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Description	

PAPER Special Section on Discrete Mathematics and Its Applications

Arranging Fewest Possible Probes to Detect a Hidden Object with Industrial Application

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SUMMARY This paper addresses the problem of arranging fewest possible probes to detect a hidden object in a specified region and presents a reasonable scheme for the purpose. Of special interest is the case where an object is a double-sided conic cylinder which represents the shape of the energy distribution of laser light used in the optical network. The performance of our scheme is evaluated by comparing the number of probes to that of an existing scheme, and our scheme shows a potential for reducing the number of probes. In other words, the time for detection is significantly reduced from a realistic point of view.

key words: *computational geometry, covering, Minkowski sum, light path alignment, dual-plane scheme*

1. Introduction

The Internet based on optical network is revolutionizing our life style. Optical network comprises not only optical fiber cables but a wide variety of optoelectronic components and photonic devices necessary to generate, modulate, guide, amplify, switch and detect light. These tiny and complex devices are assembled into a package that couples the light into or out of optical fiber. Transferring light signals through the optical devices is not as easy as transferring electric signals through metallic devices. To prevent power loss between optical devices, a delicate alignment of optical path is made at various assembly stages. For example, the accuracy required for the devices used in the long-haul optical fiber network is usually less than one micro meter. Since aligning the light path with sub-micron accuracy using normal vision system is almost impossible, the "active alignment" method (i.e. actually emitting light from one device and detecting the light power at the other device and then establishing positioning feedback) is taken to find the peak power position of the light.

The light path alignment process comprises two subsequent processes. Firstly a process called "blind search" is used to detect the light and secondly another process called "fine search," to find the peak power position. Figure 1 illustrates the blind-search process which uses an optical fiber as a sensing probe. The intensity of the laser light has a Gaussian-like distribution and can be roughly estimated using analytical methods [5]. However, practically, the intensity is profiled using a beam-analyzing equipment. When the lens converges the light, the energy distribution takes a conic shape like Fig. 2. In the blind-search process, typical assembly systems use *single-plane scheme* that defines a search plane perpendicular to the axis of the light and then shifts the sensing probe by even pitches to measure the power. If the light is not detected, the probe is shifted toward the light path direction by a preset pitch and the same search process is repeated. In the single-plane scheme, the probe moves in such a way that the search plane is covered with congruent circles whose diameter is equal to the diameter of the core of the optical fiber. This usually requires hundreds of measurement points per plane. A couple of supplemental methods are considered to reduce the search time. Most of the methods consider changing the measurement path on the search plane heuristically but do not ensure the reduction of the search time. In this paper, we are going to introduce a new blind-search method called *dual-plane scheme* that exactly reduces the number of measurement points and thus, the search time is reduced. In special cases, our scheme performs four times better than conventional single-plane

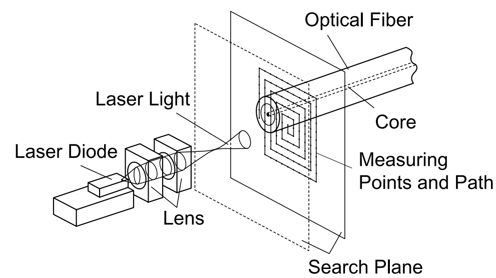

Fig. 1 Illustration of the blind-search process in the light path alignment application.

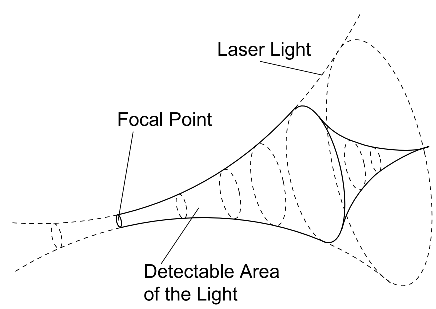

Fig. 2 The object to be detected.

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scheme. The application areas of our search scheme is not limited to optical device assembly but can be used in other sensing applications in which a rigid object is to be searched in a broad search space.

