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Tight Bound on the Gathering of Two Oblivious Mobile Robots with Inconsistent Compasses

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Abstract

We consider a system of autonomous mobile robots that can move in the two dimensional space. These robots must gather, in finite time, at a single point in the plane, not predetermined (gathering problem). We consider that the robots are equipped with compasses, although these compasses can be inconsistent. In our previous work, we proposed an algorithm that gathers two oblivious mobile robots in finite time when the compasses diverge by at most $45°$. *In this paper, we extend this work by proving a tight bound on the degree of divergence of robots' compasses for solving the gathering problem. More specifically, we present a self-stabilizing algorithm to gather, in a finite time, two oblivious robots equipped with compasses that can differ by an angle strictly smaller than* $180°$, *and we show that it is a tight bound.*

Keywords: Distributed Mobile Computing, Autonomous Robots, Cooperation and Control, Gathering, Tight Bound, Asynchrony, Inaccurate Compasses, Oblivious Computations.

1 Introduction

Background. Over the past few years, using a large number of simple and low-cost robots to accomplish some cooperative tasks in a distributed fashion has received a lot of attention. This approach is interesting for a number of reasons, including decreased costs, faster computation, fault tolerance capabilities, the possibility of extendability of the system and the reusability of the robots in different applications. Subsequently, part of the focus on the research community has been on how to coordinate such simple mobile robots so that they can

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cooperate. Studies can be found in different disciplines, from artificial intelligence to engineering [19, 20, 22, 24] (see [23] for a survey).

Recently, various aspects of this problem have been studied from the point of view of distributed computing [1, 10, 11, 16] aiming to identify the algorithmic limitations of what autonomous mobile robots can do. Conversely, as the common models of multiple robot systems assume simple and relatively weak robots, the issue of resilience to failure becomes prominent.

In our work, in particular, we focus on systems without any prior infrastructure (e.g, no Global Positioning System), where robots are deployed in adverse environment, for instance robots working on Mars, and they are required to cooperate and self-organize to build such infrastructure in spite of the unreliability of their sensors. More specifically, we focus on a very fundamental coordination problem, which is the gathering problem, where robots are required to gather at some arbitrary location which is not determined in advance and without agreement on a common coordinate system. Besides, robots are equipped with compasses that are inaccurate. While being very simple to express, this problem has the advantage of retaining the inherent difficulty of agreement, namely the question of breaking symmetry between robots. Among other things, gathering at a point means that the robots are spontaneously able to reach an agreement on an origin.

Prencipe [12] has shown that gathering oblivious robots cannot be achieved deterministically without additional assumptions. In particular, if robots can detect multiplicity (i.e., count robots that share the same location) gathering is possible for *three or more* robots. Similarly, gathering of any number of robots is possible if they share a common direction, as given by compasses, with *no errors*. Our work is motivated by the pragmatic standpoint that (1) compasses are error-prone devices in reality, and (2) multiplicity detection allows for gathering in situations with more than two robots.

In some of our recent work, we have studied gathering with *inaccurate* compasses [14], and we provided a self-stabilizing algorithm to gather, in a finite time, two oblivious robots equipped with compasses that can differ by as much as $\pi/4$. A similar result has been also presented by Imasu et al. [9] at a domestic workshop in Japan. However, the question that remains open is what is the tight bound on the degree of divergence of robots' compasses in solving the gathering problem?

Therefore, this paper presents a tight bound on the degree of compasses inaccuracies under which asynchronous mobile robots can gather relying on oblivious computations. In particular, we provide a self-stabilizing [1] algorithm whereby two asynchronous mobile robots can gather in finite time even if their compasses diverge by an angle strictly smaller than π, and we show that it is a tight bound.

Contribution. In this paper, we further study the solvability of the gathering of mobile robots in the face of compass inaccuracies. The main result is an tight bound on the degree of divergence of compasses in solving the gathering of two asynchronous oblivious mobile robots. In particular, we present an algorithm that solves the problem in a finite time when robots' compasses are inconsistent by an angle which is strictly smaller than $180°$, and we show that this bound is a tight bound. Besides, our algorithm is self-stabilizing.

Related Work. The gathering problem has been studied extensively in the literature, under different models and various assumptions [3, 4, 8, 16]. In particular, in all these studies, the problem has been solved only by making some additional assumptions regarding robots' capabilities.

[1] Self-stabilization is the property of a system which, starting in an arbitrary state, always converges toward a desired behavior [7, 13].

In their SYm model [16], referred to as semi-synchronous model, Suzuki and Yamashita proposed an algorithm to solve the gathering problem deterministically in the case where robots have unlimited visibility. For a system with two robots, they have proven that it is impossible to achieve the gathering of two *oblivious* mobile robots that have no common orientation in a finite time. The difficulty of the problem is inherent in breaking the symmetry between the two robots. Using the same model, Ando et al. [2] proposed an algorithm to address the gathering problem in systems wherein robots have limited visibility. Their algorithm *converges* toward a solution to the problem, but it does not solve it deterministically.

In his CORDA model [11], referred to as an asynchronous model, Prencipe [12] has shown that there exists no deterministic algorithm to solve the gathering problem in finite time with oblivious robots. Cieliebak et al. [4] have introduced multiplicity, and have shown that gathering is possible for three or more robots, when they are able to detect multiple robots at a single point. Finally, Flocchini et al. [8] have solved the gathering problem for any number of robots when they share a common direction, as provided by a compass[2].

