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A Petri-Net-Based Model for the Mathematical Analysis of Multi-Agent Systems

Kunihiko HIRAISHI[†a)], ***Regular Member***

SUMMARY Agent technology is widely recognized as a new paradigm for the design of concurrent software and systems. The aim of this paper is to give a mathematical foundation for the design and the analysis of multi-agent systems by means of a Petri-net-based model. The proposed model, called PN^2, is based on place/transition nets (P/T nets), which is one of the simplest classes of Petri nets. The main difference of PN^2's from P/T nets is that each token, representing an agent, is also a P/T net. PN^2's are sufficiently simple for the mathematical analysis, such as invariant analysis, but have enough modeling power.

key words: *Petri nets, multi-agent systems, object orientation, agent orientation, mobile agents*

1. Introduction

Agent technology is widely recognized as a new paradigm for the design of concurrent software and systems. The aim of this paper is to give a mathematical foundation for the design and the analysis of multi-agent systems by means of a Petri-net-based model.

Petri nets are a well-known model for concurrent and distributed systems, and there have been many results on the theory, and also on practical applications. To represent an agent as a token in Petri nets, it is necessary to introduce notion of *objects*, i.e., a collection of data types with methods that give access to the data, into attributes of tokens. There have been various Petri-net-based models introducing this concept [1]–[8]. Since most of these classes of object Petri nets are based on high-level Petri nets, i.e., they allow arbitrary transformation functions on tokens, they are too complex to be analyzed. These high-level object Petri nets aim to describe real applications in an object-oriented manner, and their research direction is different from that of this paper. We aim to propose a model that is sufficiently simple for the mathematical analysis, but has enough modeling power.

Recently, an elementary class of object Petri nets, called *elementary object systems* (EOS), has been proposed [9] to model systems in an object oriented manner keeping the model as simple as possible. The approach of this paper is similar to that of EOS. The proposed model, called PN^2 ("Petri Nets in a Petri Net"), is based on place/transition nets. Intuitively, a PN^2 is a Petri net such that each token, representing an agent, is also a place/transition net. This feature is essential to represent autonomous agents, i.e., an agent is a single process that can make decisions by itself. Each token describing an agent is available for transitions in the upper-level net when the corresponding transition of the token is enabled, and changes its state by occurrences of its own transitions. Each token can move between places, and can make its copy.

Main features of PN^2's are summarized as follows:

(i) A class of PN^2's, called infinite-sort PN^2's, has the same modeling power as Turing machines.
(ii) PN^2's allow dynamic bindings of transitions. This may cause a combinatorial number of transition bindings, but it is avoidable in the invariant analysis. In addition, more than two agents can synchronize.
(iii) PN^2's are based on *the value semantics* [10], whereas many of object Petri nets are based on *the reference semantics*. In the reference semantics, each net as a token is a reference to the object. Therefore, copying tokens does not mean copying objects. In the value semantics, each net as a token represents an independent instance of objects, and therefore duplication and vanishing of objects are allowed.

2. Preliminaries

A multi-set over a non-empty set S is a function $M : S \to \mathbb{N}$, i.e., for each $s \in S$, $M(s)$ denotes the number of occurrences of s in M. Let $\mathcal{M}_S$ denote the family of all multi-sets over S. For $M \in \mathcal{M}_S$, let $|M| = \sum_{s \in S} M(s)$. We usually denote a multi-set M by a formal sum $\sum_{s \in S} M(s)\#s$. For two multi-sets $M_1, M_2 \in \mathcal{M}_S$, we write $M_1 \subseteq M_2$ if $M_1(s) \leq M_2(s)$ for every $s \in S$. Moreover, the difference $M_2 - M_1$ is defined only when $M_1 \subseteq M_2$, and is a multi-set $\sum_{s \in S}(M_2(s) - M_1(s))\#s$. We simply write s to denote $1\#s$, and write $\emptyset$ to denote the empty multi-set.

A place/transition net (P/T net) is a triple $\pi = (P, T, A)$, where P is a finite set of places, T is a finite set of transitions, and $A : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a function representing arcs. *A marking* of π is a multi-set over P. Let Σ be a finite set of symbols. A P/T

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[†]The author is with School of Information Science, Japan Advanced Institute of Science and Technology, Ishikawa-ken, 923-1292 Japan.
a) E-mail: hira@jaist.ac.jp

net $\pi = (P, T, A)$ with a labeling function $\ell : T \to \Sigma$ is called *a labeled-P/T net* (ℓ–P/T net, for short), and is denoted by π^ℓ. As usual, the domain of the labeling function ℓ is extended to T^*. We can identify an ℓ-P/T net with a P/T net if the labeling function ℓ is an injection. A pair $\mu = (\pi^\ell, m)$ of an ℓ-P/T net and a marking is called *a marked ℓ-P/T net*.

Let $\mu = (\pi^\ell = (P, T, A)^\ell, m)$ be a marked ℓ-P/T net. A transition t is *enabled* in a marking m if for every $p \in P$: $A(p,t) \le m(p)$. An enabled transition can *occur*. An occurrence of a transition t changes the marking to m' such that $m'(p) = m(p) + A(t,p) - A(p,t)$ ($p \in P$). We write $m \xrightarrow{t} m'$ to denote this situation. Moreover, this notation is extended to finite sequences of transitions, i.e., we write $m \xrightarrow{\sigma} m'$ to denote that an occurrence of a sequence of transitions $\sigma \in T^*$ changes the marking from m to m'. We say that a marking m' is *reachable* from a marking m if $m \xrightarrow{\sigma} m'$ for some $\sigma \in T^*$. Let $\mathcal{R}(\mu)$ denote the set of all markings reachable from m. Since marked ℓ-P/T nets will be treated as tokens in the proposed model, we introduce a state transition function δ on the set of all marked ℓ-P/T nets by $\delta(\mu, t) = \mu'$, where $\mu' = (\pi^\ell, m')$. Let PT_Σ denote the class of all marked ℓ-P/T nets with labeling functions to Σ.

3. PN^2's

In this section, we show the definition of the model with some examples.