One of the earliest topics in computational geometry is the Art Gallery problem that requires the minimum number of guards to watch an art gallery of a polygonal shape [3], [4]. It has been generalized to a watchman route problem [1] for finding a shortest possible route to find an intruder hidden in an art gallery. The problem has also been extended to the problem of detecting a mobile intruder hidden in an art gallery [6]. The problem to be discussed in this paper is also an extension of the art gallery problem in yet another direction. We want to find or detect a rigid object hidden somewhere in an art gallery by arranging probes appropriately over the gallery. Computational complexity of the problem highly depends on the shapes of the object and the gallery, the degrees of freedom of the object (with/without rotation in addition to translation), and the dimension of those geometric objects. Our problem is closely related to the problem of covering a region by some simple geometric objects (see e.g., [2]).

This paper is organized as follows. In Sect. 2 we describe the basic geometric problem in a general form. Then in Sect. 3, we restrict ourselves to some special cases which are closely related to the applications mentioned above. Finally, in the last section we have some concluding remarks with open problems.

2. Problem Description

A **rigid object** $\mathcal{B}$ is hidden somewhere in a region. Implicitly we assume one point o in its interior as a **reference point** (the origin). We call a region in which o can lie **reference point region** $\mathcal{R}$. We want to arrange fewest possible probes in a **probe region** $\mathcal{P}$, in which the probes can be placed, so that we can detect $\mathcal{B}$ wherever it is hidden. We assume that we can determine whether a point q lies in the interior of $\mathcal{B}$ by a **predicate** $F(q)$ that can be computed in linear time in the length of the predicate. We also assume implicitly that a rigid object $\mathcal{B}$ has a simple shape and thus the inclusion test can be done in constant time. If the object $\mathcal{B}$ is a triangle (p_1, p_2, p_3) in the plane, then the predicate is

$$F(q):\ \triangle(p_1,p_2,q) \geq 0, \qquad \triangle(p_2,p_3,q) \geq 0,$$
$$\text{and}\quad \triangle(p_3,p_1,q) \geq 0,$$

where $\triangle(p,q,r)$ is positive if the three points are arranged in a counter-clockwise order, 0 if they lie on a line, and negative if they are in a clockwise order. We also assume that the three points p_1, p_2 and p_3 are arranged in a counter-clockwise order.

Thus, a rigid object $\mathcal{B}$ is specified as

$$\mathcal{B} = \{q|F(q)\}. \tag{1}$$

By $\mathcal{B}(p,\theta)$ we denote the object $\mathcal{B}$ translated to the point p (so the reference point is located at p) and then rotated by an angle θ in a counterclockwise direction around the reference point. Then, the corresponding predicate becomes

$$q \in \mathcal{B}(p,\theta) \Leftrightarrow F(T_{-\theta}(q-p)), \tag{2}$$

where $T_\theta(q)$ is the point determined by counterclockwise rotation of the point q around the origin by the angle θ. We assume that point inclusion is also tested in time proportional to the complexity of the object.

An object $\mathcal{B}(p,\theta)$ can be detected if at least one probe is contained in the interior of the object, that is,

$$p_i \in \mathcal{B}(p,\theta) \Leftrightarrow F(T_{-\theta}(p_i - p)) \tag{3}$$

for some probe p_i.

We say that a set of probes is feasible if they detect an object wherever it is located. We want to find a minimum feasible set of probes. A key idea behind our scheme presented in this paper is to define the **image** $\mathcal{B}^{-1}$ of a rigid object $\mathcal{B}$. If no rotation is allowed, then it is defined by

$$\mathcal{B}^{-1} = \{p|F(-p)\}. \tag{4}$$

Figure 3 shows the image of a polygonal object $\mathcal{B}$, that is point-symmetric to the original shape. For the time being, we do not allow rotation.

Now, our problems are described as follows:

Problem 1: Feasibility Test
INSTANCE: A rigid body $\mathcal{B}$ characterized by a predicate defined by polynomial inequalities with respect to a reference point o, a reference point region $\mathcal{R}$, and a probe region $\mathcal{P}$.
QUESTION: Is there a feasible set of probes? Or, in other words, is it possible to arrange the probes so that they can detect a hidden object wherever it is located?

Problem 2: Optimal Feasible Set
INSTANCE: A rigid body $\mathcal{B}$, a reference point region $\mathcal{R}$, and a probe region $\mathcal{P}$.
QUESTION: Find a minimum feasible set of probes if there exists one.