In some of our recent work [14], we studied the solvability of gathering oblivious mobile robots with limited visibility in the face of compass instabilities. In particular, we proposed an algorithm that solves the problem in a finite time, in the SYm model, where compasses are unstable for some arbitrary long periods, provided that they stabilize eventually.

Recently, Cohen and Peleg [5] also addressed the issue of analyzing the effect of errors in solving gathering and convergence problems. In particular, they studied imperfections in robot measurements, calculations and movements. They showed that gathering cannot be guaranteed in environments with errors, and illustrated how certain existing geometric algorithms, including ones designed for fault-tolerance, fail to guarantee even convergence in the presence of small errors. One of their main positive results is an algorithm for convergence under bounded measurement, movement and calculation errors. However, their work does not relate to compasses.

The gathering problem also has been studied in the presence of faulty robots by Agmon and Peleg [1] in synchronous and asynchronous settings. In particular, they proposed an algorithm that tolerates one crash-faulty robot in a system of three or more robots. They also showed that in an asynchronous environment, it is impossible to perform a successful gathering in a 3-robot system with one Byzantine[3] failure. Later on, Défago et al. [6] confirm the impossibility of gathering in systems with Byzantine robots by showing that the impossibility persists in stronger models. They also show the existence of randomized solutions for systems with Byzantine-prone robots.

Finally, a recent study on the gathering of fat robots was done by Czyzowicz et al. [18], in which they represented robots by unit discs, and they proposed an algorithm to gather at most four robots in the plane under the CORDA model.

Structure. The remainder of this paper is organized as follows. Sect. 2 describes the system model and the basic terminology. In Sect. 3, we describe our gathering algorithm based on compass inconsistencies and give a tight bound. In Sect. 4, we prove the correctness of our algorithm. Finally, Sect. 5 concludes the paper.

[2]A compass does not only indicate the North direction, but also gives a unified clockwise orientation.

[3]A robot is said to be Byzantine if it executes arbitrary steps that are not in accordance with its local algorithm [17].

2 System Model and Definitions

2.1 System Model

In this paper, we consider the CORDA model of Prencipe [11], which is defined as follows. The system consists of a set of autonomous mobile robots $\mathcal{R} = \{r_1, \cdots, r_n\}$. A robot is modelled as a unit having computational capabilities, which can move freely in the two-dimensional plane. In addition, robots are equipped with sensor capabilities to observe the positions of other robots, and form a local view of the world. The robots are modelled and viewed as points in the Euclidean plane.[4] The local view of each robot includes a unit of length, an origin, and the directions and orientations of the two x and y coordinate axes as given by a compass.

The robots are completely *autonomous*. Moreover, they are *anonymous*, in the sense that they are a priori indistinguishable by appearance, and they do not have any kind of identifiers that can be used during their computations. Furthermore, there is no direct means of communication among them.

We further assume that the robots are *oblivious*, meaning that they keep information neither on previous observations nor on past computations.

The cycle of a robot consists of four states: Wait-Look-Compute-Move.

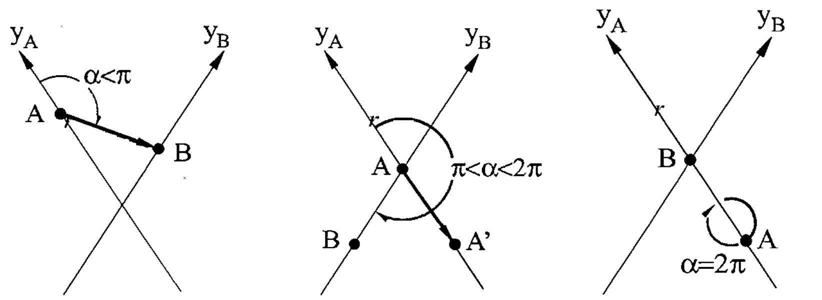
- *Wait*. In this state, a robot is idle. A robot cannot stay permanently idle (see Assumption 2) below. At the beginning all robots are in Wait state.
- *Look*. Here, a robot *observes* the world by activating its sensors, which will return a snapshot of the positions of all other robots with respect to its local coordinate system. Since each robot is viewed as a point, the positions in the plane are just the sets of robots' coordinates.
- *Compute*. In this state, a robot *performs* a local computation according to its algorithm. The algorithm is the same for all robots, and the result of the *compute* state is a destination point.
- *Move*. The robot *moves* toward its computed destination. If the destination is its current location, then the robot is said to perform a *null movement*; otherwise, it is said to execute a *real movement*. The robot moves toward the computed destination, but the distance it moves is unmeasured; neither infinite, nor infinitesimally small (see Assumption 1). Hence, the robot can only go towards its goal, but the move can end anywhere before the destination.

In this model, the (global) time that passes between two successive states of the same robot is finite, but unpredictable. In addition, no time assumption within a state is made. This implies that the time that passes after the robot starts observing the positions of all others and before it starts moving is arbitrary, but finite. That is, the actual movement of a robot can be based on a situation that was observed arbitrarily far in the past, and therefore it may be totally different from the current situation.

