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Description	

An Analytical Method for MMSE MIMO Turbo Equalizer EXIT Chart Computation

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***Abstract*— A frequency domain soft-interference canceling minimum mean-square error (MMSE) turbo equalizer is studied. A computationally efficient method for computing extrinsic information transfer (EXIT) function of the equalizer is proposed. The method is based on the Gaussian approximation for the equalizer output and it produces an approximate EXIT chart in a fraction of computational load compared to previous numerical methods. The accuracy of the method is verified for both single-input single-output (SISO) and multiple-input multiple output (MIMO) channels.**

***Index Terms*— Turbo equalizers, convergence analysis, EXIT charts, frequency-selective channels, space-time channels.**

I. Introduction

Turbo equalization [1] is one of the most compelling techniques to implement well-performing equalizers without requiring excessive computational complexity. The complexity advantage is the outcome of the separation of equalization and decoding, while the high performance is achieved by iteratively exchanging soft information between the equalizer and the channel decoder. In this paper we concentrate on the application and analysis of turbo equalization based on soft interference canceling followed by minimum mean-square error (MMSE) linear filtering. The approach was originally proposed in [2], [3] for iterative detection of coded direct-sequence code division multiple access signals, applied to channel equalization in [4], [5], and further to multiple-input multiple-output (MIMO) channel equalization in [6]. Frequency-domain approaches for MMSE turbo equalization are proposed in [7] and [8] for SISO and MIMO systems, respectively. The approach differs from decision-feedback equalization (DFE) [9] and interference cancellation MMSE [10] based approaches in that these utilize two optimized filters for feed-forward and -back filtering, whereas the approach considered here utilizes only one filter in succession of the canceling.

The performance of the system is dependent on the convergence characteristic of the equalizer. A useful tool to analyze the convergence of iterative detection algorithms is density evolution [11] that tracks the evolution of the distribution of extrinsic information in the algorithm. The method of extrinsic information transfer (EXIT) charts [12] is a simplification of density evolution and assumes a known (usually Gaussian) distribution for the soft information. The EXIT functions describing how each soft-input-soft-output block transforms input metrics into output metrics were initially obtained through numerical simulations [12]. Numerically obtained EXIT functions for the MMSE turbo equalizer have been used in e.g. [13], [14], [15] to convergence analysis in selected fixed channels. Even though these approaches give insight into the convergence behavior of the equalizer, the limitation to a single channel realization makes their direct application to frequency-selective fading channels impractical. Recently, analytical approaches to the evaluation of the convergence of turbo equalizers and iterative detectors have been proposed in e.g. [16], [17], [18]. In this paper we develop a convergence analysis technique for a frequency domain MMSE turbo equalizer. We utilize the expression for the frequency domain MMSE filter to evaluate the equalizer output mutual information with the transmitted bits and to derive an approximate EXIT function of the equalizer. In this way, the computational burden for determining the EXIT chart for a channel realization can be mitigated. Random fading channels can then be represented by a sufficiently large set of analytically computed EXIT charts, making it possible to apply statistical convergence analysis for turbo equalizers in frequency-selective fading channels.

This paper is organized as follows. The system model is described in Section II and the MMSE turbo equalizer algorithm presented in Section III. The method to compute the analytical EXIT function of the equalizer is described in Section IV and its accuracy verified in Section V through comparison with simulations. The paper concludes with a discussion and summary.

II. System Model

The system employs N_T transmit and N_R receive antennas. The transmission is bit-interleaved coded modulation (BICM) with M bits per symbol, and each transmit antenna sends an independently encoded and modulated symbol stream. In the following vectors are marked with bold lowercase, matrices with bold uppercase notations. An estimate of a variable is denoted by $\hat{(\cdot)}$. The operator $\mathrm{diag}(\cdot)$ with a vector argument denotes a diagonal matrix with the vector elements on the diagonal, and with a matrix argument a diagonal matrix with the diagonal elements those of the argument matrix.

The information bits of each transmit antenna are encoded by a channel code, interleaved and segmented into N groups of M bits $\mathbf{b}_{n_T}(n) = \{b_{n_T,1}(n), \ldots, b_{n_T,m}(n), \ldots, b_{n_T,M}(n)\}, b_{n_T,m}(n) \in \{0, 1\}$.

