, multigranular linguistic contexts, randomly linguistic preferences, satisfactory principle.**

I. Introduction

THE mathematical model of fuzzy concepts was first introduced in [55] by using the notion of partial degrees of membership, in connection with the automatic representation and manipulation of human knowledge. Since then, mathematical foundations as well as successful applications of fuzzy set theory have been developed. In particular, the application of fuzzy set theory to decision-making problems when only qualitative or uncertain information is available has been the subject of much research over the last decades, e.g., [6], [30], [31], [39], [45], [46], [50], and many others (see, e.g., [17] and [40] for a recent review).

In practice, there are many decision situations in which the information cannot be assessed precisely in a quantitative form but may be in a qualitative one, and thus, the use of a *linguistic approach* is necessary [17]. For example, in multiexpert decision-making (MEDM) situations, experts' judgements including preferences are often vaguely qualitative and cannot be estimated by exact numerical values. Therefore, a more realistic approach may be to use linguistic assessments instead of numerical values by means of linguistic variables [11], [21], [23], [25], [29], that is, variables whose values are not numbers but words or sentences in a natural or artificial language. Each linguistic value is characterized by a syntactic value or label and a semantic value or meaning. The label is a word or sentence belonging to a linguistic term set and the meaning is a fuzzy subset in a universe of discourse [56]–[58].

In linguistic decision analysis, a solution scheme must comply with the following three steps [17].

1) *Choice of the linguistic term set:* Basically, one has to choose the granularity of the linguistic term set, its labels, and their associated semantics.
2) *Choice of the aggregation operator for linguistic information:* It consists of establishing an appropriate aggregation operator for aggregating and combining the provided linguistic preference values.
3) *Choice of the best alternatives, carried out in two phases:*
    a) *Aggregation phase:* Obtaining collective linguistic preferences on the alternatives by aggregating the individual linguistic preferences by means of the chosen aggregation operator.
    b) *Exploitation phase:* Establishing a rank ordering among the alternatives according to the collective linguistic preferences for choosing the best one(s).

Essentially, the first two steps serve the aggregation phase in the third step, while the exploitation phase is determined depending on the choice of the semantic description of the linguistic term set. Roughly speaking, if the linguistic term set is semantically represented, for example, by the space of parameterized fuzzy numbers, many methods for the total ordering of fuzzy numbers that have been suggested in the literature can be used in the exploitation phase. When the semantics of the linguistic term set is based on a predefined ordered structure, techniques of linguistic approximation are necessary [9], [45]. More importantly, irrespective of the membership function based semantics or ordered structure based semantics of the linguistic term set, one has to face the problem of weighted aggregation of linguistic information. The issue of weighted aggregation has been studied extensively in, e.g., [4], [10], [12], [22]–[24], [48], [51], and [52]. Again, the linguistic aggregation process is determined depending on the semantic description of the linguistic term set. While several authors perform direct computation on a finite and totally ordered term set, the others use the membership function representation to aggregate linguistic values based

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on the extension principle [56]–[58]. As mentioned in [16], in both approaches the results usually do not exactly match any of the initial linguistic terms, so a process of linguistic approximation must be applied. This process causes loss of information and hence a lack of precision.

In this paper, we focus on the MEDM problem with linguistic information. Usually, a group decision environment is characterized by a finite set of experts (actors or decision makers) who are called to express their preference values on a predefined set of alternatives (or options). The MEDM problem is then to first aggregate preferences individually expressed to obtain collective preferences, and second, rank the alternatives in order to select the best one(s). Conventionally, there are several techniques used to linguistically evaluate the alternatives based, for example, on the specification of linguistic preference relations or linguistic assessments. This paper assumes the information is given in the form of linguistic assessments [16], [18]. To avoid the limitation mentioned above, we propose a probability-based approach with the computation solely based on the order-based semantics of the linguistic terms. It is worth noting that by performing direct computation on linguistic terms in the proposed approach, the burden of quantifying a qualitative concept is eliminated. Furthermore, as illustrated by application examples, the results yielded by this method are comparable to previous work.

The main contributions of this paper are as follows: First, we propose a new linguistic decision rule for MEDM problems which is based on a probability-based interpretation of weights and a so-called satisfactory principle (described in Section III and followed in Section IV by an experimental/comparative study). Second, we introduce a formal notion of linguistic hierarchies in terms of ordered structure-based semantics of the linguistic term sets and simultaneously present a method of transformation of a linguistic hierarchy defined in the sense of [18] to that defined in the sense of this paper. Then we show how the proposed approach can be applied to MEDM problems defined in multigranular linguistic contexts. As such, in a sense, the proposed approach can be considered as a possible extension of the proposal developed in [18] for MEDM with multigranular linguistic contexts. However, it should also be mentioned that, while the multigranular hierarchial linguistic approach with two-tuple linguistic representation in [18] results in a linguistic evaluation at the end of the decision process, which consequently, allows us to consider different aggregation schemes and different selection models, the approach based on the satisfactory principle in this paper introduces a real-valued choice function that induces a ranking order among alternatives but not a linguistic evaluation.

The paper is organized as follows. Section II begins with a brief review of descriptions of the linguistic term set in linguistic decision analysis and follows by presenting a general scheme of MEDM problems. Section III introduces a new MEDM method resulted in a satisfactory-oriented linguistic decision rule and Section IV applies the proposed method to an MEDM problem defined over the same linguistic term set. In Section V, after introducing the notion of a linguistic hierarchy, we describe how this method can be applied to solve an MEDM problem defined in a multigranular linguistic context. Finally, Section VI presents some concluding remarks.

II. Preliminaries

In this section, we first briefly recall different approaches to description of the linguistic term set with its associated semantics in linguistic decision analysis (a comprehensive overview on this given in [17]). Then we shall reformulate a general scheme for MEDM problems with linguistic information (see also [18]).

A. Description of the Linguistic Term Set in Linguistic Decision Analysis

In practice, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language instead of numerical values. This arises for different reasons [6]. First, the information may be unquantifiable due to its nature, and can be stated only in linguistic terms (e.g., when evaluating the "comfort" or "design" of a car [35], terms like "good," "medium," "bad" would be used). In other cases, precise quantitative information may not be stated because either it is unavailable or the cost of its computation is too high, so an "approximate value" may be tolerated (e.g., when evaluating the speed of a car, linguistic terms like "fast," "very fast," "slow" may be used instead of numerical values). In such situations, a linguistic approach is necessary and helpful. By scanning the literature, one can find an extensive application of linguistic approaches to many different areas of decision analysis, including group decision-making [2], [20]–[22], [27], [29]–[31], multicriteria decision-making (MCDM) [3], [5], [44], [53], consensus [14], [25], [26], software development [8], [34], [49], subjective assessment of car evaluation [35], material selection [7], personel management [28], environmental assessment [15], etc.