Finally, in the model, there are two limiting assumptions related to the cycle of a robot.

Assumption 1 *It is assumed that the distance travelled by a robot r in a move is not infinite. Furthermore, it is not infinitesimally small: there exists a constant $\delta_r > 0$, such that, if the target point is closer than δ_r, r will reach it; otherwise, r will move towards it by at least δ_r.*

[4] We assume that there are no obstacles to obstruct vision. Moreover, robots do not obstruct the view of other robots and can "see through" other robots.



(a) $0 < \alpha \leq \pi$, then A moves directly to B. :**Direct move**

(b) $\pi < \alpha < \pi + \theta$, then A moves by distance $\|\overline{AB}\|$ to A': **Side move south.**

(c) $\pi + \theta < \alpha \leq 2\pi$, then A does not move: **No move.**

Figure 1. Principle of the algorithm.

Assumption 2 *The time required by a robot r to complete a cycle (Wait-Look-Compute-Move) is not infinite. Furthermore, it is not infinitesimally small; there exists a constant $\epsilon_r > 0$, such that the cycle will require at least ϵ_r time.*

2.2 Definitions

Definition 1 (Relative north) *A relative north $\overrightarrow{N_A}(t)$ is a vector that indicates a north direction for some robot A at some time t. $\overrightarrow{N_A}$ is collocated with the local positive y-axis of robot A.*

Definition 2 (Inconsistent compasses) *Informally, compasses of a pair of robots A and B are* inconsistent *by some angle θ iff., the absolute difference between the north indicated by the compass of A, $\overrightarrow{N_A}$ and $\overrightarrow{N_B}$ is at most θ at any time t. In addition, the north directions of A and B are invariant over time. The special case when $\theta = 0$ represents perfect compasses.*

Formally, compasses are inconsistent *by some angle θ iff., the following two properties are satisfied:*

1. *Inaccuracy:* $\forall A, B \in \mathcal{R}, \forall t, |\measuredangle \overrightarrow{N_A}(t)\overrightarrow{N_B}(t)| \leq \theta$,
2. *Invariance:* $\forall A, \forall t, t', \overrightarrow{N_A}(t) = \overrightarrow{N_A}(t')$.

3 Tight Bound on Gathering with Inconsistent Compasses

In this section, we provide an algorithm for solving the gathering of two asynchronous oblivious mobile robots when their compasses diverge by an angle $\theta < \pi$.

3.1 Algorithm Overview

The algorithm is described informally as follows. Consider a local x-y coordinate system where, the positive y-axis points North and hence the positive x-axis points East. Let also the location of the robot be the origin of its local coordinate system.

Table 1. Combination of movements of robot A and B allowed by the algorithm ($\theta < \pi$).

	Robot A		
Robot B	$0 < \alpha \leq \pi$ *(direct move)*	$\pi < \alpha < \pi + \theta$ *(side move south)*	$\pi + \theta < \alpha \leq 2\pi$ *(no move)*
$0 < \alpha \leq \pi$ *(direct move)*	○	○	○
$\pi < \alpha < \pi + \theta$ *(side move south)*	○	○	○
$\pi + \theta < \alpha \leq 2\pi$ *(no move)*	○	○	not applicable

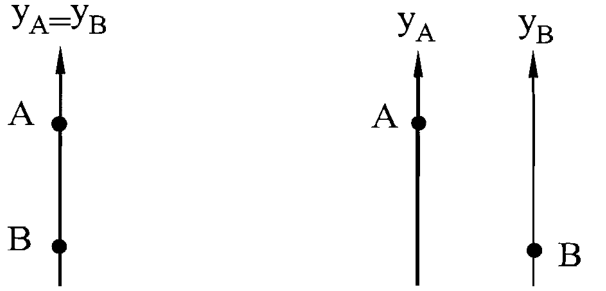
Let A be some robot, and let B be the position at which the other robot is located. We denote by α the angle between the y-axis of robot A, namely y_A and the segment $\overline{AB}$. That is, $\alpha = 0$ when B is on the positive y_A axis and $\alpha = \pi/2$ when B is on the positive x-axis of robot A. Finally, let θ be the difference in north direction indicated by the two local coordinate systems of robot A and B. In our algorithm, we assume that $0 \leq \theta < \pi$. Then, robot A decides its movement as follows:

- If the angle α between y_A and $\overline{AB}$ in clockwise direction is strictly greater than 0 and smaller than or equal to π, then robot A moves directly on the segment $\overline{AB}$ to B. We refer to this move as ***direct move***.
- If the angle α is strictly greater than π and smaller than $\pi + \theta$, then robot A moves towards its south by the distance $\|\overline{AB}\|$. We will refer to this move as ***side move south***.
- If the angle α is strictly greater than $\pi + \theta$ and smaller than or equal to 2π, then robot A does not move. We refer to this move as ***no move***.

The algorithm is given in Algorithm 1, and Table 1 summarizes the different movements of robot A and B (the table is symmetrical).