Figure 4 illustrates a set of probes arranged in the probe region $\mathcal{P}$ which is usually contained in the reference point region $\mathcal{R}$. The figure includes three objects with different

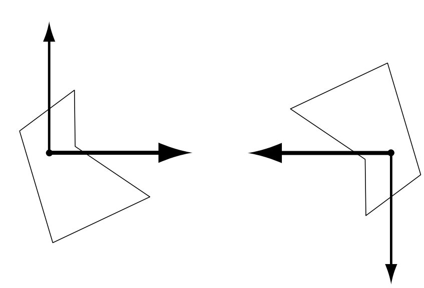
(a) (b)

Fig. 3 The image of an object: (a) given rigid object ($\mathcal{B}$) and (b) its image ($\mathcal{B}^{-1}$).

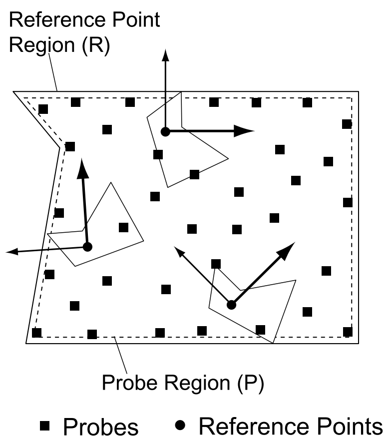

Fig. 4 Hidden objects in a reference-point region $\mathcal{R}$ and a set of probes in a probe region $\mathcal{P}$.

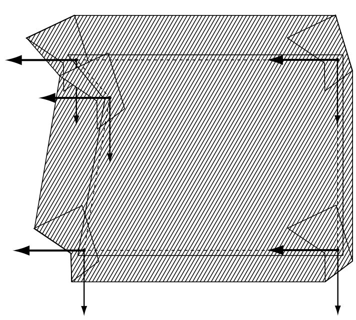

Fig. 5 Minkowski sum $\mathcal{P} \oplus \mathcal{B}^{-1}$.

angles.

From the above definitions, we have the following basic observations.

Lemma 2.1: For an arbitrary rigid object $\mathcal{B}$ the following always holds

$$p \in \mathcal{B}(q) \Leftrightarrow q \in \mathcal{B}^{-1}(p). \tag{5}$$

Proof $p \in \mathcal{B}(q) \Leftrightarrow F(p-q) \Leftrightarrow q \in \mathcal{B}^{-1}(p)$. □

Once we have the above lemma, the following two lemmas are almost obvious.

Lemma 2.2: Given $\mathcal{B}$, $\mathcal{R}$ and $\mathcal{P}$, there is a feasible set of probes if and only if the Minkowski sum $\mathcal{P} \oplus \mathcal{B}^{-1}$ contains $\mathcal{R}$ (see Fig. 5).

Proof The proof immediately follows from the definition of the Minkowski sum:

$P \oplus Q = \{p + q | p \in P \text{ and } q \in Q\}$. □

Lemma 2.3: A set $S = \{p_1, p_2, \ldots, p_n\}$ of probes in $\mathcal{P}$ covers a region $\mathcal{R}$ if and only if the union of their images covers $\mathcal{R}$, that is,

$$\mathcal{R} \subseteq \cup_{p_i \in S} \mathcal{B}^{-1}(p_i). \tag{6}$$

Proof "if" part: Eq. (6) implies that for any point $p \in \mathcal{R}$ there exists some points p_i such that $p \in \mathcal{B}^{-1}(p_i)$. By the definition, $p \in \mathcal{B}^{-1}(p_i) \Leftrightarrow p_i \in \mathcal{B}(p)$, which implies that the point p_i detects the object $\mathcal{B}(p)$ at p.

"Only if" part: If the union of the images does not cover $\mathcal{R}$, there must be a point $p \in \mathcal{R}$ which is not contained in $\mathcal{B}^{-1}(p_i)$ for any point $p_i \in S$. This means that $p_i \notin \mathcal{B}(p)$ for any $p_i \in S$ and thus the set S does not cover $\mathcal{R}$. □

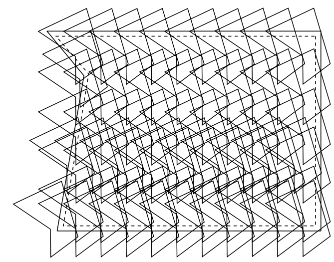

Fig. 6 A covering of the region $\mathcal{R}$ by the image $\mathcal{B}^{-1}$.