In any linguistic approach to solving a problem, the term set of a linguistic variable and its associated semantics must be defined first to supply the users with an instrument by which they can naturally express their information. In accomplishing this objective, an important aspect to analyze is the granularity of uncertainty, i.e., the level of discrimination among different countings of uncertainty or, in the other words, the cardinality of the linguistic term set used to express the information. As mentioned in [2], the cardinality of the term set must be small enough so as not to impose useless precision on the users, and it must be rich enough in order to allow a discrimination of the assessments in a limited number of degrees.

Syntactically, there are two main approaches to generating a linguistic term set. The first one is based on a context-free grammar [1], [56]–[58]. This approach may yield an infinite term set. A similar approach is to consider primary linguistic terms (e.g., *high, low*) as generators, and linguistic hedges (e.g., *very, rather, more, or less*) as unary operations. Then the linguistic term set can be obtained algebraically [37], [38]. However, according to observations in [36], the generated language does not have to be infinite, and in practice human beings can reasonably manage to keep about seven terms in mind. A second approach is to directly supply a finite term set and consider all terms as primary ones, distributed on a scale on which a total order is defined [2], [10], [16], [18], [21], [22], [53], [54]. For instance, a set of seven terms S could be given as follows:

$$S = \{s_0 = none, s_1 = very\ low, s_2 = low, s_3 = medium, \\ s_4 = high, s_5 = very\ high, s_6 = perfect\}$$

in which $s_i < s_j$ if and only if $i < j$.

For the semantic aspect, once the mechanism of generating a linguistic term set has been determined, its associated semantics must be defined accordingly. In the literature, there are three main possibilities for defining the semantics of the linguistic term set: *semantics based on membership functions and a semantic rule, semantics based on the ordered structure of the term set*, and *mixed semantics*. Usually, the first semantic approach is used when the term set is generated by means of a generative grammar. This approach consists of two elements: 1) the primary fuzzy sets designed as associated semantics of the primary linguistic terms and 2) a semantic rule M for generating fuzzy sets semantically associated with nonprimary linguistic terms from primary fuzzy sets. Often, while the primary terms are labels of primary fuzzy sets which are defined subjectively and context-dependently, the semantic rule M defines linguistic hedges and connectives as mathematical operations on fuzzy sets aimed at modifying the meaning of linguistic terms applied. The second semantic approach is based on a finite linguistic term set accompanied with an ordered structure which intuitively represents the semantical order of linguistic terms. Further, these linguistic terms are assumed to distribute on a scale (e.g., [0, 1]) either symmetrically or nonsymmetrically depending on a particular situation. The third semantic approach is a mixed representation of the previous two approaches, that is, an ordered structure of the primary linguistic terms and a fuzzy set representation of linguistic terms (see [17] for more details). In this paper we adopt the ordered structure based semantics of the linguistic term set.

B. General Scheme of MEDM Problems

There are various formulations of fuzzy MEDM problems in the literature. However, a common characteristic of these problems is a finite set of experts, denoted by $E = \{e_1, \dots, e_p\}$, who are asked to assess another finite set of alternatives (or candidates) $A = \{a_1, \dots, a_n\}$. The general scheme of MEDM problems considered in this paper follows [18], as shown in Table I, where linguistic assessments x_{ij} can be given either in the same linguistic term set or in different linguistic term sets of a linguistic hierarchy.

From the literature on linguistic decision analysis, one can find that there are two general decision models: the first model is mainly based on an aggregation-and-ranking scheme, and the second is based on consensus-reaching oriented solution schemes. The approach proposed in this paper could be considered as following the first general model.

III. Satisfactory-Oriented Linguistic Decision Rule

In this section, we shall propose a linguistic decision rule based on a *satisfactory principle* and a probability-based approach. To this end, we assume a subjective probability distribution P_E defined on the set of experts E. This assumption essentially underlies the calculating basis for the proposed choice function. Motivations for the assumption of such a probability distribution are as follows.

From a practical point of view, given a set of alternatives A, if there is an ideal expert, say e_I, whose evaluation of alternatives the decision maker (DM) completely believes in, then it is enough for the DM to use e_I's assessments to rank alternatives and select the best one(s). However, in practice this is not generally the case. Thus, numerous experts are called to express their preference values on the alternatives, on the one hand, to collect enough information for the problem from various points of view, and, on the other hand, to reduce the subjectivity of the decision. In this sense, $P_E(e_j)$, for each $j \in \{1, \dots, p\}$, may be interpreted as the probability that DM randomly selects expert e_j from the population E as a sufficient source of information for the decision-making purpose. Such a probability distribution may come from DM's knowledge of the experts. Lacking any such knowledge, a uniform distribution would be assumed. It is of interest to note that in a different but similar context, such a probability distribution is also assumed in the voting model for linguistic modeling [32], [33].

TABLE I
General MEDM Problem

Alternatives	Experts			
	e_1	e_2	$\cdots$	e_p
a_1	x_{11}	x_{12}	$\cdots$	x_{1p}
a_2	x_{21}	x_{22}	$\cdots$	x_{2p}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_n	x_{n1}	x_{n2}	$\cdots$	x_{np}

From a theoretical point of view, in traditional decision analysis, MEDM and MCDM methods often involve a measure μ on 2^E (E plays the role of criteria in MCDM) that must be a *capacity* on E [13], i.e., $\mu : 2^E \to [0,1]$ such that $\mu(\emptyset) = 0$, $\mu(E) = 1$, and $\mu(C) \le \mu(D)$ for any $C \subseteq D$. Important subclasses of capacities are probability measures (i.e., additive capacities), belief functions, possibility and necessity measures. Although in the following discussion we only deal with the case of a probability distribution assumed on E, other capacities such as possibility or necessity measures would be interesting to consider and this is left for further research.

In the tradition of linguistic decision analysis, a weighting vector

$$W = [w_1, \dots, w_p]$$

is also often associated with E such that $w_j \in [0,1]$ and $\sum_j w_j = 1$. Collective preference values for the alternatives may then be obtained via a linguistic weighted aggregation operation, for example [17], of the form

$$X_i = \bigoplus_{j=1}^{p} w_j \odot x_{ij} \tag{1}$$

where $\oplus$ and $\odot$ are, respectively, a weighted aggregation operation and a product operation of a number by a label (or its fuzzy set based semantics). Formally, (1) can be seen as a linguistic counterpart of expected utilities in decision-making under uncertainty [41], where the set of experts plays the role of states of the world, and then the weights play the role of subjective probabilities assigned to the states.

Let us return to the general MEDM problem with a probability function P_E defined on E. Assume that

$$L = \{s_0, \dots, s_g\}$$

is the linguistic term set accompanied with the ordered structure such that $s_k < s_h$ iff $k < h$, and $x_{ij} \in L$.

Under such a formulation, the problem induces n random preferences, denoted by $X_1, \dots, X_n$, each X_i for an alternative a_i with associated probability distribution P_i defined by

$$P_i(X_i = s) = P_E(\{e_j \in E | x_{ij} = s\}) \tag{2}$$

for $i = 1, \dots, n$ and $s \in L$.