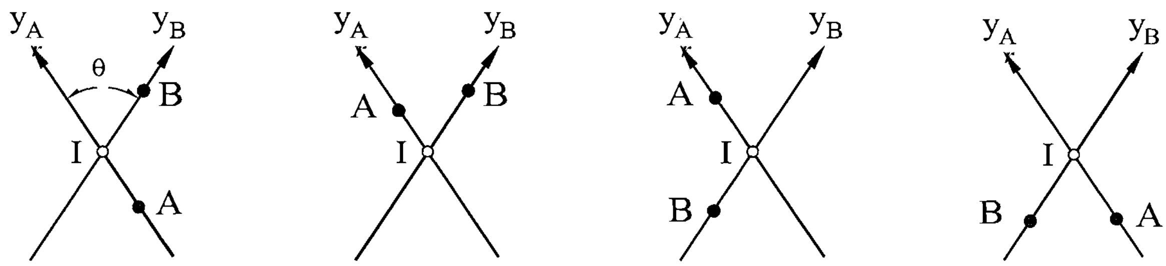
Algorithm 1 Gathering two of asynchronous robots, when compass divergence $\theta < \pi$.

if (r sees only itself) **then** {*gathering terminated*}
    Do_nothing();
else
    $B :=$ position of the other robot B;
    $y_A := y$-axis of robot A;
    $\alpha :=$ angle between y_A and $\overline{AB}$ in clockwise direction;
    if ($0 < \alpha \leq \pi$) **then** {direct move}
        robot A moves to robot B;
    else if ($\pi < \alpha < \pi + \theta$) **then** {side move south}
        robot A moves toward its south by distance $\|\overline{AB}\|$;
    else if ($\pi + \theta < \alpha \leq 2\pi$) **then** {no move}
        Do_nothing();
    end if
end if



(a) **Situation (1).** (b) **Situation (2).**

Figure 2. Situations of A and B where $\theta = 0$.



(a) **Situation (3).** (b) **Situation (4).** (c) **Situation (5).** (d) **Situation (6).**

Figure 3. Situations of A and B where $\theta \neq 0$ and both A and B are not at I.

3.2 Description of Situations

In this section, we define the different possible situations of robot A and B, when their compasses are inconsistent by $0 \leq \theta < \pi$. Without loss of generality, we consider that the north of robot B, denoted by y_B is always on the right hand side of the north of robot A, denoted by y_A. Thus, we define the following 10 situations[5]:

1. **Situation (1):** the y_A axis of robot A and y_B of robot B are equal ($y_A = y_B$), and A and B are located on the same y-axis (refer to Fig. 2(a)).

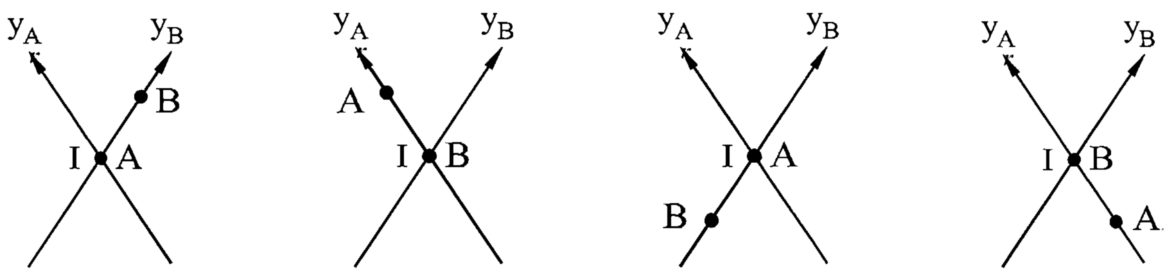
2. **Situation (2):** the y_A axis of robot A and y_B of robot B are parallel. That is, A and B are not located on the same y-axis (refer to Fig. 2(b)).

   Situations (1) and (2) refer to cases when θ is equal to zero. In the following cases, we consider that θ is other than zero. Let I be the intersection of y_A and y_B. Then, four cases arise when both A and B are not at I.

3. **Situation (3):** in this situation, A is below I, and B is above I (see Fig. 3(a)).

4. **Situation (4):** in this situation, both A and B are above I (see Fig. 3(b)).

[5]If the north of robot B is on the left hand side of the north of robot A, then by symmetry we will have the same 10 situations.



(a) **Situation (7).** (b) **Situation (8).** (c) **Situation (9).** (d) **Situation (10).**

Figure 4. Situations of A and B where $\theta \neq 0$ and either A or B is at I.

5. **Situation (5):** in this situation, A is above I, and B is below I (see Fig. 3(c)).
6. **Situation (6):** in this situation, both A and B are below I (see Fig. 3(d)).

   Finally, we distinguish the following four cases (refer to Fig. 4) when either robot A or B is at I.

7. **Situation (7):** in this situation, A is at I and B is above I.
8. **Situation (8):** in this situation, A is above I and B is at I.
9. **Situation (9):** in this situation, A is at I, and B is below I.
10. **Situation (10):** in this situation, A is below I, and B is at I.

4 Correctness

In this section, we will prove that our algorithm solves the problem of gathering two robots in a finite time, when their compasses diverge by an angle that is strictly smaller than π. To do so, we show how any possible situation is transformed into gathering in a finite time. Fig. 6 shows a diagram of all possible transitions between situations.