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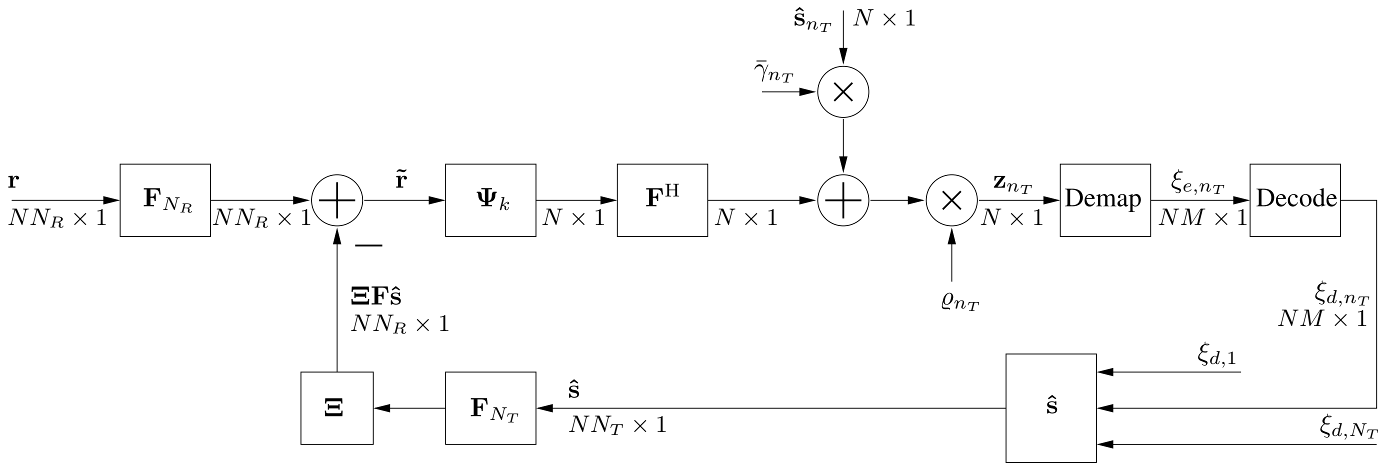

Fig. 1. A block diagram of the frequency domain MMSE turbo equalizer for antenna n_T.

Each group is mapped into a complex symbol with a symbol mapper $B : \mathbf{b}_{n_T}(n) \rightarrow s_{n_T}(n)$ consisting of a set of 2^M complex points labeled with the binary input vector $\mathbf{b}_{n_T}$. The transmitted symbols can be arranged into the vector

$$\mathbf{s} = \left[\mathbf{s}_1^{\mathrm{T}}, \ldots, \mathbf{s}_{n_T}^{\mathrm{T}}, \ldots, \mathbf{s}_{N_T}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{NN_T}, \tag{1}$$

with

$$\mathbf{s}_{n_T} = \left[s_{n_T}(1), \ldots, s_{n_T}(n), \ldots, s_{n_T}(N)\right]^{\mathrm{T}} \in \mathbb{C}^{N}, \tag{2}$$

where N is the length of the block of transmitted symbols, and n_T enumerates the transmit antennas. In this paper we normalize the constellation so that $\mathrm{E}\left\{|s_{n_T}(n)|^2\right\} = 1$.

The channel is assumed to be static within a block, and varying independently between blocks. A prefix, whose length exceeds the maximum multipath spread, is prepended to each transmitted block. After the removal of the prefix at the receiver the space-time multipath channel matrix $\mathbf{H}$ with L separable paths, N_T transmit and N_R receive antennas is given by the block-circulant matrix

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{1,N_T} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{N_R,1} & \cdots & \mathbf{H}_{N_R,N_T} \end{pmatrix} \in \mathbb{C}^{NN_R \times NN_T}. \tag{3}$$

The multipath channel response between transmit antenna n_T and receive antenna $j = 1 \ldots N_R$ is given by the vector

$$\mathbf{h}_{n_T,n_R} = \left[h_{n_T,n_R,1}, \ldots, h_{n_T,n_R,l}, \ldots, h_{n_T,n_R,L}\right]^{\mathrm{T}} \in \mathbb{C}^{L \times 1}. \tag{4}$$