Quite importantly, as mentioned in [2], the procedure of asking each expert to linguistically evaluate each alternative in terms of its performance adopts an absolute evaluation and is based on the assumption that the alternatives are independent. Therefore, if we view the collective preference values of alternatives as random preferences X_i, $i = 1, \dots, n$, we have for each i, X_i which is stochastically independent of all the others. This assumption allows us to easily compute the probabilities of comparisons of two independent probability distributions of the two random preferences. That is, we can work out the probability that one of the associated random preferences is *less than* or *equal to* the other. More particularly, for any X_i, X_j such that $i \neq j$, we have

$$P(X_i \succeq X_j) = \sum_{s \in L} P_i(X_i = s) P(s \succeq X_j) \tag{3}$$

where $P(s \succeq X_j)$ is the cumulative probability function defined by

$$P(s \succeq X_j) = \sum_{\substack{x \in L \\ s \geq x}} P_j(X_j = x). \tag{4}$$

The quantity $P(X_i \succeq X_j)$ could be interpreted as the probability of "the performance of a_i is as at least good as that of a_j" under the evaluation scheme $\langle E, P_E \rangle$. Intuitively, *it is perfectly satisfactory to select an alternative as the best if its performance is as at least good as all the others under the same evaluation scheme*. We have called this the *satisfactory principle*.

Now we are ready, based on the satisfactory principle, to propose a choice function defined as follows:

$$V(a_i) = \sum_{j \neq i} P(X_i \succeq X_j) \tag{5a}$$

$$= \sum_{j \neq i} \sum_{s \in L} \left[P_i(X_i = s) \sum_{\substack{x \in L \\ s \geq x}} P_j(X_j = x) \right]. \tag{5b}$$

Then the satisfactory-oriented linguistic decision model for the MEDM problem is defined by

$$a_{best} = \arg\max_{a_i \in A} \{V(a_i)\}. \tag{6}$$

In the following section, we shall illustrate how this model works in practice by an application taken from [16].

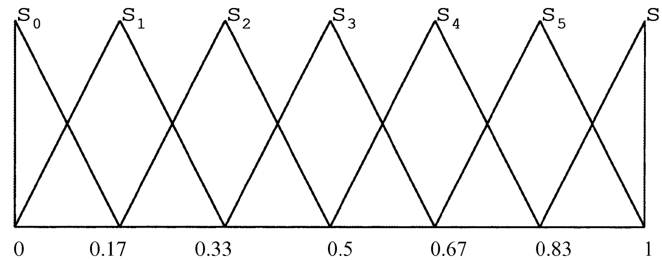

Fig. 1. Linguistic term set.

TABLE II
MEDM PROBLEM IN UPGRADING COMPUTING RESOURCES

Options	Experts			
	p_1	p_2	p_3	p_4
a_1	s_1	s_3	s_4	s_4
a_2	s_3	s_2	s_1	s_4
a_3	s_3	s_1	s_3	s_2
a_4	s_2	s_4	s_3	s_2

IV. MEDM PROBLEM DEFINED OVER A COMMON LINGUISTIC TERM SET

A. Problem Description

A distribution company needs to upgrade its computing system, so it hires a consulting company to survey the different possibilities existing on the market, to decide which is the best option for its needs. The options (alternatives) are the following:

a_1	a_2	a_3	a_4
UNIX	WINDOWS – NT	AS/400	VMS

The consulting company has a group of four consultancy departments

p_1	p_2	p_3	p_4
Cost analysis	System analysis	Risk analysis	Technology analysis

Each department in the consulting company provides an evaluation vector expressing its assessment of options. These evaluations are assessed in the set S of seven linguistic terms (graphically, shown in Fig. 1)

$$S = \{s_0 = none, s_1 = very\ low, s_2 = low, s_3 = medium, s_4 = high, s_5 = very\ high, s_6 = perfect\}$$

in which $s_i < s_j$ if and only if $i < j$, and are given in Table II.

As usual, the selection model used to solve this problem consists of two steps.

1) Obtain a *collective performance value* for each option.
2) Apply a selection process based on the obtained collective performance vector.

TABLE III
RANDOM PREFERENCES FOR OPTIONS

Random Preferences	L						
	s_0	s_1	s_2	s_3	s_4	s_5	s_6
X_1	0	0.25	0	0.25	0.5	0	0
X_2	0	0.25	0.25	0.25	0.25	0	0
X_3	0	0.25	0.25	0.5	0	0	0
X_4	0	0	0.5	0.25	0.25	0	0

In parallel, the satisfactory-oriented linguistic decision model also uses a two-step scheme but, instead:

1) calculate a value of the choice function for each option by (5);
2) carry out the selection process by the decision rule (6).

B. Satisfactory-Oriented Linguistic Decision Model

In this part, we shall apply the satisfactory-oriented linguistic decision model to solve the above problem. In this example, we assume a uniform distribution $P_E = [0.25, 0.25, 0.25, 0.25]$ defined on $E = \{p_1, p_2, p_3, p_4\}$. The decision model consists of the following three steps:

- First, from the information given to the problem, we obtain random preferences X_i for options a_i, $i = 1$, 2, 3, 4, respectively, as shown in Table III.
- Then, by using (5), we obtain values of the choice function for options a_is as follows:

$V(a_1)$	$V(a_2)$	$V(a_3)$	$V(a_4)$
2.1875	1.75	1.4375	1.9375

which ranks options a_is in the order

$$a_1 \succ a_4 \succ a_2 \succ a_3.$$

- Using the decision rule (6), we obtain as the solution set of options

$$\{a_1\}$$

In other words, the best option for the distribution company when upgrading its computing system is a UNIX-based system.

C. Comparative Study

Here, as a comparative analysis, we shall briefly recall various solutions of the problem solved with existing methods, namely, solutions based on the extension principle, the symbolic approach, and the two-tuple fuzzy linguistic representation model (for more details, see [16]). All these methods are based on an aggregation-and-ranking scheme.

1) Solution Based on the Extension Principle: By this method, a linguistic aggregation operator based on the extension principle acts according to the scheme

$$S^p \xrightarrow{\tilde{F}} F(\mathcal{R}) \xrightarrow{app_1(\cdot)} S$$

where $\tilde{F}$ is an aggregation operation based on the extension principle, $F(\mathcal{R})$ is the set of fuzzy sets over the set of real numbers $\mathcal{R}$, and $app_1(\cdot)$ is a linguistic approximation operation that names each resulted fuzzy set by a linguistic label taken from S.