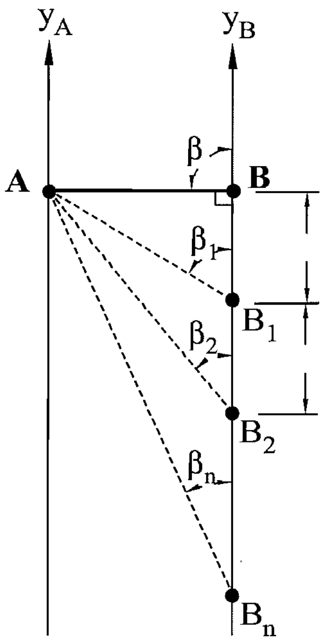
Assume without loss of generality that y_B is to the right of y_A, and $0 \leq \theta < \pi$, then trivially, we derive the following lemmas:

Lemma 1 *The situations* $1 - 10$ *(Section 3.2) form a list of all possible positions of robot A and B.*

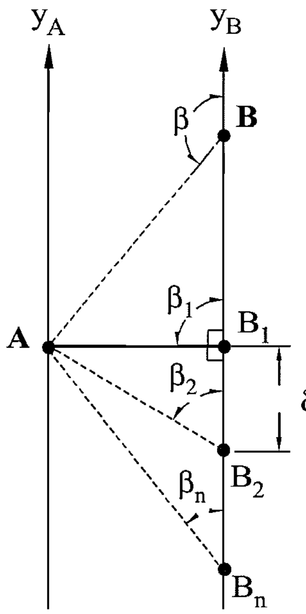
Lemma 2 *Under Algorithm 1, there exists no situation where both robots A and B perform **no move**.*

In the remainder of this paper, we denote by α_A the angle from y_A to robot B in clockwise direction with respect to the local coordinate system of A, and by α_B the angle from y_B to robot A in clockwise direction with respect to the local coordinate system of B. We also denote by I, the intersection of y_A and y_B.

Lemma 3 *In a finite number of cycles, **Situation** (1) and **Situation** (10) are transformed into gathering.*



(a) Initially, β is equal to $\pi/2$.



(b) Initially, β is an obtuse angle.

Figure 5. Robot B stops (forever) at B_n in finite number of steps.

PROOF. Let two robots A and B be in *Situation* (1). Without loss of generality, let A be above B. According to the algorithm, as long as A is above B, B performs *no move* because $\alpha_B = 2\pi$. Consider now the movement of robot A. We have $\alpha_A = \pi$. Then, A performs a *direct move* to B. By Assumption 1, in one cycle, A travels at least δ_r. Consequently, A reaches B in a finite number of steps.

The proof of transformation of *Situation* (10) to gathering is similar to the proof of *Situation* (1), and thus omitted here. $\square_{\text{Lemma 3}}$

Lemma 4 *In a finite number of cycles,* ***Situation*** (2) *is transformed into* ***Situation*** (1) *or to gathering.*

PROOF. By the algorithm, B moves on y_B by the distance $\|\overline{AB}\|$ toward its south, and A performs a direct move to B. First, it is easy to see that if A moves to the position of B, and B has already left its position (by moving on y_B toward its south), then, since the cycle of A and B is finite, they will reach **Situation** (1) in finite time.

Assume now the worst case, where B is activated infinitely many times, however A is not. Since, by the algorithm B moves on y_B toward its south by $\|\overline{AB}\|$, then, we need to show that there will be a time after which B stops (forever) moving toward its south, and this happens in finite time.

Let β be the angle between the segment $\overline{AB}$ and y_B in clockwise direction. The proof consists of showing that: (1) β is monotonically decreasing when B moves, and (2) β becomes less than $\pi - \theta$ in finite number of steps.

Consider first the situation in Figure 5(a), where $\overline{AB}$ is perpendicular to y_B.

Assume that B is activated at time t, while A is not. By Assumption 1, in the worst case, B moves toward its destination on y_B by $\delta_r > 0$. Let B_1 be the new destination of B. Consider the triangle $\triangle(A, B, B_1)$, then it is easy to see that the angle at B_1 is less than the angle at B. Let B_2 be a new destination of B, which is at distance δ_r from B_1. Then, it is also easy to observe that the angle at B_2 is less than the angle at B_1. We thus, conclude that β is is monotonically decreasing when B moves on y_B toward south.

Now we will show that β becomes less than $\pi - \theta$ in finite number of steps. Let n be the maximal number of steps that robot B takes in order for β to become less than $\pi - \theta$.

We assume the worst case where in every step (cycle), B moves on y_B by δ_r. Let B_n be the position at which robot B stops after n cycles. This means that, at B_n, $\beta < \pi - \theta$. Then, we get:

$$\begin{aligned} \tan(\pi - \theta) &= \overline{AB}/\overline{BB_n} \\ &= \overline{AB}/n.\delta_r \end{aligned}$$

Thus, $n = \overline{AB}/\tan(\pi - \theta).\delta_r$. We have, $\overline{AB} > 0$ by hypothesis, and it is a constant. Also, δ_r is a constant. By definition, $\pi - \theta$ is a strictly positive value. Consequently, $\tan(\pi - \theta) > 0$, and thus n is finite.

Now consider the situation in Figure 5(b), where the angle formed by $\overline{AB}$ and y_B is an obtuse angle ($\pi/2 < \beta < \pi$). Let B_1 be the perpendicular to y_B starting at A.