Each $\mathbf{H}_{n_T,n_R} \in \mathbb{C}^{N \times N}$ in (3) denotes a circulant matrix with the multipath channel response (4) on its first column. The block-circulant channel matrix (3) can be block-diagonalized by the block-Fourier matrix

$$\mathbf{F}_{N_R} = \mathbf{I}_{N_R} \otimes \mathbf{F} \in \mathbb{C}^{NN_R \times NN_R}, \tag{5}$$

where $\mathbf{I}_{N_R}$ is an identity matrix of dimension N_R, $\otimes$ denotes the Kronecker product, and where the Fourier matrix $\mathbf{F} \in \mathbb{C}^{N \times N}$ has each element defined as $[\mathbf{F}]_{i,j} = N^{-\frac{1}{2}} e^{-\jmath \frac{2\pi}{N}(i-1)(j-1)}, \jmath = \sqrt{-1}, i, j = 1 \ldots N$. The block-circulant channel matrix can then be de-composed into

$$\mathbf{H} = \mathbf{F}_{N_R}^{\mathrm{H}} \mathbf{\Xi} \mathbf{F}_{N_T}. \tag{6}$$

The frequency domain channel matrix is a block matrix with diagonal blocks and is given by

$$\mathbf{\Xi} = \begin{pmatrix} \mathbf{\Xi}_{1,1} & \cdots & \mathbf{\Xi}_{1,N_T} \\ \vdots & \ddots & \vdots \\ \mathbf{\Xi}_{N_R,1} & \cdots & \mathbf{\Xi}_{N_R,N_T} \end{pmatrix} \in \mathbb{C}^{NN_R \times NN_T}. \tag{7}$$

For both (3) and (7), one column of blocks is denoted with the subscript n_T (as in $\mathbf{H}_{n_T}$). When the symbols (1) pass through the frequency selective channel, the received signal is given by

$$\mathbf{r} = \mathbf{H}\mathbf{b} + \mathbf{w} \in \mathbb{C}^{NN_R \times 1}, \tag{8}$$

where $\mathbf{w} \in C^{NN_R \times 1}$ is the vector of circularly symmetric zero-mean complex Gaussian noise with covariance $\mathrm{E}\left[\mathbf{w}\mathbf{w}^{\mathrm{H}}\right] = \mathbf{I}\sigma_0^2$. The receiver SNR (in dB) is defined as $\mathrm{SNR}_{[dB]} = -10 \log_{10}\left(\sigma_0^2\right)$.

III. Frequency-Domain Block MMSE MIMO Turbo Equalizer

The derivation of the soft-canceling MMSE turbo algorithm has been presented in detail [2] for DS-CDMA systems and for channel equalization by using MMSE estimation theory in [14]. A frequency domain algorithm derivation in the spirit of [14] is provided in [8]. A brief overview of the algorithm is given in the following along with a block diagram in Fig. 1.

Prior information is provided by the N_T channel decoders in the form of likelihood ratios ξ_{d,n_T} which are stacked into the vector $\boldsymbol{\xi}_d \in \mathbb{R}^{NMN_T \times 1}$. The expected value the transmitted symbols $\hat{\mathbf{s}} = \mathrm{E}\left[\mathbf{s}|\boldsymbol{\xi}_d\right] \in \mathbb{C}^{NN_T \times 1}$ is computed using the prior information assuming independent encoded bits. The vector of symbol estimates is used in constructing an estimate of the received signal, which is subtracted from the received signal, creating the residual

$$\tilde{\mathbf{r}} = \mathbf{r} - \mathbf{H}\mathrm{E}\left[\mathbf{s}|\boldsymbol{\xi}_d\right] \in \mathbb{C}^{NN_R \times 1}. \tag{9}$$