3.1 Definition

A PN^2 is a 9-tuple $PN^2 = (P, \Sigma, T, E, {}^\bullet\cdot, \cdot^\bullet, \cdot_\Sigma, \cdot_T, s_0)$, where

1. P is a finite set of *places*;
2. Σ is a finite set of *symbols*;
3. T is a finite set of *transitions*;
4. E is a finite set of *transition components*;
5. ${}^\bullet\cdot : E \to \mathcal{M}_P \backslash \{\emptyset\}$;
6. $\cdot^\bullet : E \to \mathcal{M}_P$;
7. $\cdot_\Sigma : E \to \Sigma$;
8. $\cdot_T : E \to T$;
9. s_0 is *the initial configuration*, where a configuration is a mapping $s : P \to \mathcal{M}_{PT_\Sigma}$.

Example 1: Figure 1 is a graphical representation of a $PN^2 = (P, \Sigma, T, E, {}^\bullet\cdot, \cdot^\bullet, \cdot_\Sigma, \cdot_T, s_0)$ defined as follows:

1. $P = \{p_1, p_2, p_3\}$;
2. $\Sigma = \{a, b, c, d\}$;
3. $T = \{t_1, t_2, t_3\}$;
4. $E = \{e_1, e_2, e_3, e_4, e_5\}$;
5. ${}^\bullet e_1 = p_1$, ${}^\bullet e_2 = p_3$, ${}^\bullet e_3 = p_2 + p_3$, ${}^\bullet e_4 = p_2$, ${}^\bullet e_5 = p_2$;
6. $e_1^\bullet = p_2$, $e_2^\bullet = p_2 + p_3$, $e_3^\bullet = p_2 + p_3$, $e_4^\bullet = p_1$, $e_5^\bullet = \emptyset$;
7. $e_{1\Sigma} = a$, $e_{2\Sigma} = c$, $e_{3\Sigma} = d$, $e_{4\Sigma} = b$, $e_{5\Sigma} = c$;
8. $e_{1T} = t_1$, $e_{2T} = t_1$, $e_{3T} = t_2$, $e_{4T} = t_3$; $e_{5T} = t_3$;
9. $s_0 : p_1 \mapsto \mu_1$; $p_2 \mapsto \emptyset$; $p_3 \mapsto \mu_2$, where μ_1 and μ_2 are the marked ℓ-P/T nets shown in the figure.

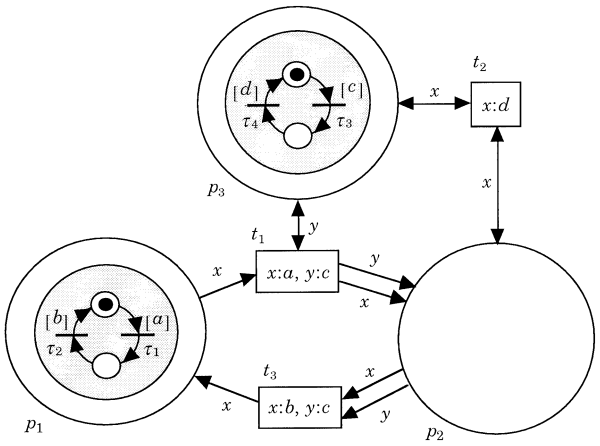

Fig. 1 An example of PN^2's.

Each transition $t \in T$ consists of a set of transition components $E_t := \{e \in E \mid e_T = t\}$. In the graphical representation of PN^2's, we associate labels $x, y, z, \cdots$ with each arc to distinguish transition components, and write $x : a, y : b, \cdots$ to indicate that $x_\Sigma = a$, $y_\Sigma = b$, $\cdots$ for transition components $x, y, \cdots$.

In the definition of PN^2, the 6-tuple $(P, T, E, {}^\bullet\cdot, \cdot^\bullet, \cdot_T)$ defines a P/T net. We call this *the upper-level net*. On the other hand, marked ℓ-P/T nets in the configuration is called *lower-level nets*.

We also define a subclass of PN^2's, called *finite state agent nets* (FSAN's), which will be useful in describing real applications. An FSAN is a PN^2 such that each marked ℓ-P/T net in the lower-level is a deterministic finite automaton (DFA), or a one-safe state machine in Petri net terminology.

3.2 State Transition Rule

Let T_Σ denote the set of all transitions of marked ℓ-P/T nets in PT_Σ. *A transition binding* for a transition $t \in T$ is a pair of functions $b : E_t \to PT_\Sigma$ and $w : E_t \to T_\Sigma$ such that for each $e \in E_t$:

(i) $w(e)$ is a transition of $b(e)$;
(ii) $w(e)$ has a label e_Σ;
(iii) $w(e)$ is enabled in $b(e)$.

A transition binding (b, w) is *enabled* in a configuration s if for each $p \in P$,

$$\sum_{e \in E_t} {}^\bullet e(p) \# b(e) \subseteq s(p).$$

A transition binding can *occur* if it is enabled. An occurrence of a transition binding (b, w) changes the configuration to s' such that for each $p \in P$,

$$s'(p) = s(p) - \sum_{e\in E_t} {}^\bullet e(p)\#b(e) + \sum_{e\in E_t} e^\bullet(p)\#\delta(b(e), w(e)).$$

Since ${}^\bullet e \neq \emptyset$ for every transition component e (see 5. in the definition of PN^2), no new ℓ-P/T nets are introduced by any occuring of transition bindings. Let B_t denote the set of all transition bindings for a transition t and let $B = \cup_{t\in T} B_t$.

We write $s \overset{(b,w)}{\rightarrow} s'$ to denote that an occurrence of a transition binding (b, w) changes the configuration from s to s', and this notation is extended to finite sequences of transition bindings. We say that a configuration s' is *reachable* from a configuration s if $s \overset{\sigma}{\rightarrow} s'$ for some $\sigma \in B^*$. Let $\mathcal{R}(PN^2)$ denote the set of all configurations reachable from the initial configuration.