The Lemma 2.3 implies that our problem reduces to that of finding the minimum number of images $\mathcal{B}^{-1}$ to cover the entire reference point region $\mathcal{R}$.

Figure 6 shows a covering of the region $\mathcal{R}$ by the image $\mathcal{B}^{-1}$. The set of corresponding reference points gives us a feasible set of probes.

3. Application to the Blind-Search Process in the Light Path Alignment

In this paper we consider a rather special case in which a rigid object is a double-sided conic cylinder and the reference region (which is equal to the probe region) is a cuboid of $L \times L$ wide and of unit height. We assume that the height of the cylinder in one side is exactly 1 so that each half of the cylinder is tall enough to cover the height of the cuboid. The conic cylinder shown in Fig. 7 is a simplified model of the light energy distribution around the focal point of the laser beam. It is double-sided due to converging and diverging ends of the energy distribution, as depicted in Fig. 8. We will show that the knowledge obtained in the previous section can be effectively applied to the blind-search process in the light path alignment application.

Figure 9 shows how a double-sided conic cylinder is embedded in the light energy distribution. Such simplification may sacrifice efficiency of the search compared with that using the actual shape, but simplicity of our model is also very important for practical implementation.

Our objective is to arrange the smallest number of probes in the probe region (it is equal to the reference region which is a cuboid in this case) so that any hidden object (a focal point of a laser beam in this case) can be detected wherever it is located in the reference region. The traditional heuristic method is characterized as a *single-plane scheme* in which all the probes must be located on a single plane which is parallel to the base face (rectangle in this case) of the reference region. To cover the entire cuboid by those cylinders we are forced to cover the base face (rectangle) by circles given as the cross section of the conic cylinder with the plane parallel to the rectangle at the distance 1 from the

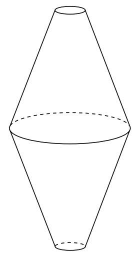

Fig. 7 Axis-parallel conic cylinder.

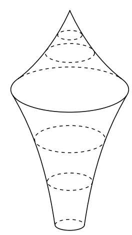

Fig. 8 An example of light energy distribution.

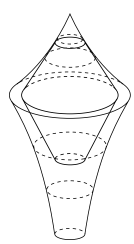

Fig. 9 A double-sided conic cylinder.

central cross section of the cylinder (that is the part of the largest radius). For simplicity, we assume that the height of the cylinder in one side is exactly 1 and the largest and smallest radii of the circles given as cross sections are R and r, respectively. So, under the assumption we have to cover the rectangle by circles of the smallest radius r. Obviously it is not advantageous.

The performance of the blind-search based on this single-plane scheme can be improved if we use the morphic characteristic of the light energy distribution. We would like to introduce here a new scheme called *dual-plane scheme* in which probes are located in two different planes. We will prove an advantage of the dual-plane scheme over the single-plane one by showing that the total number of the probes is considerably reduced. Figure 10 illustrates the concept of our scheme.

The reference point region $\mathcal{R}$ is an axis-parallel cuboid of height 1. Formally, an object $\mathcal{B}$ is defined by

$$\begin{aligned}\mathcal{B} &= \{(x, y, z) |\sqrt{x^2 + y^2} \\ &\le (r - R)|z| + R, -1 \le z \le 1\},\end{aligned} \tag{7}$$

where r and R are the radii of the top and bottom circles of the object. Since $\mathcal{B}$ is symmetric, $\mathcal{B}^{-1} = \mathcal{B}$.

It has an axis that is parallel to the z-axis and it is bounded by a conic surface. Note that, the cross section

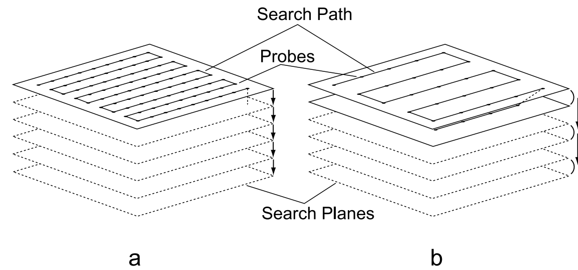

Fig. 10 (a) Conventional single-plane scheme and (b) our dual-plane scheme.