The output vector of the time-domain equalizer for the transmit antenna n_T, giving the MMSE estimates of the transmitted symbols, can be expressed as

$$\mathbf{z}_{n_T} = \left(\mathbf{I} + \mathbf{D}_{n_T}\mathbf{B}_{n_T}\right)^{-1}\left[\mathbf{D}_{n_T}\hat{\mathbf{s}}_{n_T} + \mathbf{H}_{n_T}^{\mathrm{H}}\boldsymbol{\Sigma}^{-1}\tilde{\mathbf{r}}\right] \in \mathbb{C}^{N\times 1}, \tag{10}$$

where we have used the diagonal matrices

$$\mathbf{D}_{n_T} = \mathrm{diag}\left\{\mathbf{H}_{n_T}^{\mathrm{H}}\boldsymbol{\Sigma}^{-1}\mathbf{H}_{n_T}\right\} \in \mathbb{R}^{N\times N} \tag{11}$$

$$\mathbf{B}_{n_T} = \mathrm{diag}\left\{\left|\hat{\mathbf{s}}_{n_T}\right|^2\right\} \in \mathbb{R}^{N\times N}, \tag{12}$$

and where $\boldsymbol{\Lambda} = \mathrm{E}\left[|\hat{\mathbf{s}} - \mathbf{s}|^2\right] \in \mathbb{R}^{N\times N}$ is the diagonal covariance matrix of the transmitted symbols after soft interference cancellation, $\mathbf{H}_{n_T}$ is the n_Tth column of $\mathbf{H}$, and

$$\boldsymbol{\Sigma} = \mathrm{E}\left\{\tilde{\mathbf{r}}\tilde{\mathbf{r}}^{\mathrm{H}}\right\} \tag{13}$$

$$= \mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^{\mathrm{H}} + \sigma_0^2\mathbf{I} \in \mathbb{C}^{NN_R\times NN_R}. \tag{14}$$

If the channel decomposition (6) is applied into (10), the filtering equation can be converted to the frequency domain. The remaining problem is, that the symbol error covariance matrix $\boldsymbol{\Lambda}$ is a full matrix and the conversion does not directly offer any advantage in computational complexity. However, the symbol error covariance matrix is hermitian symmetric and circulant, and can be approximated by a block-diagonal matrix with diagonal blocks given by

$$\boldsymbol{\Delta} = \mathbf{F}_{N_T}\boldsymbol{\Lambda}\mathbf{F}_{N_T}^{\mathrm{H}} \in \mathbb{C}^{NN_T\times NN_T} \tag{15}$$

$$\approx \mathrm{diag}\left\{\bar{\lambda}_1, \ldots, \bar{\lambda}_{n_T}, \ldots, \bar{\lambda}_{N_T}\right\} \otimes \mathbf{I}_N = \boldsymbol{\Delta}_a, \tag{16}$$

where the scalar

$$\bar{\lambda}_{n_T} = \frac{1}{N}\sum_{n=1}^{N}\lambda_{n_T,n} \tag{17}$$

represents the average residual symbol interference after cancellation. With the approach above, (10) can be approximated by a frequency domain filter whose output $\mathbf{z}_{n_T,a}$ is given by

$$\mathbf{z}_{n_T,a} = \left(1 + \bar{\gamma}_{n_T}\varrho_{n_T}\right)^{-1}\left[\bar{\gamma}_{n_T}\hat{\mathbf{b}}_{n_T} + \mathbf{F}^{\mathrm{H}}\boldsymbol{\Psi}_{n_T}\mathbf{F}_{N_R}\tilde{\mathbf{r}}\right] \in \mathbb{C}^{N\times 1}, \tag{18}$$

where the following definitions have been used

$$\bar{\gamma}_{n_T} = \frac{1}{N}\mathrm{tr}\left\{\boldsymbol{\Xi}_{n_T}^{\mathrm{H}}\left(\boldsymbol{\Xi}\boldsymbol{\Delta}_a\boldsymbol{\Xi}^{\mathrm{H}} + \sigma_0^2\mathbf{I}\right)^{-1}\boldsymbol{\Xi}_{n_T}\right\} \tag{19}$$

$$\varrho_{n_T} = \frac{1}{N}\sum_{n=1}^{N}\left|\hat{s}_{n_T}(n)\right|^2 = 1 - \bar{\lambda}_{n_T} \tag{20}$$

$$\boldsymbol{\Psi}_{n_T} = \boldsymbol{\Xi}_{n_T}^{\mathrm{H}}\left(\boldsymbol{\Xi}\boldsymbol{\Delta}_a\boldsymbol{\Xi}^{\mathrm{H}} + \sigma_0^2\mathbf{I}\right)^{-1} \in \mathbb{C}^{N\times NN_R}. \tag{21}$$