Example 2: We consider a PN^2 in Fig. 1. Denoting each configuration s by a vector $[s(p_1), s(p_2), s(p_3)]$, the configuration changes as follows.

$$[\mu_1, \emptyset, \mu_2] \overset{(b_1,w_1)}{\rightarrow} [\emptyset, \mu_1' + \mu_2', \mu_2'] \overset{(b_2,w_2)}{\rightarrow} [\emptyset, \mu_1' + \mu_2, \mu_2] \overset{(b_3,w_3)}{\rightarrow} [\mu_1, \emptyset, \mu_2],$$

where $\mu_1' = \delta(\mu_1, \tau_1)$, $\mu_2' = \delta(\mu_2, \tau_3)$, $\mu_1 = \delta(\mu_1', \tau_2)$, $\mu_2 = \delta(\mu_2', \tau_4)$, and (b_i, w_i) $(i = 1, 2, 3)$ are unique transition bindings for transition t_i such that

$$b_1 : e_1 \mapsto \mu_1, e_2 \mapsto \mu_2;\ \ w_1 : e_1 \mapsto \tau_1, e_2 \mapsto \tau_3,$$
$$b_2 : e_3 \mapsto \mu_2';\ \ w_2 : e_3 \mapsto \tau_4,$$
$$b_3 : e_4 \mapsto \mu_1', e_5 \mapsto \mu_2;\ \ w_3 : e_4 \mapsto \tau_2, e_5 \mapsto \tau_3.$$

Example 3: We consider an AGV (Automated Guided Vehicle) system consisting of a graph representing tracks on which two AGV's are moving (Fig. 2). AGV_1 visits A and D alternately, and AGV_2 visits C and F alternately. The track graph is partitioned into three zones in which at most one AGV is allowed to exists at any moment.

We model this system by an FSAN. Figure 3 is the upper-level net representing the track. AGV_1 and AGV_2 are modeled by DFA's shown in Fig. 4, which are initially put on place A and C, respectively. Each controller that supervises a zone is modeled by a DFA in Fig. 5, which is put on place Z_i $(i = 1, \cdots, 4)$. When an AGV moves from one zone to another, it communicates with zone controllers of both zones. The state of each zone controller is changed by this movement.

Behavior of each AGV is determined by interaction with *the environment* consisting of the track graph and other agents. In this system, AGV_1 and AGV_2 will behave as follows:

$$\begin{aligned} AGV_1 :&\ A \rightarrow B \rightarrow (C \text{ or } E) \rightarrow D \rightarrow C \rightarrow B \\ &\rightarrow (A \text{ or } E \rightarrow F \rightarrow A) \rightarrow \cdots, \\ AGV_2 :&\ C \rightarrow B \rightarrow (A \text{ or } E) \rightarrow F \rightarrow A \rightarrow B \\ &\rightarrow (C \text{ or } E \rightarrow D \rightarrow C) \rightarrow \cdots. \end{aligned}$$

Reduction of states in lower-level nets is an interesting problem, and is also practically important. Be-

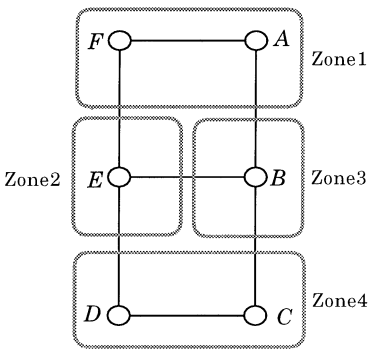

Fig. 2 Track graph and zones.

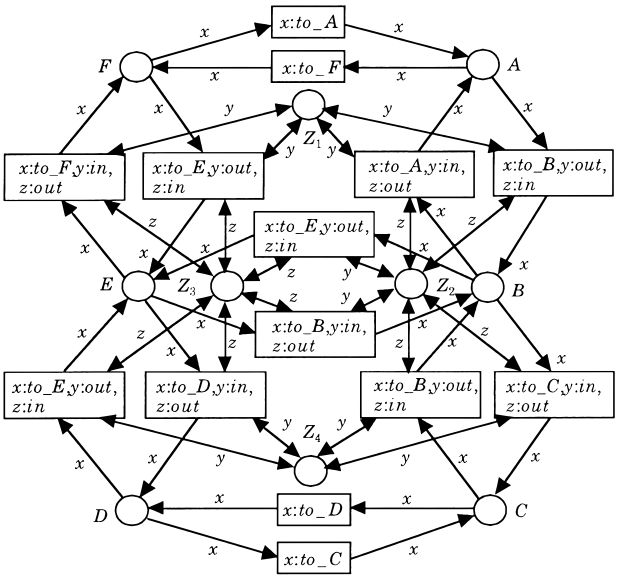

Fig. 3 The upper-level net.

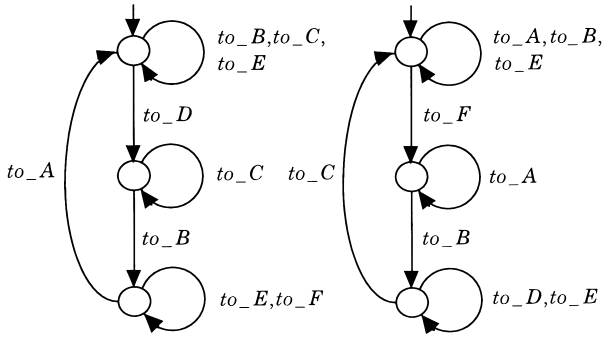

Fig. 4 DFA's representing AGV_1 and AGV_2.

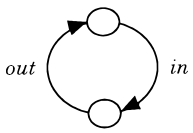

Fig. 5 A DFA for zone control.

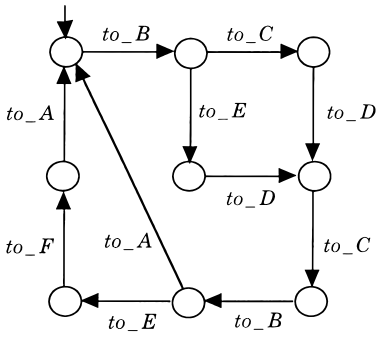

Fig. 6 Another representation of AGV_1.

havior of AGV in Fig. 6 is the same as that in Fig. 4, but it has more states. This problem is closely related to *decentralized supervisory control problem* of discrete event systems [12].

4. Modeling Power of PN^2's

In this section, we show some results on the modeling power of PN^2's.