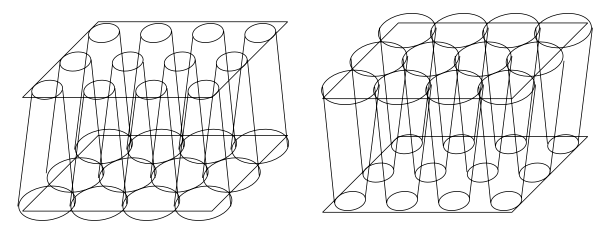

Fig. 11 Covering the cuboid region $\mathcal{R}$ by two sets of conic cylinders.

of $\mathcal{B}$ at any plane perpendicular to the z-axis is a disk. A radius of such a disk is largest at $z = 0$ and smallest at $z = 1$ and $z = -1$. The largest and smallest radii are denoted by R and r respectively.

Consider the following arrangement of conic cylinders. An idea here is to place those cylinders on the two planes $z = 0$ and $z = 1$ (see Fig. 11). Precisely, two sets of probe locations are determined as follows.

$$\begin{aligned}S_0(k) &= \{(0, 0, 0), (0, 2d, 0), (0, 4d, 0), \ldots, \\ &\quad (2d, 0, 0), (2d, 2d, 0) \ldots, \\ &\quad (2kd, 2kd, 0)\}, \\ S_1(k) &= \{(d, d, 1), (d, 3d, 1), (d, 5d, 1), \ldots, \\ &\quad (3d, d, 1), (3d, 3d, 1), \ldots, \\ &\quad ((2k + 1)d, (2k + 1)d, 1)\}.\end{aligned}$$

The set of conic cylinders whose center points (reference points) are located on the plane $z = 0$ are called 0-cylinders and those on the plane $z = 1$ 1-cylinders. 0-cylinders are located on a regular grid of space $2d$. 1-cylinders are also located in a similar manner, but their centers are characterized by odd integers times d. The space parameter d is determined by

$$d = \frac{R + r}{2}. \tag{8}$$

The other parameter k is determined to be a smallest integer such that the corresponding set of cylinders cover the entire cuboid. To find such an smallest integer k, we have to consider different cases (See Fig. 12). For simplicity, we assume that the base area is a square of side length L. In case $kd < L \le (k + 1)d$, the cuboid is covered with cylinders corresponding to $S_0(k) \cup S_1(k)$ so that total number of cylinders needed is $|S_0(k)|^2 + |S_1(k)|^2 = 2k^2 = 2(\lfloor L/(2d) \rfloor)^2$. In other case $(k + 1)d < L \le (k + 2)d$, the cuboid is covered with the

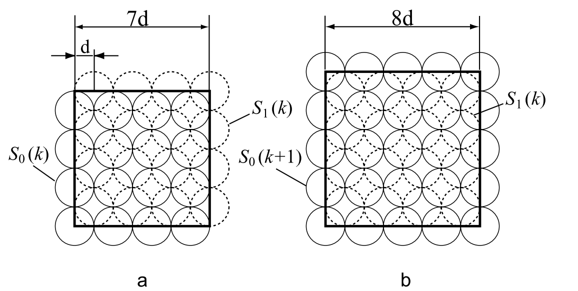

Fig. 12 Change in parameter k with different size of base area; (a) upper right corner of cuboid lies on the center of $S_1(k)$ (b) the same corner lies on the center of $S_0(k+1)$.

ones corresponding to $S_0(k+1) \cup S_1(k)$ so that the total number of disks for covering becomes $|S_0(k+1)|^2 + |S_1(k)|^2 = (k+1)^2 + k^2 = (\lfloor L/(2d) \rfloor + 1)^2 + (\lfloor L/(2d) \rfloor)^2$.