The physical meaning of the scalar (19) is not immediately obvious, but considering (18) it can be seen as one to the two multiplicative terms of the desired symbol estimate amplitude at the equalizer output. In the case of full prior information, it reduces to an expression equal to the MMSE SINR as given later in (25). The scalar (20) represents the average energy of the prior symbol estimates. The matrix given by (21) defines the frequency domain filter, consisting of a single coefficient per frequency bin, for the residual signal $\tilde{\mathbf{r}}$. The filter output for antenna n_T for each symbol can be approximated as the scalar

$$z_{n_T,a}(n) = \mu_{n_T,a}s_{n_T}(n) + w, \tag{22}$$

where $w \sim \mathcal{N}\left(0, \mu_{n_T,a}(1 - \mu_{n_T,a})\right)$ and

$$\mu_{n_T,a} = \bar{\gamma}_{n_T}\left(1 + \bar{\gamma}_{n_T}\varrho_{n_T}\right)^{-1}. \tag{23}$$

The SNR of the equivalent channel (22) is given by

$$\mathcal{L}_{n_T,a} = \frac{\mu_{n_T,a}}{1 - \mu_{n_T,a}} \tag{24}$$

$$= \frac{\bar{\gamma}_{n_T}}{1 + \bar{\gamma}_{n_T}\left(\varrho_{n_T} - 1\right)}. \tag{25}$$

IV. Equalizer Convergence

In this section we derive an approximate EXIT function of a frequency domain MMSE turbo equalizer assuming a BICM transmission. The mutual information between the likelihood $\boldsymbol{\xi}_{d,n_T}$ provided by the decoder n_T and the transmitted data $\mathbf{b}_{n_T}$ for transmit antenna n_T is denoted as [19]

$$I_{d,n_T} = N^{-1}\lim_{N\to\infty} I\left(\mathbf{b}_{n_T}; \boldsymbol{\xi}_{d,n_T}\right), \tag{26}$$

and the mutual information between the de-mapper output likelihood $\boldsymbol{\xi}_{e,n_T}$ and the transmitted data as

$$I_{e,n_T} = N^{-1}\lim_{N\to\infty} I\left(\mathbf{b}_{n_T}; \boldsymbol{\xi}_{e,n_T}\right). \tag{27}$$

The EXIT function of the equalizer provides the mapping $I_{e,n_T} = f\left(I_{d,n_T}\right)$, which is conditioned on the channel response and receiver noise variance.

When decoder feedback is transformed into soft symbol estimates for interference cancellation, the feedback mutual information is embodied in the values of $\bar{\lambda}_{n_T}$ and ϱ_{n_T} given by (17) and (20), respectively. Assuming the decoder feedback likelihoods ξ_{d,n_T} are i. i. d. and Gaussian distributed, we can use the expected value of $\bar{\lambda}_{n_T}$ and ϱ_{n_T} over the distribution of ξ_{d,n_T} as a basis for computing the equalizer output for infinite block length. In practice, a look-up table for the translation of mutual information to $\mathrm{E}\left\{\bar{\lambda}_{n_T}\right\}$ and $\mathrm{E}\left\{\varrho_{n_T}\right\}$ can be pre-computed by Monte-Carlo integration to avoid repetitive computations. The dependence of $\bar{\lambda}_{n_T}$ and ϱ_{n_T} on the prior information, and, thus, the look-up table, is modulation and mapping specific.

The mutual information between transmitted bits and the de-mapper output is computed as the average bitwise mutual information of the equivalent Gaussian channel output, given by (22), as [20]

$$I_{e,n_T} = C_B\left(\mu_{n_T,a}, B\right) - \frac{1}{2M}\sum_{j=1}^{M}\sum_{k=0}^{1}C_{B_k^j}\left(\mu_{n_T,a}, B\right), \tag{28}$$

where $C_{B_k^j}$ is the constellation constrained capacity (CCC) of the jth sub-constellation consisting of the points where the bit j takes the value k. The CCC of a constellation in an AWGN channel with variance $\mu_{n_T,a}\left(1 - \mu_{n_T,a}\right)/2$ per dimension is defined as

$$C_B\left(\mu_{n_T,a}, B\right) = M - 2^{-M}\times$$
$$\sum_{i=1}^{M}\mathrm{E}\left\{\log_2\sum_{j=1}^{M}\exp\left(-\frac{\left|\mu_{n_T,a}\left(b_i - b_j\right) + v\right|^2 - |v|^2}{\mu_{n_T,a}\left(1 - \mu_{n_T,a}\right)/p}\right)\right\}. \tag{29}$$

where $p = 1$ for complex and $p = 2$ for real modulations. The sub-constellations are defined by the mapping rule assumed