4.1 Simulating Marked P/T Nets

Marked P/T nets can be considered as a special class of PN^2's by treating each token as a marked ℓ-P/T net having one transition and no places, i.e., it takes only one state and the transition is always enabled. As usual, we draw "•" to denote such a token (e.g., Fig. 9).

4.2 Bounded/Finite-Sort PN^2's

A PN^2 is called *bounded* if there exists a nonnegative integer k such that for every $s \in \mathcal{R}(PN^2)$ and every $p \in P$: $|s(p)| \leq k$, and is called *unbounded* otherwise. A PN^2 is called *finite-sort* if all marked ℓ-P/T nets contained in the initial configuration s_0 are bounded[†], and is called *infinite-sort* otherwise. When a PN^2 is finite-sort, there exists a marked P/T net $\widetilde{PN}^2 = (\widetilde{P}, \widetilde{T}, \widetilde{A}, \widetilde{m}_0)$ that simulates the behavior of the PN^2. We show the construction.

First we define the following sets.

- $\overline{PT_\Sigma}$ is the set of all marked ℓ-P/T net (π^ℓ, m) such that $m \in \mathcal{R}(\mu_0)$ for some $\mu_0 = (\pi^\ell, m_0)$ in the intial configuration.
- $\overline{B_t}$ is the set of all transition binding (b, w) for transition t that is valid, i.e., for all $e \in E_t$: $b(e) \subseteq \overline{PT_\Sigma}$. Let $\overline{B} = \overline{B_t}$.

By the assumption, both $\overline{PT_\Sigma}$ and $\overline{B}$ are finite and effectively computable. Note that in general not all marked ℓ-P/T nets in $\overline{PT_\Sigma}$ appear in reachable configurations.

For each place $p \in P$ and each marked ℓ-P/T net $\mu \in \overline{PT_\Sigma}$, we prepare a place p_μ, and for each transition $t \in T$ and each transition binding $(b, w) \in \overline{B_t}$, we prepare a transition $t_{(b,w)}$, where arcs are connected as follows:

- $\widetilde{A}(p_{b(e)}, t_{(b,w)}) = {}^\bullet e(p)$ and $\widetilde{A}(t_{(b,w)}, p_{\delta(b(e),w(e))}) = e^\bullet(p)$ $(e \in E_t)$;
- Otherwise, $\widetilde{A}(\cdot, t_{(b,w)}) = \widetilde{A}(t_{(b,w)}, \cdot) = 0$.

The initial marking $\widetilde{m}_0$ is defined by $\widetilde{m}_0(p_\mu) = s_0(p)(\mu)$. Then the state transition diagrams of PN^2 and $\widetilde{PN}^2$ are isomorphic to each other.

Note that even if a PN^2 is bounded, it is not always possible to construct a marked P/T net that simulates the behavior of the PN^2. An example will be shown in the next subsection.

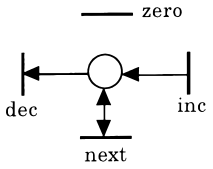

Fig. 7 Construction of a PN^2 representing the register machine (1).

4.3 Infinite-Sort PN^2's

We show that the class of infinite-sort PN^2's has the same level of modeling power as Turing machines. To prove this, *register machines* are used. A register machine is a computer-like machine with a number of registers which are used to store arbitrarily large numbers. A program is written to manipulate the registers. It was proved that a register machine with the following set of instructions is equivalent to Turing machines [11].

1. $P(n)$: increase register n by 1.
2. $D(n)$: decrease register n by 1 (only if register n is nonzero).
3. $j(n)[s]$: jump to statement s if register n is zero.

We now show that the register machine can be converted to an infinite-sort PN^2. We represents n registers used in a program by n places $r_1, \cdots, r_n$. We also use $m + 1$ places $p_0, \cdots, p_m$ to represent the program counter, where m is the number of statements in the program. In the initial configuration, we put a marked ℓ-P/T net shown in Fig. 7 on p_0 and also on each r_i, $i = 1, \cdots, n$. Each instruction in the program is represented by a transition shown in Fig. 8.

The most important part of this construction is the transition from p_i and r_n to p_s. Occurring of this transition are possible only when both nets in p_i and r_n are identical. Transition *zero* in the lower-level net

[†]A marked ℓ-P/T net μ is bounded if there exists a nonnegative integer k such that for every $m \in \mathcal{R}(\mu)$ and every $p \in P$: $m(p) \leq k$.

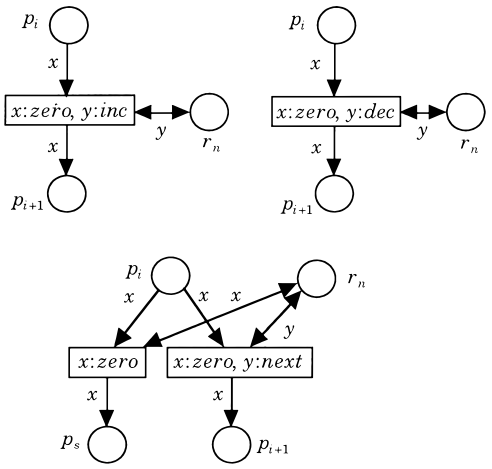

Fig. 8 Construction of a PN^2 representing the register machine (2).

is always enabled because it has no input places, however its firing is restricted by the upper-level net. The marked ℓ-P/T net in place p_i has no tokens, and the marked ℓ-P/T net in place r_n has tokens corresponding to the value of the register. If the value is 0, then both nets are identical and only the left transition is enabled. Otherwise, only the right transition is enabled. This enables *zero testing*.

4.4 Accepting Languages

PN^2's can represent arbitrary large numbers with zero-testing ability. Given a marked ℓ-P/T net $\mu = (\pi^\ell, m_0)$, $L(\mu) = \{\ell(\sigma) \mid \sigma \in T^*,\ \exists m \in F : m_0 \xrightarrow{\sigma} m\}$ is called the language accepted by μ, where F is a finite set of accepting markings.

