Title	直観主義論理に基づく理論体系の保守的拡大に関する研究
Author(s)	立澤，正博
Citation	
Issue Date	2008-09
Type	Thesis or Dissertation
Text version	author
URL	http://hdl.handle.net/10119/4758
Rights	
Description	Supervisor:石原哉，情報科学研究科，修士

A study of some conservative extensions of systems based on intuitionistic logic

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August 8, 2008

Keywords: intuitionistic logic, conservative extension, negative translation, \$-translation.

In intuitionistic logic, formulas are understood intuitively by the BHK-interpretation as follows:

- A proof of $A \wedge B$ is given by presenting a proof of A and a proof of B.
- A proof of $A \vee B$ is given by presenting either a proof of A or a proof of B.
- A proof of $A \rightarrow B$ is a method transforming every proof of A into a proof of B.
- There is no proof of $\perp$.
- A proof of $\forall x A$ is a method transforming every object d into a proof of $A(d)$.
- A proof of $\exists x A$ is given by presenting a construction of an object d and a proof of $A(d)$.

Claiming existence of some object in intuitionistic logic is understood the same as giving a construction of the object under this interpretation.

In intuitionistic logic, "explicit existence" which means there is some construction of the object as the meaning of formulas indicates in BHK-interpretation differs from usual classical "mathematical existence", and "explicit existence" is dealt with systematically.

On the other hand, programs of computer are constructive essentially and the Curry-Howard isomorphism which makes correspondence of the typed λ-calculus of a kind of computation models to constructive proofs based on intuitionistic logic is known. This means that the analysis of the proof of a mathematical proposition leads the analysis of the property of the corresponding program, and also that a program can be extracted from a constructive proof.

Therefore, it is important from a point of view of information science that a mathematical proposition has constructive proof.

In this study, some classes on which classical logic is a conservative extention of intuitionistic logic, namely classes of formulas such that if they are proved in classical logc then they can be proved in intuitionistic logic, are defined by the syntactical form of the elements of them.

Usually, it is more easy to prove a proposition classicaly than intuitionistically, but such syntactical definition guarantees propositions which have suitable form that it is enough to get a constructive proof by getting a classical proof. The combination of this and Curry-Howard isomorphism leads that a program can be extracted from not only constructive proof but also classical proof for some propositions which have suitable syntactical form.

In this study, such classes are defined by two translations which look like each other but different.

At first, the negative translation ^{g} is defined inductively as a translation from formulas into formulas as follows:

· $\bot^g := \bot$.

· $P^g := \neg\neg P$ (where P is an atomic formula except $\bot$).

· $(A \wedge B)^g := A^g \wedge B^g$.

· $(A \vee B)^g := \neg(\neg A^g \wedge \neg B^g)$.

· $(A \to B)^g := A^g \to B^g$.

· $(\forall x A)^g := \forall x A^g$.

· $(\exists x A)^g := \neg \forall x \neg A^g$.

By using this negative translation, classical logic can be embedded into minimal logic. That is,

if a formula A is proved in classical logic then A^g is proved in minimal logic.

Here, a syntactical definition of a class on which classical logic is a conservative extension of intuitionistic logic is given by defining a class W_i of A such that $A^g \to A$ is proved in intuitionistic logic.

In the next place, the \$-translation $^\$$ is defined inductively as a translation from formulas into schemas which include a place holder \$ as follows:

· $\bot^\$:= \$$.

· $P^\$:= \neg_\$ \neg_\$ P$ (where P is an atomic formula except $\bot$).

· $(A \wedge B)^\$:= A^\$ \wedge B^\$$.

· $(A \vee B)^\$:= \neg_\$ \neg_\$ (A^\$ \vee B^\$)$.

· $(A \to B)^\$:= A^\$ \to B^\$$.

· $(\forall x A)^\$:= \forall x A^\$$.

· $(\exists x A)^\$:= \neg_\$\neg_\$\exists x A^\$$.

This equals to the translation which is given by replacing of $\perp$ in ^{g} by \$ and

if a formula A is proved in classical logic then $A^\$$ is proved in minimal logic.

Here, by defining a class $\mathcal{I}$ of A such that $A^\$ \to \neg_\$\neg_\$ A$ (where, $\neg_\$ A \equiv A \to \$$) is proved in intuitionistic logic syntactically, and if $A \in \mathcal{I}$ is proved in classical logic then a formula $(A \to A) \to A$ given by replacing of \$ in $\neg_\$\neg_\$ A$ by A is proved in intuitionistic logic, and hence A is proved in intuitionistic logic too. Using this $\mathcal{I}$, a syntactical definition of a class $\mathcal{K}$ on which classical logic is a conservative extension of intuitionistic logic is given.

In this study, already known classes $\mathcal{W}_i$ and $\mathcal{K}$ are not extended but the relationship of $\mathcal{W}_i$ and $\mathcal{K}$ such as

$$\mathcal{W}_i \not\subseteq \mathcal{K}, \quad \mathcal{K} \not\subseteq \mathcal{W}_i, \quad \mathcal{W}_i \cap \mathcal{K} \neq \emptyset$$

becomes clear.

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