By $C^0_{i,j}$ and $C^1_{i,j}$ we denote the intersections of the cuboid with the 0-cylinder and 1-cylinder, respectively, whose reference points are located at the points (id, jd, z), that is,

$$\begin{aligned} C^0_{i,j} &= \{(x, y, z) | 0 \le z \le 1, \\ &\quad (x - id)^2 + (y - jd)^2 \le (R - (R - r)z)^2\}, \\ &\quad (i, j) = (0, 0), (0, 2d), (0, 4d), \ldots, \\ &\quad (2d, 0), (2d, 2d), \ldots, (2kd, 2kd) \text{ and} \\ C^1_{i,j} &= \{(x, y, z) | 0 \le z \le 1, \\ &\quad (x - id)^2 + (y - jd)^2 \le (r + (R - r)z)^2\}, \\ &\quad (i, j) = (d, d), (d, 3d), (d, 5d), \ldots, \\ &\quad (3d, d), (3d, 3d), \ldots, ((2k + 1)d, (2k + 1)d). \end{aligned}$$

Lemma 3.1: A cuboid of a square base face and of height 1 can be covered by 0-cylinders and 1-cylinders placed at the locations specified by $S_0(k)$ and $S_1(k)$, respectively, on the planes $z = 0$ and $z = 1$, respectively.

Proof We shall show how the cross section of the cuboid at $z = z_0$, $0 \le z_0 \le 1$ is covered by those cylinders. When $z_0 > 1/2$, the 1-cylinders cover more space than 0-cylinders. The radius $r_1(z_0)$ of a 1-cylinder at $z = z_0$ is given by

$$r_1(z_0) = r + (R - r)z_0. \tag{9}$$

Similarly, the radius $r_0(z_0)$ of the circle of a 0-cylinder at $z = z_0$ is given by

$$r_0(z_0) = R - (R - r)z_0. \tag{10}$$

If $r_1(z_0) > \sqrt{2}d$, that is, $z_0 > (\sqrt{2}d - r)/(R - r)$, then the cross section of the cuboid at $z = z_0$ is covered by 1-cylinders. For $d < r_1(z_0) \le \sqrt{2}d$, that is, $1/2 < z_0 \le (\sqrt{2}d - r)/(R - r)$, the farthest points from a center (id, jd) of a 1-cylinder $C^1_{i,j}$ are $\{((i \pm 1)d, (j \pm 1)d)\}$ (See Fig. 13).

Consider the intersection at $z = z_0$ between two cylinders $C^1_{i,j}$ and $C^1_{i,j+2}$, which is given by $(id + \sqrt{r_1(z_0)^2 - d^2}, (j + 1)d)$. This point is covered by the cross section of the cylinder (or exactly disk) $C^0_{i+1,j+1}$ because

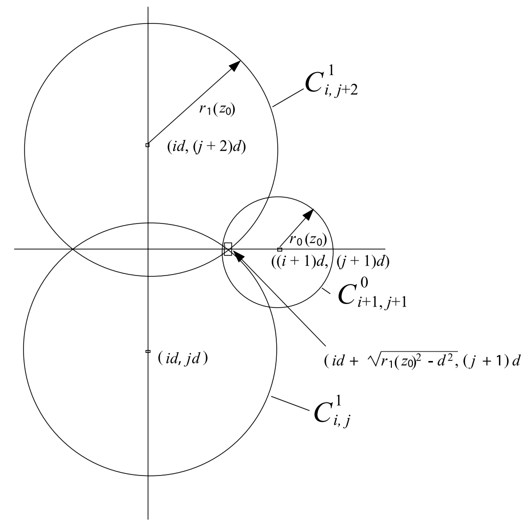

Fig. 13 Cross sections of 0-cylinders and 1-cylinders when $1/2 < z_0 \le (\sqrt{2}d - r)/(R - r)$.

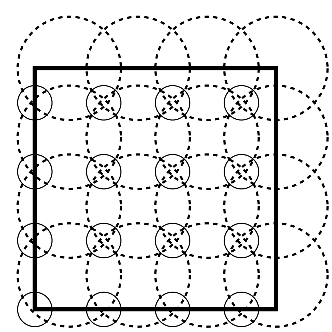

Fig. 14 Cross section at $z = \frac{3}{4}$.

$$\begin{aligned} &\{(i + 1)d - id - \sqrt{r_1(z_0)^2 - d^2}\}^2 \\ &\quad + \{(j + 1)d - (j + 1)d\}^2 - r_0(z_0)^2 \\ &= \{d - \sqrt{r_1(z_0)^2 - d^2}\}^2 - (2d - r_1(z_0))^2 \\ &= -2d\{\sqrt{r_1(z_0)^2 - d^2} + 2(d - r_1(z_0))\} \\ &= -2d(2d - 2r_1(z_0) + \sqrt{r_1(z_0) - d^2}) < 0. \end{aligned}$$