TABLE I
APPROXIMATION PARAMETERS FOR (32), GRAY MAPPING

Modulation	H_1	H_2	H_3
BPSK[21]	0.3073	0.8935	1.1064
QPSK	0.3073	0.8935	1.1064
8PSK	0.2516	0.7274	1.2392
16QAM	0.2224	0.6783	1.3617

TABLE II
SISO CHANNEL COEFFICIENTS

Nr.	1	2	3	4
Value	1.06 - j0.28	-1.77 - j1.42	2.69 - j2.19	-2.27 + j0.57
Nr.	5	6	7	8
Value	0.42 - j0.99	-0.85 - j0.78	1.98 - j1.06	1.56 + j3.11
Nr.	9	10		
Value	2.12 - j0.85	-0.92 + 0.57		

and the value of $C_{B_k^j}$ can be computed with (29) by setting $M \rightarrow M-1$ and defining $b_{[i,j]}$ for the sub-constellation. The expectation in (29) is taken over the two-dimensional Gaussian distribution of the zero-mean complex noise variable v. In the special case of real valued binary modulation (29) specializes into the J-function of [12].

By defining a parameter σ

$$\sigma = \begin{cases} 2\sqrt{2\mathcal{L}_{n_T,a}} & \text{real modulation} \quad (30) \\ 2\sqrt{\mathcal{L}_{n_T,a}} & \text{complex modulation} \quad (31) \end{cases}$$

we can approximate the function (28) for any modulation mapping, in order to be used in numerical computations, by (c.f. [21])

$$I_{e,n_T}(\sigma) \approx \left(1 - 2^{-H_1 \sigma^{2H_2}}\right)^{H_3} \quad (32)$$

where the mapping-specific parameters H_1, H_2 and H_3 can be obtained by least-squared curve fitting. The parameter values for BPSK and Gray-mapped QPSK, 8PSK and 16QAM are listed in Table I. In principle, similar parameters can be found for any symbol mapping to apply the analysis.

V. ACCURACY OF ANALYSIS

A. SISO Case

The accuracy of the proposed method for computing the approximate EXIT function for the frequency domain equalizer was tested by comparing the analytical results with EXIT functions obtained by numerical simulations using histograms.

Two scenarios were considered, the first of which was a single-input-single-output case with the absolute value of the channel coefficients given by Table II. The equalizer was simulated with a block length of 16384 coded BPSK symbols over 10 blocks per each EXIT function point. The measured mutual information at the equalizer output was compared to the value computed using the J-function approximation (32).

The result indicates that in a SISO case the analysis is very accurate when $\mathbf{I}_{d,n_T} \rightarrow 1$, regardless of the SNR. The analysis is relatively less accurate when prior information is low. A closer examination of the test simulation results reported in

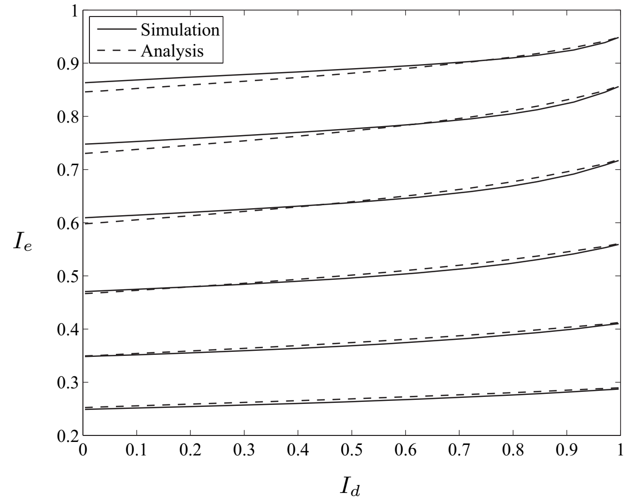

Fig. 2. Analytical and simulated EXIT functions, and the analysis error at (bottom to top) $-3, -1, 1, 3, 5, 7$dB SNR.