In this case, from above, we can conclude that from B_1 to B_n, β is monotonically decreasing, and β takes n finite steps to become smaller than $\pi - \theta$. Besides, by considering the triangle $\triangle(A, B, B_1)$, it is easy to show that β is monotonically decreasing while B is moving toward B_1. In addition, robot B takes finite number of steps to reach B_1 because $\|\overline{BB_1}\|$ is less than $\|\overline{AB}\|$, which is finite, and B travels at least δ_r in one cycle. Consequently, from B to B_n, β is monotonically decreasing and, β becomes less than $\pi - \theta$ in finite number of steps.

Now since B has stopped moving in finite number of steps at B_n, eventually A will do a direct move to B. Since the distance $\|\overline{AB}\|$ is finite, and by Assumption I, A travels at least $\delta_r > 0$ in one cycle, thus, A reaches B in a finite number of cycles. $\square_{\text{Lemma 4}}$

Lemma 5 *In a finite number of cycles, **Situation** (6) is transformed into **Situation** (10) or to gathering.*

The proof of this lemma is similar to the proof of Lemma 4, and thus omitted.

Lemma 6 *In a finite number of cycles, **Situation** (3) is transformed into **Situation** (5) or to gathering.*

PROOF.

Let A and B be in Situation (3). By the algorithm, A performs a direct move to B, and B performs a direct move to A. Assume first that A performs a look operation at time t, while B does not. Subsequently, if B does not perform any look operation while A is moving toward it, then, A will gather with B at position B in finite time. Similarly, both robots gather at A in finite time if B is activated while A is not.

Now we will show how Situation (3) can be transformed to Situation (5). The proof consists of showing that there will be a finite time, where A is to the left of y_B, and B is to the left of y_A.

Let y'_A be the parallel to y_A passing through B. Let also y'_B be the parallel to y_B passing through A.

Assume that A performs a look operation at time t, and B also performs a look operation at time $t' \geq t$. Then, by the algorithm, A moves to $B(t)$, and B moves to $A(t')$. By Assumption 2, the cycle of a robot is finite, and the distance $\|\overline{AB}\|$ is finite. Then, in a finite number of steps, A and B exchange positions on the segment $\overline{AB}$. Let t'' be the time when this happens. Then, at time t'', B is to the left of y'_A, and also A is to the left of y'_B. Let also $I' = y_A(t'') \cap y_B(t'')$, thus at time t'', A is above I', and B is below I'. This terminates the proof. $\square_{\text{Lemma 6}}$

Lemma 7 *In a finite number of cycles,* ***Situation*** (4) *is transformed into* ***Situation*** (5) *or* ***Situation*** (8) *or* ***Situation*** (9) *or to gathering.*

PROOF.

By the algorithm, A executes a direct move to B, and B performs a side move to south. We distinguish the following cases depending on the activation of A and B:

- *Transformation to Situation* (5) *or* (8): Let B perform a look operation at time t, while A remains inactive. Assume also that $\|\overline{AB}\| > \|\overline{IB}\|$, where $I = y_A \cap y_B$. Then, first, if B stops at I, then A and B enter Situation (8), where B is at I and A above I. Trivially, this transformation is done in finite time by Assumption 2. Now, if B stops after the point I, then B is below I. Subsequently, A and B enter Situation (5), where A is above I, and B is below I. This transformation is also done in finite time by the same argument.

- *Transformation to Situation* (9): Let B performs a look operation at time t. Then, B executes a side move south. Let t' be the time when robot B passes by I. Suppose that A also performs a look operation at time t' and sees B at position I. Then, A performs a direct move to I (since at t', $\alpha_A = \pi$). Let t'' be the time when A finishes its move to I. Consequently, at time t'', A and B enter Situation (9), where A is at I, and B is below I.

- *Transformation to gathering*: This case is trivial. A and B gather in finite time at B by Assumption 1 if A performs a look operation before B, and during the movement of A to B, robot B does not perform any look operation.

$\square_{\text{Lemma 7}}$

Lemma 8 *In a finite number of cycles,* ***Situation*** (8) *is transformed into* ***Situation*** (5) *or* ***Situation*** (9) *or to gathering.*

The proof of this lemma is similar to the proof of Lemma 7, and thus omitted here.

Lemma 9 *In a finite number of cycles,* ***Situation*** (5) *is transformed into* ***Situation*** (6) *or* ***Situation*** (9).

PROOF. Let A and B be in Situation (5). Then, by the algorithm, both A and B execute side move south. The proof is straightforward. If A stops at I, then we get A and B in Situation (9) because B remains below I. This transformation is done in finite number of steps by Assumption 1. If A stops after I, then A is below I. Since B is also below I, then A and B reach Situation (6) in a finite number of steps by similar arguments. $\square_{\text{Lemma 9}}$

Lemma 10 *In a finite number of cycles, **Situation** (7) is transformed into **Situation** (9) or to gathering.*

PROOF.

Let A and B be in Situation (7), where B is to the right of y_A, and A is on y_B. By the algorithm, A performs a direct move to B, and B performs a direct move to A. Trivially, if one robot, say A, is activated and moves to the position of B, while B does not perform any look operation during the movement of A toward it, then both A and B gather at B in finite time by Assumption 2.