Assuming a membership function representation of triangular type of linguistic terms (see Fig. 1) and the arithmetic mean as an aggregation operator, we obtain collective performance values for alternatives a_i, for $i = 1, \ldots, n$, by

$$C_i = \left(\frac{1}{p} \sum_{j=1}^{p} a_{ij}, \frac{1}{p} \sum_{j=1}^{p} b_{ij}, \frac{1}{p} \sum_{j=1}^{p} c_{ij} \right)$$

where $x_{ij} = (a_{ij}, b_{ij}, c_{ij})$ (see Table I). Applying this to the problem, we get the collective performance vector in the form of triangular type fuzzy sets as shown at the bottom of the next page. Clearly, these fuzzy sets do not exactly match any linguistic term in S, and therefore, a linguistic approximation process based, for example, on Euclidean distance [16] must be applied. The linguistic approximation process yields the collective performance vector as

$app_1(C_1)$	$app_1(C_2)$	$app_1(C_3)$	$app_1(C_4)$
s_3	s_3	s_2	s_3

Finally, the selection criterion is used to obtain the solution set of options as follows:

$$\{a_1, a_2, a_4\}.$$

Clearly, this method lacks precision and does not yield a good solution—see the equation at the bottom of the page.

2) Solution Based on the Symbolic Approach: This method uses a linguistic aggregation operator based on the symbolic approach according to the scheme

$$S^p \xrightarrow{C} [0, g] \xrightarrow{app_2(\cdot)} \{0, \ldots, g\} \longrightarrow S$$

where C is, for example, the convex combination of linguistic terms defined in [12], and $app_2(\cdot)$ is, for example, the rounding operator which approximates an index in $[0, g]$ by one in $\{0, \ldots, g\}$ associated to a term in S. With the weighting vector

C_1	C_2	C_3	C_4
$(0.33, 0.5, 0.66)$	$(0.25, 0.42, 0.58)$	$(0.21, 0.38, 0.54)$	$0.3, 0.45, 0.625$

being $W = [0.25, 0.25, 0.25, 0.25]$, this method applied to the above problem yields the collective performance vector as

C_1	C_2	C_3	C_4
s_3	s_3	s_2	s_3

Again, the solution set of options is

$$\{a_1, a_2, a_4\}$$

which also lacks precision and, therefore, is not a good solution.

3) Solution Based on the Two-Tuple Fuzzy Linguistic Representation Model: To avoid the loss of information caused by approximate computational models, the two-tuple fuzzy linguistic representation model has been proposed in [16]. In this model, information is represented by means of two-tuples of the form (s, α), where $s \in S$ and $\alpha \in [-0.5, 0.5)$, i.e., linguistic information is encoded in the space $S \times [-0.5, 0.5)$. Under such a representation, if a value $\beta \in [0, g]$ representing the result of a linguistic aggregation operation, then the two-tuple that expresses the equivalent information to β is obtained by means of the following transformation:

$$\begin{aligned} \Delta: \ \ [0,g] &\longrightarrow S \times [-0.5, 0.5) \\ \beta &\longmapsto (s_i, \alpha) \end{aligned}$$

with

$$\begin{cases} i = \text{round}(\beta) \\ \alpha = \beta - i \end{cases}$$

and, inversely, a two-tuple $(s_i, \alpha) \in S \times [-0.5, 0.5)$ can be equivalently represented by a numerical value in $[0, g]$ by means of the following transformation:

$$\begin{aligned} \Delta^{-1}: \ \ S \times [-0.5, 0.5) &\longrightarrow [0, g] \\ (s_i, \alpha) &\longmapsto \Delta^{-1}(s_i, \alpha) = i + \alpha. \end{aligned}$$

Furthermore, traditionally numerical aggregation operators have been also extended for dealing with linguistic two-tuples in [16]. For example, let $x = \{(r_1, \alpha_1), \dots, (r_n, \alpha_n)\}$ be a set of linguistic two-tuples, the two-tuple arithmetic mean $\overline{x}^e$ is computed as

$$\overline{x}^e = \Delta\left(\sum_{i=1}^{n} \frac{1}{n}\Delta^{-1}(r_i, \alpha_i)\right).$$

The comparison of linguistic information represented by two-tuples is defined as follows. Let (s_i, α_1) and (s_j, α_2) be two two-tuples, then:

- if $i < j$, then (s_i, α_1) is less than (s_j, α_2);
- if $i = j$, then:
  1) if $\alpha_1 = \alpha_2$ then (s_i, α_1) and (s_j, α_2) represent the same information;
  2) if $\alpha_1 < \alpha_2$ then (s_i, α_1) is less than (s_j, α_2);
  3) if $\alpha_1 > \alpha_2$ then (s_i, α_1) is greater than (s_j, α_2).

Let us apply this model to the above problem. Representing performance values x_{ij} given in Table II in the form of two-tuples as $(x_{ij}, 0)$ respectively and using the two-tuple arithmetic mean, we obtain the collective performance values $\overline{x}_i^e$, i =1,2,3,4, for options a_i, respectively, as

$\overline{x}_1^e$	$\overline{x}_2^e$	$\overline{x}_3^e$	$\overline{x}_4^e$
$(s_3, 0)$	$(s_3, -0.5)$	$(s_2, 0.25)$	$(s_3, -0.25)$

which also ranks options a_is in the order

$$a_1 \succ a_4 \succ a_2 \succ a_3.$$

Using the selection criterion, we get as the solution set of options

$$\{a_1\}.$$

As desired, this solution coincides with that obtained by our proposed method. It is worth noting here that as the proposed approach is solely based on the ordered structure-based semantics of the linguistic term set, it is quite natural in terms of interpretation.

V. MEDM Problem Defined Over a Linguistic Hierarchy

In this section, we discuss how the satisfactory-oriented linguistic decision rule could be applied to the MEDM problem in multigranular linguistic contexts. Before doing so, it is necessary to introduce the notion of a linguistic hierarchy in terms of ordered structure based semantics of the linguistic term sets.

A. Linguistic Hierarchies

Linguistic hierarchies arise quite naturally in problems for which one needs to deal with multiple sources of linguistic information. For example, in the context of linguistic decision analysis, a linguistic hierarchy can be used when linguistic assessments are assessed in linguistic term sets with different granularity of uncertainty and/or semantics [18].

A linguistic hierarchy of a linguistic variable X, denoted by $\mathcal{T}_X$, is a hierarchical tree consisting of a finite number of levels labeled as $t_0, t_1, \dots, t_m$, which is defined as follows:

- Level t_0 is the root of the tree labeled by X—the name of the linguistic variable.
- Each level t_i, for $i = 1, \dots, m$, is a finitely linguistic term set of X, denoted by L_i, accompanied with a total order such that:
  i) $|L_i| < |L_{i+1}|$, for any $i = 1, \dots, m-1$;
  ii) for each $i = 1 \dots, m-1$, there exists only a mapping $\Gamma_i : L_i \to 2^{L_{i+1}} - \{\emptyset\}$ fulfilling $\Gamma_i(s_j^i) \cap \Gamma_i(s_{j'}^i) = \emptyset$ for any $s_j^i \neq s_{j'}^i$, $\bigcup_{s_j^i \in L_i} \Gamma_i(s_j^i) = L_{i+1}$;
  iii) if $s_j^i < s_{j'}^i$ in L_i then $s_k^{i+1} < s_{k'}^{i+1}$ in L_{i+1} for any $s_k^{i+1} \in \Gamma_i(s_j^i)$ and $s_{k'}^{i+1} \in \Gamma_i(s_{j'}^i)$.