It was shown that $L_{ww^R} = \{ww^R \mid w \in \Sigma^*\}$ cannot be accepted by any marked ℓ-P/T nets [11]. It is a non-regular, context-free language. The reason is described as follows. Any marked ℓ-P/T net cannot have k^r possible markings after firing r transitions, where k is a constant. This implies that any marked ℓ-P/T net cannot simulate a pushdown stack.

We can show a PN^2 that accepts L_{ww^R} (Fig. 9). Let $\mu(i)$ denote the marked ℓ-P/T net initially put on p_3 with i tokens. The set of accepting markings is $F = \{[\emptyset, \emptyset, \mu(0), \emptyset, 1]\}$. By an input 0010, the configuration changes to

$$p_2 \mapsto \mu(1) + \mu(2) + \mu(4);\ p_3 \mapsto \mu(4);\ p_4 \mapsto \mu(3).$$

It realizes a pushdown stack.

There exists a coloured Petri nets [13] equivalent to the above PN^2 (Fig. 10). Procedures for increment/decrement of the stack pointer are described as arc expressions in the coloured Petri net, while they are represented by transitions of the lower-level nets in the PN^2.

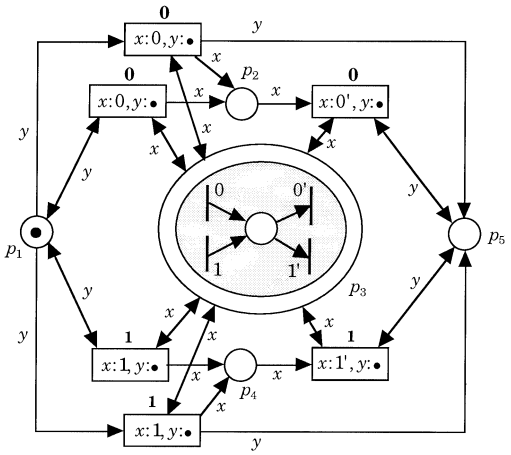

Fig. 9 A PN^2 accepting L_{ww^R}.

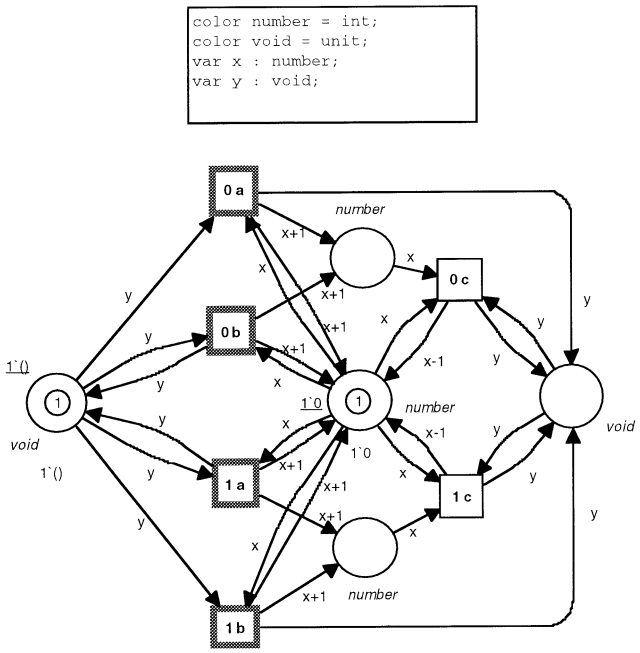

Fig. 10 A CPN accepting L_{ww^R}.

5. State Equation and Invariant Analysis of PN^2's

In this section, we first define state equations of PN^2's, and after that discuss invariant analysis using incidence matrices.

5.1 Injective-PN^2's

A PN^2 is called *injective* if every marked ℓ-P/T net in the intial configuration has an injective labeling function. For any PN^2, we can construct an equivalent injective-PN^2 by

(i) Relabeling of transitions in the lower level nets if more than one transitions have the same label;

(ii) Duplication of transitions in the upper level net corresponding to the relabeling.

Therefore, there are no differences in the modeling power between PN^2's and injective-PN^2's.

In injective-PN^2's, we can omit the second part of transition bindings, therefore a transition binding for a transition t is defined by a function $b : E_t \to PT_\Sigma$ such that for each $e \in E_t$, the unique transition having label e_Σ is enabled in $b(e)$. In what follows, we consider injective-PN^2's only to simplify the discussion. Moreover, we identify a label with the transition having the label.

5.2 State Equation

We show how the state equation of an injective-PN^2 is defined. We first define *firing count vectors*. Suppose that $T = \{t_1, \cdots, t_n\}$ and $P = \{p_1, \cdots, p_m\}$. In what follows, we will use an index i to denote transitions, and j to denote places. In addition, we will write just i (j) in subscripts to indicate the transition t_i (place p_j), e.g., we will write E_i instead of E_{t_i}, and B_i instead of B_{t_i}.

A firing count vector is a $|T|$-dimensional column vector $x = [x_1, \cdots, x_n]^t$, where each x_i is a multi-set over B_i, and it indicates how many times each transition binding to occur. In this formulation, it is necessary to consider all elements in $\overline{B}$, and $|\overline{B}|$ may increase exponentially in the size of the net. We show this problem is avoidable by introducing a different representation of firing count vectors.

Let $E_i = \{e_i^1, \cdots, e_i^{|E_i|}\}$. We represent each x_i by a $|E_i|$ dimensional vector $\hat{x}_i = [x_i^1, \cdots, x_i^{|E_i|}]$ such that

(i) for each $k = 1, \cdots, |E_i|$:

$$x_i^k = \sum_{b \in B_i} x_i(b) \# b(e_i^k)$$

(ii) for every $k, k' \in \{1, \cdots, |E_i|\}$, $|x_i^k| = |x_i^{k'}|$.

The second requirement is necessary for the vector to be decomposed into a multi-set over B_i. The reason why this compact representation of firing count vectors is possible is that there is no interference among transition components of each transition.

The following example shows how we can avoid to deal with the combinatorial number of transition bindings.