The last inequality is verified as follows. Let $r_1 = r_1(z_0)$ and $f(r_1) = 2d - 2r_1 + \sqrt{r_1 - d^2}$.
Differentiating the function f w.r.t. r_1 we have
$f'(r_1) = -2 + \frac{r_1}{\sqrt{r_1^2 - d^2}}$.
So, the function $f(r_1)$ takes an extreme value when $r_1 = \frac{2\sqrt{3}}{3}d$. The extreme value is positive since $f(\frac{2\sqrt{3}}{3}d) = (2 - \sqrt{3})d > 0$

We also see that $f(d) = 0$ and $f(\sqrt{2}d) = (3 - 2\sqrt{2})d > 0$. All these observations suggest $f(r_1) > 0$. If $d < r_1(z_0) \le \sqrt{2}d$, then $-2d(2d - 2r_1(z_0) + \sqrt{r_1(z_0) - d^2}) < 0$. □

Figure 14 shows the cross section of cuboid at $z = 3/4$.

Now, we can compare the performance of our dual-plane scheme with that of the single-plane scheme in which probes are arranged so that the smallest circles cover the rectangular reference point region. We discuss here in the

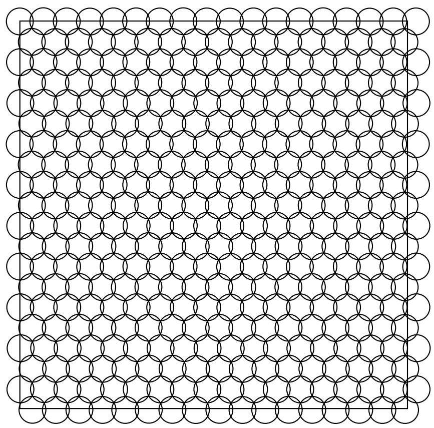

Fig. 15 Hexagonal arrangement of circles.

case (b) in Fig. 12. The number of circles required by our scheme is

$$(k+1)^2 + k^2 = (\lfloor L/(2d) \rfloor + 1)^2 + \lfloor L/(2d) \rfloor^2. \tag{11}$$

On the other hand, the number of circles required by the single-plane scheme is

$$(k'+1)^2 + k'^2 = (\lfloor L/(2r) \rfloor + 1)^2 + \lfloor L/(2r) \rfloor^2. \tag{12}$$

Thus, the ratio is given by

$$\begin{aligned} &(k'^2 + (k'+1)^2)/(k^2 + (k+1)^2) \\ &\quad \simeq \lfloor L/(2r) \rfloor^2 / \lfloor L/(2d) \rfloor^2 \simeq (d/r)^2 \\ &\quad = ((R+r)/(2r))^2 \\ &\quad = (1 + R/r)^2/4. \end{aligned} \tag{13}$$

So, if $R = 3r$, then the ratio is $(1+3)^2/4 = 4$. Although the shape of the light energy distribution varies according to the kind of light sources and the configuration of optics, the ratio $R/r = 3$ is reasonable from the experience of the first author. This ratio 4 means that the dual-plane scheme can reduce the number of probes by a factor of 4 if the larger radius is thrice of the smaller one.

The performance of the single-plane scheme can be improved by arranging circles of radius r on a hexagonal grid as shown in Fig. 15. In this case, the number of circles to cover the cross section is given by

$$\frac{L^2}{1.5r \times \sqrt{3}r} = \frac{2\sqrt{3}}{9}\frac{L^2}{r^2}. \tag{14}$$

Thus, the ratio is improved to

$$\frac{\sqrt{3}}{9}\left(1 + \frac{R}{r}\right)^2, \tag{15}$$

which is 3.08 when $R = 3r$.

4. Conclusions

In this paper, we have considered the problem of arranging fewest possible probes to find a hidden geometric object in a given region. This problem is closely related to an industrial application of the light path alignment problem. To reduce the time of the blind-search process in the application, we introduced the *dual-plane scheme* using morphic characteristic of the object. Our scheme succeeded in improving the performance of the search process. In practical applications we have to deal with small rotation of objects with additional freedom of two rotations, which has been left as an open problem.

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