Let us denote $|L_i| = n(i)$ and

$$L_i = \left\{s_0^i, s_1^i, \dots, s_{n(i)-1}^i\right\}$$

for $i = 1, \dots, m$, and $s_j^i < s_{j'}^i$ iff $j < j'$.

For example, consider a linguistic variable *assessment* with its hierarchical tree depicted as in Fig. 2 [19]. It is then easy to

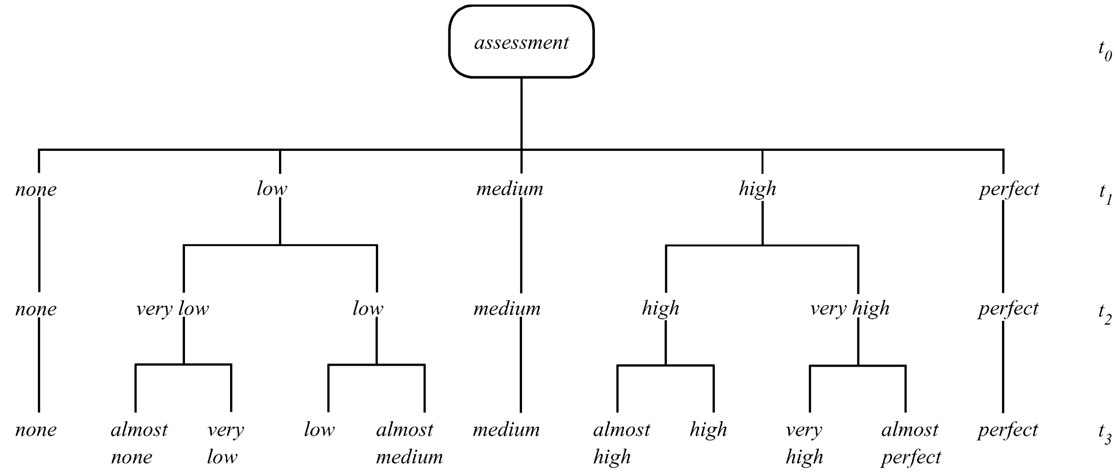

Fig. 2. Linguistic hierarchy $\mathcal{T}_{assessment}$.

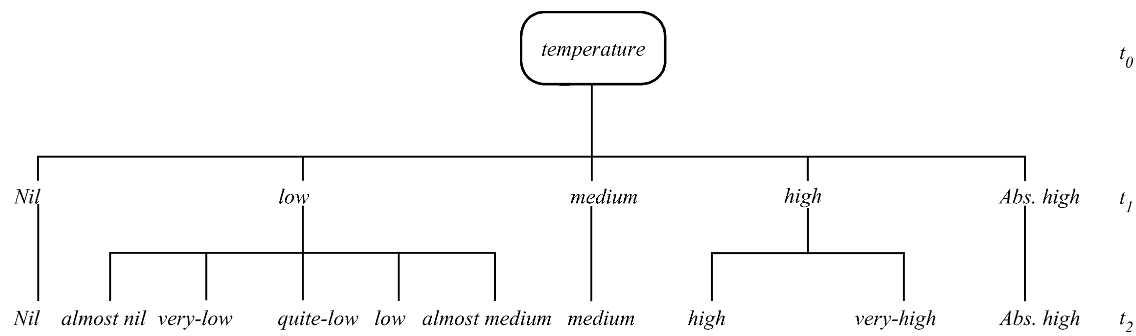

Fig. 3. Linguistic hierarchy $\mathcal{T}_{temperature}$.

see that the hierarchical tree of linguistic terms of *assessment* satisfies the conditions of a linguistic hierarchy defined above.

Intuitively, for each $i = 1, \ldots, m-1$, the mapping Γ_i plays a role as a semantic derivation from the term set L_i to its refinement L_{i+1}. That is, for $s_j^i \in L_i$, the terms in $\Gamma_i(s_j^i)$ have meanings derived from s_j^i to serve the purpose of representing qualitative information in a more assessable format. Further, for each mapping $\Gamma_i, i = 1, \ldots, m-1$, there exists a pseudoinversion $\Gamma_i^- : L_{i+1} \to L_i$ defined by

$$\Gamma_i^-\left(s_j^{i+1}\right) = s_j^i \text{ such that } s_j^{i+1} \in \Gamma_i\left(s_j^i\right).$$

In the following, we will use Γ_i and Γ_i^- to define transformations between levels of the linguistic hierarchy. Obviously, if the two-tuple fuzzy linguistic representation model defined in [16] is used, one can utilize transformation functions between levels of the hierarchy proposed in [18] for the normalization process. However, this is not considered in this paper.

In [18], the authors define a linguistic hierarchy associated with a fuzzy set representation of linguistic term sets. Clearly, without a fuzzy set representation, the above notion of a linguistic hierarchy is semantically consistent with that defined in [18], except that the condition on granularity between two consecutive levels, which says that

$$n(i+1) = 2 \cdot n(i) - 1, \quad \text{for } i = 1, \ldots, m-1$$

is not assumed. By relaxing this assumption together with the semantic derivation mapping Γ_i, we can also construct linguistic hierarchies that capture the ordered structure based semantics of nonsymmetrically distributed term sets [47]. Such linguistic term sets underlie the assumption that a subdomain of the reference domain may be more informative that the rest of the domain. For example, suppose that we require more precise behavior when the temperature is *low* in a temperature control system; then the density of linguistic labels in subdomain of *low temperature* would be greater than the density in the subdomain of *high temperature* (see Fig. 3 for an example of such a linguistic hierarchy).

To apply the proposed approach to the MEDM problem in a multigranular linguistic context with a linguistic hierarchy, LH, defined in the sense of [18], we now show how to obtain the corresponding hierarchical structure of term sets, denoted by $\mathcal{T}_{LH}$, as defined above from LH. Assume that

$$LH = \bigcup_{i=1}^{m} L\left(t_i, n(i)\right)$$

where $L(t_i, n(i)) = \{s_0^{n(i)}, \ldots, s_{n(i)-1}^{n(i)}\}$ and $n(i+1) = 2 \cdot n(i) - 1$. In addition, as defined in [18], the linguistic term sets $L(t_i, n(i))$ have an odd value of granularity representing the central label, namely $s_{(n(i)-1)/2}^{n(i)}$, the value of *indifference*. We then define the hierarchical tree $\mathcal{T}_{LH}$ in terms of ordered structure based semantics of the linguistic term sets derived from LH as follows.

- For each $i = 1, \ldots, m$, the linguistic term set L_i at level t_i of $\mathcal{T}_{LH}$ is defined by $L_i = L(t_i, n(i))$.