Now consider that both A and B are activated simultaneously. We will show that they will reach Situation (9) in finite time. The proof consists of showing that there will be a finite time, where B arrives at the left of y_A.

Assume that A performs a look operation at time t, and B also performs a look operation at time $t' \geq t$. Then, by the algorithm, A moves to $B(t)$, and B moves to $A(t')$. By Assumption 2, the cycle of a robot is finite, and the distance $\|\overline{AB}\|$ is finite. Then, in a finite number of steps, A and B exchange positions on the segment $\overline{AB}$. Let t'' be the time when this happens, and let $I' = y_A(t'') \cap y_B(t'')$. Then, at time t'', B is to the left of $y_A(t'')$. Since $A \in y_B$, then $A(t'') = I(t'')$. Thus, at time t'', B is below I', and A is at I', which represents Situation (9). This terminates the proof. $\Box_{\text{Lemma 10}}$

Lemma 11 *In a finite number of cycles, **Situation** (9) is transformed into **Situation** (6) or **Situation** (10).*

PROOF. Let A perform a look operation at time t. Then, A performs a side move south. Let t' be the time when A finishes its move. At time t', A is below I because A must move at least by δ_r toward its target, according to Assumption 1. Then, if B does not perform any look operation between t and t', A and B enter Situation (6) (both below I).

Now, assume that B performs a look operation at time $t'' > t$, and that at t'', A already has left I. Then, by the algorithm B executes a direct move to $A(t'')$. Let t_f be the time when B finishes its move. Consequently, at time t_f, B is at I, and A is below I, which represents Situation (10). Since this transition is done in a finite number of steps, the lemma holds. $\Box_{\text{Lemma 11}}$

Theorem 1 *Algorithm 1 correctly solves the gathering of two asynchronous mobile robots in finite time as long as their compasses diverge by $\theta < \pi$.*

PROOF. Lemma 1 states the different situations of robot A and B when $0 \leq \theta < \pi$. From Lemma 3 to Lemma 11, we show that every situation is transformed to gathering in finite time. Also, the diagram of all possible transitions between situations depicted in Fig. 6 shows no cycles. Thus, the theorem holds. $\Box_{\text{Theorem 1}}$

Now, we can directly derive the following corollary from Theorem 1, and the fact that the problem is impossible when θ is equal to π because it is as if robots do not have compasses.

Corollary 1 *$\theta < \pi$ is a tight bound on solving the gathering of two oblivious mobile robots with inconsistent compasses.*

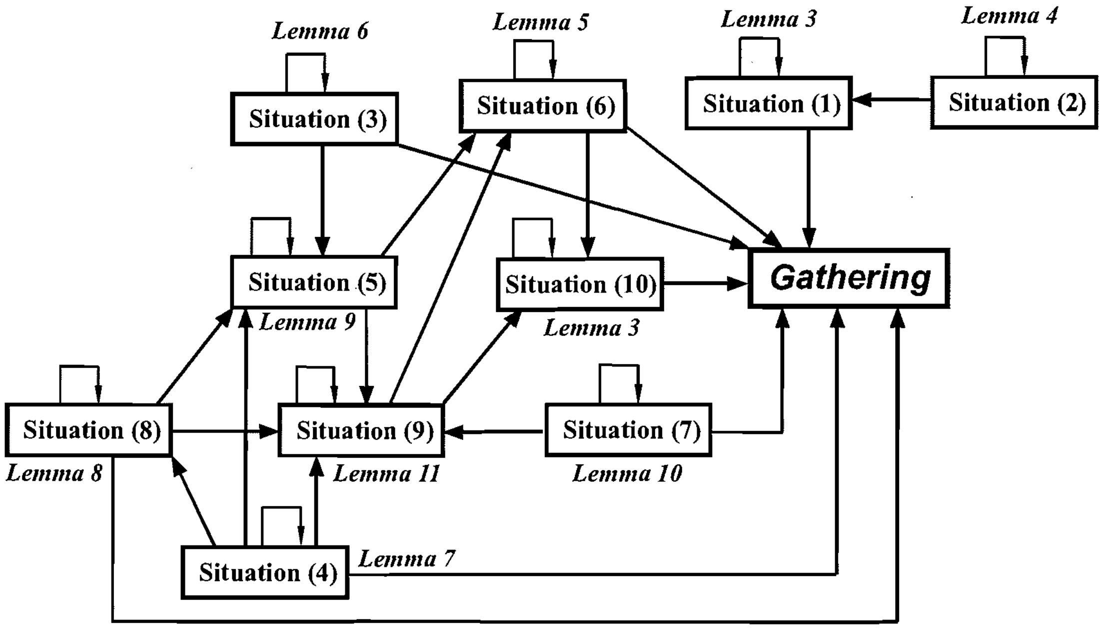


Figure 6. Diagram of possible transitions between situations.

5 Conclusion

In this paper, we presented a tight bound on the degree of divergence of robots' compasses for solving the gathering of two asynchronous memory-less mobile robots. In particular, we gave an algorithm that solves the problem in finite time when compasses can be inconsistent by an angle strictly smaller than 180°, and we show that this bound is a tight bound. Also, our algorithm is self-stabilizing.

The natural problem of generalizing our algorithm to an arbitrary finite number of robots remains open. We conjecture that a smaller bound on the degree of divergence of the compasses is required. Another interesting issue to investigate is to consider the variance in the north directions indicated by compasses over time, and how it affects the solvability of the gathering problem. This also remains an open question.