Example 4: Suppose that $E_i = \{e_i^1, \cdots, e_i^h\}$, $e_{i\Sigma}^k = a$ ($k = 1, \cdots, h$), and the input place of t_i contains r marked ℓ-P/T nets having an enabled transition with label a (Fig. 11). Then the number of possible transition bindings for t_i is $O(r^h)$. In the above representation, however, the space necessary to represent x_i is $O(rh)$.

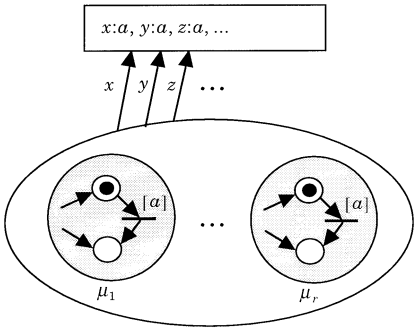

Fig. 11 Combinatorial number of transition bindings.

For a sequence of transition bindings $\sigma \in B^*$, let $\psi(\sigma) = [x_1, \cdots, x_n]^t$ denotes a firing count vector such that for each transition t_i, x_i is the sum of transition bindings for t_i occurring in σ, and let $\hat{\psi}(\sigma) = [\hat{x}_1, \cdots, \hat{x}_n]^t$ denotes the different representation of $\psi(\sigma)$ decribed above. The vector $\hat{\psi}(\sigma)$ keeps sufficient information to determine the final marking.

Lemma 1: Suppose that $s \stackrel{\sigma_1}{\to} s'$, $s \stackrel{\sigma_2}{\to} s''$, and $\hat{\psi}(\sigma_1) = \hat{\psi}(\sigma_2)$, then $s' = s''$.

We now define two matrices I^- and I^+, called *the input incidence matrix* and *the output incidence matrix*. Each of these contains a row for each place and a column for each transition. Each component $I^-(p_j, t_i)$ ($I^+(p_j, t_i)$) corresponds to ${}^\bullet e_i^k$ ($e_i^{k\bullet}$) of $e_i^k \in E_i$, and is defined as follows:

- $I^-(p_j, t_i)$ is a row vector

  $$[w_{j,i,1}^- \# Id, \ \cdots, \ w_{j,i,|E_i|}^- \# Id],$$

  where $w_{j,i,k}^- \in \mathbb{N}$, and Id is the identity function.
- $I^+(p_j, t_i)$ is a row vector

  $$[w_{j,i,1}^+ \# \delta_{e_{i\Sigma}^1}, \ \cdots, \ w_{j,i,|E_i|}^+ \# \delta_{e_{i\Sigma}^{|E_i|}}],$$

  where $w_{j,i,k}^+ \in \mathbb{N}$, and δ_τ is a function on $\mathcal{M}_{PT_\Sigma}$ such that $\delta_\tau(M) = \sum_{\mu \in PT_\Sigma} M(\mu) \# \delta(\mu, \tau)$.

Multiplication between incidence matrices and a firing count vector is defined as follows.

$$I^-(p_j, t_i) \cdot \hat{x}_i = \sum_{k=1}^{|E_i|} w_{j,i,k}^- \# Id(x_i^k) = \sum_{k=1}^{|E_i|} w_{j,i,k}^- \# x_i^k,$$

$$I^+(p_j, t_i) \cdot \hat{x}_i = \sum_{k=1}^{|E_i|} w_{j,i,k}^+ \# \delta_{e_{i\Sigma}^k}(x_i^k).$$

A configuration s is represented by a column vector $[s_1, \cdots, s_m]^t$ such that each s_i is a multi-set over PT_Σ. Hence, we obtain the following state equation.

Proposition 2: if $s \stackrel{\sigma}{\to} s'$, then $s' = s + I^+\hat{\psi}(\sigma) - I^-\hat{\psi}(\sigma)$.

Example 5: The incidence matrices of the PN^2 in Fig. 1 are

$$I^- = \begin{bmatrix} [Id,\emptyset] & [\emptyset] & [\emptyset,\emptyset] \\ [\emptyset,\emptyset] & [Id] & [Id,Id] \\ [\emptyset,Id] & [Id] & [\emptyset,\emptyset] \end{bmatrix},$$

$$I^+ = \begin{bmatrix} [\emptyset,\emptyset] & [\emptyset] & [\delta_b,\emptyset] \\ [\delta_a,\delta_c] & [\delta_d] & [\emptyset,\emptyset] \\ [\emptyset,\delta_c] & [\delta_d] & [\emptyset,\emptyset] \end{bmatrix}.$$

The firing count vector of the sequence of transition bindings in Example 2 is

$$\hat{x} = [[\mu_1,\mu_2],[\mu_2'],[\mu_1',\mu_2]]^t.$$

Firing count vectors and incidence matrices can be represented by integer vectors/matrices. To denote the size of matrices, we use the following numbers:

$$\#_{all} = |\overline{PT_\Sigma}|,\ \#_{sum} = \sum_{i=1}^{n}\sum_{k=1}^{|E_i|}\#_{i,k},$$

where $\#_{i,k} = |\overline{PT_\Sigma}(e^k_{i\Sigma})|$ and $\overline{PT_\Sigma}(a),\ a \in \Sigma$ denotes the set of all marked ℓ-P/T nets of $\overline{PT_\Sigma}$ in which a transition with label a is enabled.

Remark 1: *For FSAN's, the above values are computed as follows. Suppose that the initial configuration contains DFA_l with the set of states Q_l $(l = 1,\cdots,r)$. For DFA_l and a label $a \in \Sigma$, let $Q_l[a]$ denote the set of states in Q_l at which a is enabled. Then*

$$\#_{all} = \sum_{l=1}^{r}|Q_l|,\ \#_{i,k} = \sum_{l=1}^{r}|Q_l[e^k_{i\Sigma}]|.$$