- For each $i = 1 \ldots, m-1$, we define the mapping $\Gamma_i : L_i \rightarrow 2^{L_{i+1}} - \{\emptyset\}$ by

$$\Gamma_i\left(s_j^{n(i)}\right) = \begin{cases} \left\{s_{2j}^{n(i+1)}, s_{2j+1}^{n(i+1)}\right\}, & \text{if } j < \frac{n(i)-1}{2} \\ \left\{s_{\frac{n(i+1)-1}{2}}^{n(i+1)}\right\}, & \text{if } j = \frac{n(i)-1}{2} \\ \left\{s_{2j-1}^{n(i+1)}, s_{2j}^{n(i+1)}\right\}, & \text{if } j > \frac{n(i)-1}{2}. \end{cases}$$

It is easily seen that the hierarchy $\mathcal{T}_{LH}$ of the linguistic term sets L_i fulfills conditions of a hierarchical structure of linguistic term sets as defined above.

B. Problem Description

In multigranular linguistic contexts, the experts may have in mind different granularities of linguistic assessments drawn from a linguistic hierarchy. Therefore, their assessments may be represented in different term sets of the hierarchy. Formally, the problem is stated as follows:

- $A = \{a_1, \ldots, a_n\}$ is the set of alternatives;
- $E = \{e_1, \ldots, e_p\}$ is the set of experts;
- P_E is a probability distribution on E;
- M is a $n \times p$ matrix of preference values as shown in Table I; each column $[x_{1j}, \ldots, x_{nj}]^T$ describes linguistic assessments of expert e_j on the alternatives in which linguistic values x_{ij}, for $i = 1, \ldots, n$, are drawn from the linguistic term set $L_{k(j)}$ of a level $t_{k(j)} (k(j) \geq 1)$ in a certain linguistic hierarchy $\mathcal{T}_X$.

C. General Solution Scheme

The satisfactory-oriented approach to solving this general problem consists of the following three steps.

i) Unify the multigranular linguistic information into that represented in a uniform linguistic term set and then obtain random preferences for the alternatives.
ii) Use (5) to calculate values of the choice function for the alternatives.
iii) Make a selection based on the decision rule (6).

Once the first step in the solution scheme has been carried out, the last two steps are straightforward by making use of the choice function (5) and the decision rule (6), respectively. The first step consists of the following two processes: a normalization that unifies the multigranular linguistic information provided and a derivation of random preferences for the alternatives for decision making.

1) Linguistic Information Normalization: First we must select a linguistic term set, say L_{k^*}, from $\mathcal{T}_X$ to unify the multigranular linguistic information. As mentioned above, we can use mappings Γ_k, Γ_k^- to define transformations between levels of the linguistic hierarchy as follows.

Let L_k and L_{k+a}, $a \in \mathbb{N}^+$, be the term sets of levels t_k and t_{k+a}, respectively. Then linguistic information represented in L_k is transformed into L_{k+a} by means of the following mapping:

$$\begin{aligned} \Phi_{k+a}^k : \ & L_k \longrightarrow 2^{L_{k+a}} \\ & s_i^k \longmapsto \Phi_{k+a}^k\left(s_i^k\right) \end{aligned}$$

TABLE IV
UNIFIED PREFERENCE MATRIX

Alternatives	Experts			
	e_1	e_2	$\cdots$	e_p
a_1	$\Phi_{k^*}^{k(1)}(x_{11})$	$\Phi_{k^*}^{k(2)}(x_{12})$	$\cdots$	$\Phi_{k^*}^{k(p)}(x_{1p})$
a_2	$\Phi_{k^*}^{k(1)}(x_{21})$	$\Phi_{k^*}^{k(2)}(x_{22})$	$\cdots$	$\Phi_{k^*}^{k(p)}(x_{2p})$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_n	$\Phi_{k^*}^{k(1)}(x_{n1})$	$\Phi_{k^*}^{k(2)}(x_{n2})$	$\cdots$	$\Phi_{k^*}^{k(p)}(x_{np})$

where $\Phi_{k+a}^k(s_i^k)$ is recursively defined by

$$\Phi_{k+a}^k\left(s_i^k\right) = \bigcup_{s_j^{k+a-1} \in \Phi_{k+a-1}^k\left(s_i^k\right)} \Gamma_{k+a-1}\left(s_j^{k+a-1}\right).$$

For example, consider two term sets L_1 and L_3 from the linguistic hierarchy $\mathcal{T}_{assessment}$ in Fig. 2, we obtain

$$\begin{aligned} \Phi_3^1(none) &= \{none\} \\ \Phi_3^1(low) &= \{al.\ none, very\ low, low, al.\ medium\} \\ \Phi_3^1(medium) &= \{medium\} \\ \Phi_3^1(high) &= \{al.\ high, high, very\ high, al.\ perfect\} \\ \Phi_3^1(perfect) &= \{perfect\}. \end{aligned}$$

Intuitively, the transformation Φ_{k+a}^k originates from the practical observation that while expert e_j designed $L_{k(j)}$ as the most appropriate term set to linguistically express information according to his view, if he is provided another term set with finer granularity $L_{k(j)+a}$ for representing information, he may be hesitant to distinguish among terms in $L_{k(j)+a}$ that are semantically derived from the same term in $L_{k(j)}$. Thus, instead of using $s_i^{k(j)} \in L_{k(j)}$, he may use $\Phi_{k+a}^k(s_i^{k(j)})$ to equivalently express the information in $L_{k(j)+a}$.

Similarly, making use of pseudo-inversion Γ_k^-, the linguistic information represented in L_{k+a} can be transformed into L_k by means of the following mapping:

$$\begin{aligned} \Phi_k^{k+a} : \ & L_{k+a} \longrightarrow L_k \\ & s_i^{k+a} \longmapsto \Gamma_k^- \circ \ldots \circ \Gamma_{k+a-1}^-\left(s_i^{k+a}\right) \end{aligned}$$

where $\circ$ denotes the composition. This transformation can be intuitively interpreted as follows. While expert e_j designed $L_{k(j)+a}$ as the most appropriate term set to linguistically express information; i.e., he can distinguishably use terms in $L_{k(j)+a}$. If he is provided another term set with coarser granularity $L_{k(j)}$ for representing information, so that instead of using terms $s_i^{k(j)+a}$ in $L_{k(j)+a}$, he is only able to use terms $\Phi_k^{k+a}(s_i^{k(j)+a})$ in $L_{k(j)}$ to express the information and, consequently, in some cases he may have to accept a loss of information.