References

[1] Agmon, N., Peleg, D.: Fault-tolerant Gathering Algorithms for Autonomous Mobile Robots. In: SIAM J. Comput. **36**(1), (2006) 56–82

[2] Ando, H., Oasa, Y., Suzuki, I., Yamashita, M.: Distributed memoryless point convergence algorithm for mobile robots with limited visibility. IEEE Trans. on Robotics and Automation **15**(5) (1999) 818–828

[3] Cieliebak, M.: Gathering non-oblivious mobile robots. In: Proc. 6th Latin American Symp. on Theoretical Informatics (LATIN'04). (2004) 577–588

[4] Cieliebak, M., Flocchini, P., Prencipe, G., Santoro, N.: Solving the robots gathering problem. In: Proc. Intl. Colloquium on Automata, Languages and Programming (ICALP'03). (2003) 1181–1196

[5] Cohen, R., Peleg, D.: Convergence of Autonomous Mobile Robots With Inaccurate Sensors and Movements. In: Proc. 23rd International Symposium on Theoretical Aspects of Computer Science (STACS'06). LNCS (2006)

[6] Défago, X., Gradinariu, M., Messika, S., Raipin-Parvédy, P.: Fault-tolerant and self-stabilizing mobile robots gathering. In: Proc. 20th Intl. Symp. on Distributed Computing (DISC'06). LNCS 4167 (2006) 46–60

[7] Dolev, S.: Self-Stabilization. MIT Press (2000)

[8] Flocchini, P., Prencipe, G., Santoro, N., Widmayer, P.: Gathering of asynchronous robots with limited visibility. Theor. Comput. Sci. **337**(1–3) (2005) 147–168

[9] Imazu, H., Itoh, N., Katayama, Y., Inuzuka, N., Wada, K.: A Gathering Problem for Autonomous Mobile Robots with Disagreement in Compasses (in Japanese). In: 1st Workshop on Theoretical Computer Science in Izumo, Japan (2005) 43–46

[10] Prencipe, G.: Instantaneous Actions vs. Full Asynchronicity: Controlling and Coordinating a Set of Autonomous Mobile Robots. In Proc. 7th Italian Conference on Theoretical Computer Science (ICTCS'01). (2001) 154–171

[11] Prencipe, G.: CORDA: Distributed coordination of a set of autonomous mobile robots. In: Proc. 4th European Research Seminar on Advances in Distributed Systems (ERSADS'01), Bertinoro, Italy (2001) 185–190

[12] Prencipe, G.: On the feasibility of gathering by autonomous mobile robots. In: Proc. Colloquium on Structural Information and Communication Complexity (SIROCCO'05). (2005) 246–261

[13] Schneider, M.: Self-stabilization. ACM Computing Surveys **25**(1) (1993) 45–67

[14] Souissi, S., Défago, X., Yamashita, M.: Using Eventually Consistent Compasses to Gather Oblivious Mobile Robots with Limited Visibility. In: Proc. 8th Intl. Symp. on Stabilization, Safety, and Security of Distributed Systems (SSS'06). LNCS 4280 (2006) 471–487

[15] Souissi, S., Défago, X., Yamashita, M.: Gathering Asynchronous Mobile Robots with Inaccurate Compasses. In: Proc. 10th International Conference On Principles Of Distributed Systems (OPODIS '06). LNCS. 4305 (2006) 333–349

[16] Suzuki, I., Yamashita, M.: Distributed anonymous mobile robots: Formation of geometric patterns. SIAM Journal on Computing **28**(4) (1999) 1347–1363

[17] Tel, G.: Introduction to Distributed Algorithms. Cambridge University Press (2001)

[18] Czyzowicz, J., Gasieniec, L., Pelc, A.: Gathering Few Fat Mobile Robots in the Plane. In: Proc. 10th International Conference On Principles Of Distributed Systems (OPODIS'06). LNCS. 4305 (2006) 350–364

[19] Beni, G., Hackwood, S.: Coherent Swarm Motion under Distributed Control. In: Proc. International Symposium on Distributed Autonomous Robotic Systems (DARS'92). (1992) 39–52

[20] Matarić, M. J.: From Local Interactions to Collective Intelligence. In: Proc. The Biology and Technology of Intelligent Autonomous Agents. 144 (1995) 275–295

[21] Martinoli, A., Mondada, F.: Collective and Cooperative Group Behaviours: Biologically Inspired Experiments in Robotics. In: Proc. 4th Symposium on Experimental Robotics (ISER'95). (1995) 3–10

[22] Balch, T., Arkin, R. C.: Behavior-based Formation Control for Multi-robot Teams. IEEE Trans. on Robotics and Automation, **14** (6) (1998) 926–939

[23] Cao, Y. U., Fukunaga, A. S., Kahng, A. B.: Cooperative mobile Robotics: Antecedents and Directions. Journal of Autonomous Robots. **4** (1997) 1-23

[24] Y. Kawauchi, Y., Inaba, M., Fukuda, T.: A principle of distributed decision making of Cellular Robotic System (CEBOT). In: Proc. IEEE International Conference on Robotics and Automation. **3** (1993) 833–838