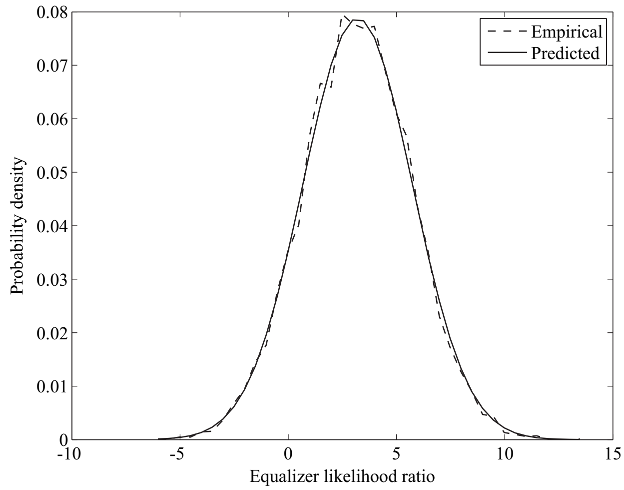

Fig. 3. Empirical and estimated equalizer output likelihood distribution at -3dB SNR.

Fig. 2 confirm that the relative error of the analysis compared to the simulation is less than 2.5% in the tested case.

B. MIMO Case

The second verification case was a two-by-two spatially uncorrelated MIMO scenario, and the utilized channel was randomly generated assuming 20 channel coefficients with equal average energy. First, the likelihood ratio distribution of the equalizer output for this case was compared to that of the estimated equivalent Gaussian channel model given by (22). The comparison is depicted in Fig. 3 for SNR -3dB and prior information level of $I_{d,1} = I_{d,2} = 0.5$ and demonstrates excellent agreement between the empirical and estimated channels.

Due to the two transmit antennas with independent channel coding, the EXIT function becomes two-variate with two inputs and two outputs. Such a system can be visualized with a three-dimensional EXIT chart, where each of the two function outputs is represented by a surface. In Fig. 4, such a surface is shown for both simulated system and an analytically computed

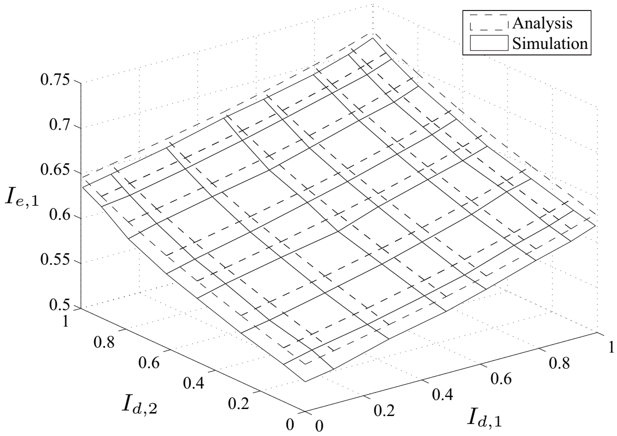

Fig. 4. Analytical and simulated EXIT functions at -3dB SNR.

equivalent. In this case, the relative error can be as high as 11% at low SNR, but reduces quickly at higher SNRs. In general, the accuracy of the method can be considered sufficient for generic evaluations of convergence. If absolute performance evaluations in terms of bit or frame error rate are required, the accuracy should be further evaluated. However, the analysis is accurate with high prior information levels that co-incide with the desired operation point where succesful channel decoding is possible. This suggests the analysis should be able to accurately predict the fixed point of equalizer convergence for the cases when the system is operating in a desired manner. However, if the EXIT function prediction is inaccurate for low and intermediate prior information levels, the analysis may give misleading results about the algorithm reaching that fixed point.

VI. Discussion and Summary

The accuracy of the proposed computation method relies on the Gaussian approximation accuracy. In cases, where the Gaussian approximation is known to be less accurate, care must be taken when applying the method. These cases include channels with a small number of dominant paths and channels with spatial correlation.

One of the possible applications of the method presented in this paper is channel code optimization for turbo equalization [22] in fading channels. This can be based on randomly generated sets of analytical EXIT functions which are jointly used for designing the channel code. Another potential application can be found by noting the effective equalizer output SNR is dependent on the parameters of the spatial channel. Thus, the analysis provides a method for studying the dependency between channel parameters and turbo equalizer convergence.

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