To avoid the loss of information during the normalization process, we choose the common linguistic term set, L_{k^*}, that has the highest granularity among those used by experts; namely

$$k^* = \max_j k(j).$$

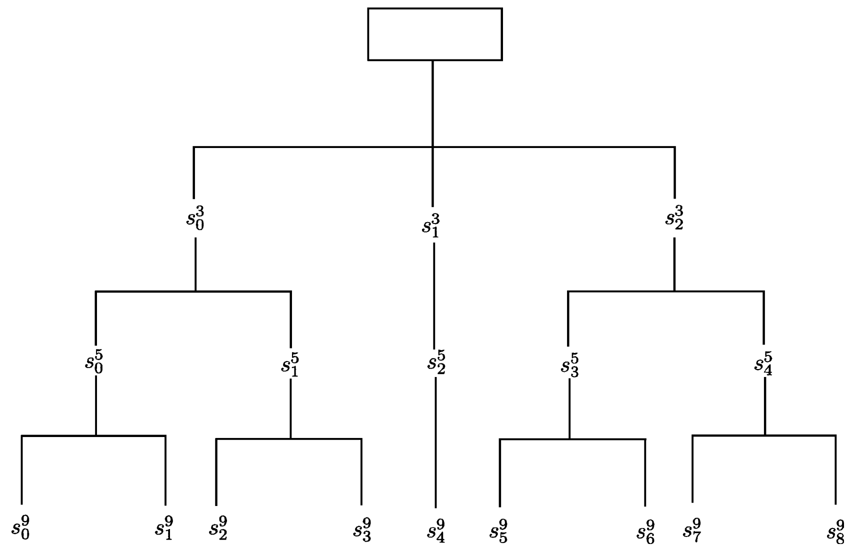

Fig. 4. Derived linguistic hierarchy $\mathcal{T}_{LH}$.

Then the linguistic information represented in $L_{k(j)}$ by experts e_j is unified into L_{k^*} by means of transformations $\Phi_{k^*}^{k(j)}$ to obtain the unified preference matrix as shown in Table IV.[1]

2) Derivation of Random Preferences: In general, the unified preference matrix obtained from the normalization process does not directly induce random preferences X_i for alternatives a_i, $i = 1, \ldots, n$, as defined in (2), but random set preferences Ξ_i $(i = 1, \ldots, n)$, each for the respective alternative, defined as follows:

$$P_i(\Xi_i = S) = P_E\left(\left\{e_j \in E | \Phi_{k^*}^{k(j)}(x_{ij}) = S\right\}\right) \qquad (7)$$

for $i = 1, \ldots, n$ and $S \in 2^{L_{k^*}}$. Formally, for each $i = 1, \ldots, n$, the probability distribution P_i of Ξ_i is nothing but a *basic probability assignment* in the sense of Shafer [42].

Now, in order to build probability distributions of the random preferences X_i's needed for decision making from these random set preferences, Ξ_is, we can fortunately use the so-called *pignistic transformation* [43] to obtain the *least prejudiced distributions*[2] of X_is for the alternatives. This is done, with an abuse of notation, as follows:

$$P_i(X_i = s) = \sum_{s \in S \subseteq L_{k^*}} \frac{P_i(\Xi_i = S)}{|S|} \qquad (8)$$

for $i = 1, \ldots, n$ and $s \in L_{k^*}$.

For the sake of illustration and comparison, in the following we shall solve the MEDM problem defined in a linguistic hierarchy taken from [18]. In this application, similar to [18], we have chosen as multigranular linguistic context the linguistic hierarchy LH whose term sets are

$$\begin{aligned} L_1 &= L(1,3) = \left\{s_0^3, s_1^3, s_2^3\right\} \\ L_2 &= L(2,5) = \left\{s_0^5, s_1^5, s_2^5, s_3^5, s_4^5\right\} \\ L_3 &= L(3,9) = \left\{s_0^9, s_1^9, s_2^9, s_3^9, s_4^9, s_5^9, s_6^9, s_7^9, s_8^9\right\}. \end{aligned}$$

[1]Note that $\Phi_{k^*}^{k(j)}$ is the identical transformation if $k(j) = k^*$.

[2]The term *pignistic probability distribution* has been used in [43] in the context of belief modeling. Here, we borrow the terminology from [32], which we think is more appropriate for our context.

According to the transformation method specified above on how to obtain the corresponding hierarchical tree derived from LH in terms of ordered structure based semantics of the linguistic term sets, we get $\mathcal{T}_{LH}$ as depicted in Fig. 4.

D. Application Example

1) Problem Description: Assume that an investment company wants to invest an amount of money in a business enterprize. There are four possible investment alternatives:

- a_1 is a car manufacturer;
- a_2 is a computer company;
- $a_3$is a food company;
- a_4 is a weapon manufacturer.

The investment company has a group of four consultancy departments:

- e_1 is the risk analysis department;
- e_2 is the growth analysis department;
- e_3 is the social-political analysis department;
- e_4 is the environmental impact analysis department.

Each department is directed by an expert who is in charge of linguistically providing assessments on the alternatives according to his own view. These experts provide their preferences, over the set of alternatives, drawn from the different term sets of the linguistic hierarchy. More particularly:

- e_1 provides his preferences in L_3;
- e_2 provides his preferences in L_2;
- e_4 provides his preferences in L_1;
- e_4 provides his preferences in L_3;

The linguistic information given by these experts is shown in Table V.

2) Decision Model: The decision model for solving the problem uses the following three-step procedure.

i) *Normalization and Transformation*: First we choose, as discussed above, the term set L_3 as the common term set for unifying the provided linguistic information. This results in the unified linguistic preference matrix shown in Table VI.

TABLE V
MULTIGRANULAR LINGUISTIC MEDM PROBLEM

Alternatives	Experts			
	e_1	e_2	e_3	e_4
a_1	s_4^9	s_3^5	s_1^3	s_4^9
a_2	s_6^9	s_4^5	s_2^3	s_5^9
a_3	s_3^9	s_3^5	s_2^3	s_3^9
a_4	s_5^9	s_3^5	s_1^3	s_5^9

TABLE VI
UNIFIED LINGUISTIC PREFERENCE MATRIX

Alternatives	Experts			
	e_1	e_2	e_3	e_4
a_1	s_4^9	$\{s_5^9, s_6^9\}$	$\{s_4^9\}$	s_4^9
a_2	s_6^9	$\{s_7^9, s_8^9\}$	$\{s_5^9, s_6^9, s_7^9, s_8^9\}$	s_5^9
a_3	s_3^9	$\{s_5^9, s_6^9\}$	$\{s_5^9, s_6^9, s_7^9, s_8^9\}$	s_3^9
a_4	s_5^9	$\{s_5^9, s_6^9\}$	$\{s_4^9\}$	s_5^9

As assumed in [18], all the experts have the same importance in the decision process, therefore we define a uniform distribution $P_E = [0.25, 0.25, 0.25, 0.25]$ on $E = \{e_1, e_2, e_3, e_4\}$. Then the problem with unified linguistic information induces random set preferences Ξ_i $(i = 1, \dots, 4)$, for the respective alternatives, defined by (7). Using (8), we obtain the least prejudiced distributions of the random preferences X_i's needed for decision making as shown in Table VII and in the equation at the bottom of the page.

ii) *Aggregation*: From the derived random preferences, making use of (5) we obtain the values of the choice function for the alternatives as

$V(a_1)$	$V(a_2)$	$V(a_3)$	$V(a_4)$
1.109 375	2.773 437 5	1.242 187 5	1.796 875

which ranks alternatives a_i's in the order

$$a_2 \succ a_4 \succ a_3 \succ a_1.$$

iii) *Exploitation*: Finally, according to decision rule (6), we obtain the solution as

$$\{a_2\}$$

i.e., the best investment option is the computer company.