Now we show integer representation of firing count vectors and incidence matrices. Each x_i^k is represented by a $\#_{i,k}$-dimensional nonnegative integer vector each components of which corresponds to a marked ℓ-P/T net in $\overline{PT_\Sigma}(e^k_{i\Sigma})$. Suppose $\overline{PT_\Sigma}(e^k_{i\Sigma}) = \{\mu^1_{i,k},\cdots,\mu^{\#_{i,k}}_{i,k}\}$. Then the vector is

$$[x_i^k(\mu_i^{k,1}),\cdots,x_i^k(\mu_i^{k,\#_{i,k}})].$$

Each $w^-_{j,i,k}\#Id$ $(w^+_{j,i,k}\#\delta_{e^k_{i\Sigma}})$ in $I^-(p_j,t_i)$ $(I^+(p_j,t_i))$ is represented by a nonnegative integer matrix having rows corresponding to $\overline{PT_\Sigma}$, and columns corresponding to $\overline{PT_\Sigma}(e^k_{i\Sigma})$. Suppose $\overline{PT_\Sigma} = \{\mu_1,\cdots,\mu_{\#_{all}}\}$ and $\overline{PT_\Sigma}(e^k_{i\Sigma}) = \{\mu^1_{i,k},\cdots,\mu^{\#_{i,k}}_{i,k}\}$. Then the matrix for $w^-_{j,i,k}\#Id$ is $[v^-_{rl}]$, where

$$v^-_{rl} = \begin{cases} w^-_{j,i,k} & (\mu^l_{i,k} = \mu_r) \\ 0 & (\text{otherwise}) \end{cases}$$

The matrix for $w^+_{j,i,k}\#\delta_{e^k_{i\Sigma}}$ is $[v^+_{rl}]$, where

$$v^+_{rl} = \begin{cases} w^+_{j,i,k} & (\delta_{e^k_{i\Sigma}}(\mu^l_{i,k}) = \mu_r) \\ 0 & (\text{otherwise}) \end{cases}$$

The size of the incidence matrices is $m\cdot\#_{all}\times\#_{sum}$.

Example 6: $I = I^+ - I^+$ of the PN^2 in Fig. 1 is represented by the following integer matrices.

$$I = \begin{array}{c} \\ \mu_1 \\ \mu_1' \\ \mu_2 \\ \mu_2' \\ \mu_1 \\ \mu_1' \\ \mu_2 \\ \mu_2' \\ \mu_1 \\ \mu_1' \\ \mu_2 \\ \mu_2' \end{array} \begin{array}{c} \begin{array}{ccccc} \mu_1 & \mu_2 & \mu_2' & \mu_1' & \mu_2 \end{array} \\ \left[\begin{array}{ccc|cc} -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{array}\right] \end{array}.$$

Then the state equation for the occurrences of transition bindings in Example 2 is $s = s + I\cdot x$, where $s = [1,0,0,0 \mid 0,0,0,0 \mid 0,0,1,0]^t$ and $x = [1,1 \mid 1 \mid 1,1]^t$. Note that the occurrences of transition bindings in x does not change the configuration. The vector x is a T-invariant, which will be formally defined in the next subsection.

5.3 Invariants

Computing invariants is one of important and effective procedures in the analysis of marked P/T nets, because the existence of invariants is necessary for the system to be stable. Moreover, invariants are obtained by solving systems of linear homogeneous equations. This implies that no calculation on integers is necessary. Using the integer matrix representation, an invariant of an injective-PN^2 are obtained by solving a system of linear homogeneous equations, and therefore various methods developed for P/T nets can be used.

A T-invariant of an injective-PN^2 is a firing count vector x such that any sequence $\sigma \in \overline{B}^*$, $\psi(\sigma) = x$ does not change the configuration. In the different representation, it is a firing count vector $\hat{x} = [[x_1^1,\cdots,x_1^{|E_1|}],\cdots,[x_n^1,\cdots,x_n^{|E_n|}]]^t$ such that

(i) $I^+\hat{x} = I^-\hat{x}$;
(ii) For each $i \in \{1,\cdots,n\}$, $|x_i^1| = |x_i^2| = \cdots = |x_i^{|E_i|}|$.

In the integer matrix representation, each x_i^k is a nonnegative integer matrix

$$[x_i^{k,1},\cdots,x_i^{k,\#_{i,k}}]^t,\ x_i^{k,l} \in \mathbb{N}.$$

Then the condition (ii) is written as follows.

$$\sum_{l=1}^{\#_{i,1}} x_i^{1,l} = \sum_{l=1}^{\#_{i,2}} x_i^{2,l} = \cdots = \sum_{l=1}^{\#_{i,|E_{t_i}|}} x_i^{|E_{t_i}|,l}$$
$$(i = 1,\cdots,n).$$

We can easily construct an integer matrix I_{Tinv} such that $I_{Tinv}\cdot x = \mathbf{0}$ is equivalent to the conditions (i) and (ii). The size of I_{Tinv} is $(m\cdot\#_{all} + \sum_{i=1}^{n}|E_i| - n)\times\#_{sum}$.

Example 7: A T-invariant of the PN^2 in Fig. 1 is

$$\hat{x} = [x_1^1 = \mu_1, x_1^2 = \mu_2, x_2^1 = \mu_2', x_2^1 = \mu_1', x_2^2 = \mu_2]^t.$$

In the nonnegative integer matrix representation,

$$\hat{x} = [1, 1 \mid 1 \mid 1, 1]^t.$$

In P/T nets, a P-invariant is a vector of weights so that the weighted sum of the number of tokens in each place is constant by any occurrence of transitions. P-invariants of an injective-PN^2 are similarly defined. Let x_b denotes the firing count vector of a single occurrence of transition binding b. *A weight function* w is a function on $\mathcal{M}_{PT_\Sigma}$ such that

$$w\left(\sum_{\mu \in PT_\Sigma} n_\mu \# \mu\right) := \sum_{\mu \in PT_\Sigma} c_\mu \cdot n_\mu \# \mu, \ (c_\mu \in \mathbb{N}).$$

A P-invariant is a row vector $y = [y_1, \cdots, y_m]$ of weight functions such that for every $b \in \overline{B}$:

$$y \cdot (I^+ \cdot x_b - I^- \cdot x_b) = \emptyset.$$

In the integer matrix representation, each y_i is represented by an integer vector $[y_i^1, \cdots, y_i^{\#_{all}}]$, and x_b by a nonnegative integer vector

$$[[x_1^1, \cdots, x_1^{|E_i|}], \cdots, [x_n^1, \cdots, x_n^{|E_i|}]]^t$$

such that each x_i^k is an $\#_{i,k}$-dimensional unit vector, i.e., it has value 1 for one component and 0 for others.