To show how consistency in the results of different term sets used for the normalization process and how a loss of information could happen, in the following we provide the unified linguistic preference matrix in other term sets of the linguistic hierarchy, as well as the corresponding solutions obtained.

- **In** L_1: The unified linguistic preference matrix is

	e_1	e_2	e_3	e_4
a_1	s_1^3	s_2^3	s_1^3	s_1^3
a_2	s_2^3	s_2^3	s_2^3	s_2^3
a_3	s_0^3	s_2^3	s_2^3	s_0^3
a_4	s_2^3	s_2^3	s_1^3	s_2^3

Values of the choice function of alternatives a_i's are

$V(a_1)$	$V(a_2)$	$V(a_3)$	$V(a_4)$
1.3125	3	1.5	2.5625

and the ranking order is

$$a_2 \succ a_4 \succ a_3 \succ a_1.$$

- **In** L_2: The unified linguistic preference matrix is

	e_1	e_2	e_3	e_4
a_1	s_2^5	s_3^5	s_2^5	s_2^5
a_2	s_3^5	s_4^5	$\{s_3^5, s_4^5\}$	s_3^5
a_3	s_1^5	s_3^5	$\{s_3^5, s_4^5\}$	s_1^5
a_4	s_3^5	s_3^5	s_2^5	s_3^5

Values of the choice function of alternatives a_i's are

$V(a_1)$	$V(a_2)$	$V(a_3)$	$V(a_4)$
1.1875	2.921 875	1.359 375	2.1875

and the ranking order is

$$a_2 \succ a_4 \succ a_3 \succ a_1.$$

Ξ_1	$\{s_4^9\} : 0.75$	$\{s_5^9, s_6^9\} : 0.25$		
Ξ_2	$\{s_5^9\} : 0.25$	$\{s_6^9\} : 0.25$	$\{s_7^9, s_8^9\} : 0.25$	$\{s_5^9, s_6^9, s_7^9, s_8^9\} : 0.25$
Ξ_3	$\{s_3^9\} : 0.5$	$\{s_5^9, s_6^9\} : 0.25$	$\{s_5^9, s_6^9, s_7^9, s_8^9\} : 0.25$	
Ξ_4	$\{s_4^9\} : 0.25$	$\{s_4^9\} : 0.25$	$\{s_5^9, s_6^9\} : 0.25$	

TABLE VII
DERIVED RANDOM PREFERENCES

Random Preferences	L_3								
	s_0^9	s_1^9	s_2^9	s_3^9	s_4^9	s_5^9	s_6^9	s_7^9	s_8^9
X_1	0	0	0	0	0.75	0.125	0.125	0	0
X_2	0	0	0	0	0	0.3125	0.3125	0.1875	0.1875
X_3	0	0	0	0.5	0	0.1875	0.1875	0.0625	0.0625
X_4	0	0	0	0	0.25	0.625	0.125	0	0

TABLE VIII
RESULTS USING THE TWO METHODS

Methods		Collective Values				Ranking Order
		$a_1 : V(a_1)$	$a_2 : V(a_2)$	$a_3 : V(a_3)$	$a_4 : V(a_4)$	
2-Tuples	L_1	$(s_1^3, 0.125)$	$(s_2^3, -0.3125)$	$(s_1^3, 0.25)$	$(s_1^3, 0.25)$	$a_2 \succ \{a_3, a_4\} \succ a_1$
	L_2	$(s_2^5, 0.25)$	$(s_3^5, 0.375)$	$(s_3^5, -0.5)$	$(s_3^5, -0.5)$	$a_2 \succ \{a_3, a_4\} \succ a_1$
	L_3	$(s_5^9, -0.5)$	$(s_7^9, -0.25)$	$(s_5^9, 0)$	$(s_5^9, 0)$	$a_2 \succ \{a_3, a_4\} \succ a_1$
New	L_1	1.31	3	1.5	2.56	$a_2 \succ a_4 \succ a_3 \succ a_1$
	L_2	1.19	2.92	1.36	2.19	$a_2 \succ a_4 \succ a_3 \succ a_1$
	L_3	1.11	2.77	1.24	1.8	$a_2 \succ a_4 \succ a_3 \succ a_1$

From these results, we can see that normalizing linguistic information by means of semantic derivation mappings and their pseudoinversions yields a consistent ranking order among the alternatives. At the same time, from unified linguistic preference matrices we can also see a loss of information during the normalization process. That is, the coarser the linguistic term set used for linguistic information normalization is, the more information can be lost. That is why we have chosen the term set with highest granularity among term sets used by experts for the normalization process.

3) A Comparative Analysis: In the preceding part, we solved an MEDM problem with linguistic assessments in a multigranular linguistic context. A model based on linguistic two-tuples for solving the same problem has been proposed in [18]. As a comparative study, we report the results using two methods, as shown in Table VIII.[3]

From these results, we can see that both methods yield a consistent ranking order among alternatives irrespective of which term set is used for linguistic information normalization. However, although the solutions to the problem are the same for both methods, the ranking order between the alternatives is different. More particularly, while both the alternatives a_4 and a_3 have the same ranking order in case of the two-tuple method, a strict order between these two alternatives, namely a_3 is dominated by a_4, is produced by our proposed method. Let us look at the original linguistic information given by the experts on these alternatives, i.e.,

$$a_3 = \left[s_3^9, s_3^5, s_2^3, s_3^9\right]$$
$$a_4 = \left[s_5^9, s_3^5, s_1^3, s_5^9\right].$$

Roughly, by doing a pairwise comparison, and keeping in mind the assumption that all the experts have the same importance in the decision process [18], it would be more suitable to rank alternative a_4 over a_3. This shows that the proposed method produces a more suitable ranking order among the alternatives for decision making than the two-tuple method.

VI. CONCLUSION

In this paper we have proposed a new model based on the so-called satisfactory-oriented approach for the MEDM problem under linguistic assessments. Basically, this approach is based on the ordered structure based semantics of linguistic term sets. We would like to emphasize that, while fuzzy set based semantics of linguistic terms is often defined subjectively and context-dependently, ordered structure based semantics may be accepted universally. Further, by performing direct computation on linguistic terms in the proposed approach, the burden of quantifying a qualitative concept is eliminated. This is also especially necessary and useful in situations where fuzzy set based semantics are inapplicable due to the nature of the linguistic information (e.g., when evaluating the "comfort" of a car [35], the "intellect" of people, etc.).

[3]Note that there is a mistake in the result of two-tuple method given in [18]—the correction of this is shown in Table VIII.

We have also discussed how to apply the satisfactory-oriented method to the MEDM problem under linguistic assessments in multigranular linguistic contexts. The same application examples applied have shown that this method gives better and comparable results with known results using previous methods (i.e., the extension principle, symbolic, and linguistic two-tuple methods).

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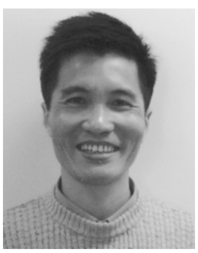

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