The number of transition bindings in $\overline{B}$ is $\sum_{i=1}^n \prod_{k=1}^{|E_i|} \#_{i,k}$. However, it is not necessary to solve the equation for all of the transition bindings. Let I_i^k denote the $\#_{all} \times \#_{i,k}$ submatrix of I corresponding to e_i^k, and let $I_i^{k,l}$ denote the l-th column vector of I_i^k. Then a vector y of weights is a P-invariant if and only if:

(i) $y \cdot \sum_{k=1}^{|E_i|} I_i^{k,1} = 0 \ (i = 1, \cdots, n)$;
(ii) $y \cdot I_i^{k,l} = y \cdot I_i^{k,l+1} \ (i = 1, \cdots, n, \ k = 1, \cdots, |E_i|, \ l = 1, \cdots, \#_{i,k} - 1)$.

We can easily construct an integer matrix I_{Pinv} such that $y \cdot I_{Pinv} = \mathbf{0}$ is equivalent to the conditions (i) and (ii). The size of I_{Pinv} is $m \cdot \#_{all} \times (n + \#_{sum} - \sum_{i=1}^n |E_i|)$.

Example 8: A P-invariant of the PN^2 in Fig. 1 is

$$y = [1, 0, 0, 0 \mid 0, 1, 0, 0 \mid 0, 0, 1, 1],$$

because

$$y \cdot (I_1^{1,1} + I_1^{2,1}) = y \cdot I_2^{1,1} = y \cdot (I_3^{1,1} + I_3^{2,1}) = 0.$$

Note that the condition (ii) is not necessary since $\#_{i,k} = 1$ for all i, k.

6. Discussion

As written in the introduction, the simplest way to represent agents in Petri-net-based models is to introduce notion of *objects*, i.e., a collection of data types with methods that give access to the data, into attributes of tokens. Representation of an agent as an object is necessary to describe the agent independently of other agents and environments where agents interact. Since most of classes of object Petri nets are based on high-level Petri nets, they are too complex to be analyzed. There exists a trade-off between expressiveness and analysis. The aim of proposing PN^2 is to give a model that is sufficiently simple for the mathematical analysis, but has enough modeling power.

The purpose of *elementary object systems* (EOS) [9] is similar to that of PN^2's, i.e., to model systems in an object oriented manner keeping the model as simple as possible. We make a comparison between PN^2's and EOS.

Modeling power: Infinite-sort PN^2's can have infinitely many states, and has the same modeling power as Turing machines, while EOS do not allow to have infinitely many states. Each lower-level nets in EOS is an elementary net system, i.e., it has finitely many reachable markings, and duplication of agents is not allowed in the upper-level net.

Agent communication: In PN^2's, communication links are defined by the function $\cdot_\Sigma$, which defines links by specifying *labels* of lower-level nets. In EOS, it is necessary to specify the interaction relation, which is a binary relation between the set of transitions in the upper-level net and the set of transitions in lower-level nets. Both representations are equivalent except that communication links in EOS are restricted to those between two agents. PN^2's allow links in which more than two agents participate.

EOS also provide another type of interaction between two objects in the same place, called object/object interaction. Communication links for this interaction is defined by a symmetric relation on the set of transitions in lower-level nets. In PN^2's, the same type of interaction can be represented by adding transitions with self-loops to the place.

In EOS, the object/object interaction relation is universal in all places of the upper-level net, while PN^2's can describe different interaction relation in each place.

Semantics: PN^2's are based on the value semantics, whereas EOS are basically based on the reference semantics[†]. As a result, enabling of transitions in EOS is restrictive. It does not allow duplication of agents, but PN^2 can do.

[†]Relationship between value semantics and reference semantics in EOS is discussed in [10]

Mathematical analysis: The main purpose of proposing EOS is to give formal semantics of object systems, and the research direction is different from that of PN^2's. Therefore, no work has been done on the mathematical analysis. On the other hands, PN^2's are designed to enable mathematical analysis on numerical matrices such as invariant analysis. In addition, allowing dynamic bindings of transitions may cause a combinatorial number of transition bindings, but it is avoidable in the invariant analysis.

Process algebra is an alternative approach to model concurrent systems. Several models based on process algebra have been proposed to represent mobility of agents, such as *π-calculus* [14], *CHOCS (Calculus of Higher Order Communicating Systems)* [15], and *ambient calculus* [16]. π-calculus realizes the mobility of agents by movement of links between processes. Each process can send link names to other processes, and communication links between agents are dynamically constructed. CHOCS is a higher order process algebra. In CHOCS, each process can send and receive processes. This feature enables to represent mobile codes in a distributed environment. An ambient is a bounded place where agents interact. In ambient calculus, each agent can enter into or exit from an ambient, and can also dissolve an ambient. Communication between agents are described as the movement of messenger agents.

These models are oriented to expressiveness of agent-based systems. In contrast to these models, PN^2's represent the mobility by movement of agents (lower-level nets) in a virtual environment (the upper-level net), and are oriented to mathematical analysis such as the state equation and invariant analysis. Existence of invariants are necessary for the system to be stable. Such system-theoretic view is important in the design of large and complex systems.

7. Conclusion

The proposed model PN^2's are one of simplest Petri-net-based models for representing the following features of multi-agent systems: internal states of an agent, dynamic interaction between agents, and multiple environments in which agents act.

This paper is just a proposal. In the future, we will study theoretical analysis on the model and applications such as JAVA based software.

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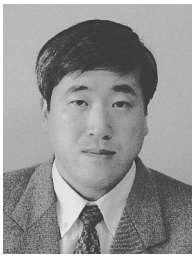

Kunihiko Hiraishi received from the Tokyo Institute of Technology the B.E. degree in 1983, the M.E. degree in 1985, and D.E. degree in 1990. In 1985 he joined the IIAS-SIS, Fujitsu Limited. Since 1993 he has been an Associate Professor at Japan Advanced Institute of Science and Technology. His current interests include discrete event systems and computational models for concurrent systems. He is a member of the IEEE, IPSJ, and SICE.