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Description	

Hybrid Turbo Multiuser Detection for OFDM Transmission with Spatially-Correlated Channels

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***Abstract*— This letter proposes a novel hybrid turbo multi-user (MU) detection technique based on group-wise soft interference canceling minimum mean-square error filtering (SC-MMSE) combined with maximum a posteriori (MAP) signal detection for OFDM MU systems. A new user grouping algorithm is developed that exploits the knowledge of the pairwise spatial correlation among the channels of each group, of which aim is to reduce noise enhancement due to the MMSE interference suppression of highly correlated user signals. It is shown that the proposed detector can achieve significant performance improvements over SC-MMSE in the presence of high spatial channel correlation.**

***Index Terms*— Hybrid turbo multi-user detection, OFDM MU-SIMO transmission, Spatially-correlated channels, EXIT analysis.**

I. INTRODUCTION

THE primary goal of this letter is to derive a hybrid group-wise signal detection technique for OFDM multi-user single-input multiple-output (MU-SIMO) communication systems. The conceptual basis of the proposed technique is simply that the MAP detection is used to detect the users signals in the same group, and that the SC-MMSE principle [1] is used to separate the groups. The proposed technique is referred to as Hy SC-MMSE-MAP for the notational convenience. Account is taken of the spatial correlation of multiple users in seeking the user grouping, by which performance sensitivity to the spatial correlation is minimized without invoking the necessity of detecting all users by MAP that requires computational complexity of an exponential order of the number of the simultaneous users. Convergence property of the proposed technique is compared with the case where all users are detected by SC-MMSE through extrinsic information transfer (EXIT) chart analysis, and is shown to be consistent with the performance simulation results.

The organization of this letter follows the standard flow of algorithm proposal articles. Section III is the major contribution of this letter, where the algorithm for the Hy SC-MMSE-MAP signal detection technique as well as the grouping technique is presented.

II. SYSTEM MODEL

Consider a single cell time- and frequency-synchronous MU OFDM system, in which the base station having M receive antennas receives signals from N active users, each equipped with a single transmit antenna. For each user, the transmission is assumed to be bit-interleaved coded modulation (BICM), where the information block of length K with symbols taking from $\{-1,+1\}$ is binary encoded, randomly interleaved, and OFDM-modulated. For the ease of formulation, we assume binary phase keying (BPSK) onwards, but the extension to more generic modulation format is rather straightforward. Employing a cyclic prefix of sufficient length, the received signals of the ith ($i = 1, ..., N_p$) OFDM slot can be expressed as[1]

$$\mathbf{y}[i,q] = \mathbf{H}[i,q]\mathbf{b}[i,q] + \mathbf{n}[i,q], \tag{1}$$

where $\mathbf{y}[i,q]$ and $\mathbf{b}[i,q]$ represent, respectively, the received and transmitted signals of all users at the qth ($q = 0, ..., Q-1$) sub-carrier, $\mathbf{H}[i,q]$ is the $M \times N$ matrix of complex channel frequency responses, which is assumed to be time-invariant over N_p OFDM symbols and perfectly known at the receiver, and $\mathbf{n}[i,q]$ is the noise vector of i.i.d. complex Gaussian random variables with zero mean and identical covariance $\sigma_0^2\mathbf{I}_M$ for all sub-carriers. In this letter, we employ the Kronecker spatial fading correlation model [2], given by

$$\mathbf{H}[i,q] = \sum_{l=0}^{L-1} \mathbf{R}^{1/2}\mathbf{G}_l[i]\mathbf{S}^{1/2} \exp\left(\frac{-j2\pi lq}{Q}\right), \tag{2}$$

where $\mathbf{R}$, and $\mathbf{S}$ are the receive and transmit correlation matrices, respectively, $\mathbf{G}_l[i]$ is an $M \times N$ matrix comprised of i.i.d. complex Gaussian entries with zero mean and unit variance, assumed to be independent for different l, and L is the number of path components of the frequency-selective fading channels. For notational simplicity, we will henceforth omit the sub-carrier index q and the OFDM slot index i.

III. HYBRID TURBO DETECTION FOR OFDM MODULATION

The turbo receiver performs iterative detection and decoding employing soft-input-soft-output (SISO) algorithms. It consists of the Hy SC-MMSE-MAP detector and a set of N channel decoders, each separated by interleavers and deinterleavers. Performing iterative processing, extrinsic log-likelihood-ratios (LLRs) about the coded bits are exchanged between these stages several times following the turbo principle [3]. The

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[1] *Notation:* The $K \times K$ dimensional identity matrix is written as $\mathbf{I}_K$, and the $K \times 1$ all-one vector as $\mathbf{e}_K$. diag$\{\mathbf{x}\}$ denotes the diagonal matrix with values of the vector $\mathbf{x}$ on the diagonal, $diag\{\mathbf{A}\}$ is the operator which extracts the diagonal elements of matrix $\mathbf{A}$, $\|\cdot\|$ represents the Euclidean-norm, and $|\mathcal{X}|$ the cardinality of a set $\mathcal{X}$. The relative complement of two sets $\mathcal{A}$ and $\mathcal{B}$ is denoted by $\mathcal{A} - \mathcal{B}$.

Hy SC-MMSE-MAP detector is based on group-wise soft-cancellation MMSE filtering and MAP detection. It first separates the N users into H disjoint groups $\{\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_H\}$, such that each group $\mathcal{G}_h = (a_1, ..., a_{G_h})$ contains G_h integers which correspond to indexes of elements in $\mathbf{b}$. For each group $\mathcal{G}_h$, soft cancellation of multiple-access interference components that originate from the remaining $H-1$ groups is performed by utilizing the extrinsic information fed back from channel decoding. The signal vector representing the SC output can be expressed as

$$\tilde{\mathbf{y}}_{\mathcal{G}_h} = \mathbf{y} - \mathbf{H}\bar{\mathbf{b}} + \mathbf{H}_{\mathcal{G}_h}\bar{\mathbf{b}}_{\mathcal{G}_h} = \tilde{\mathbf{y}} + \mathbf{H}_{\mathcal{G}_h}\bar{\mathbf{b}}_{\mathcal{G}_h}, \tag{3}$$

where $\bar{\mathbf{b}} = [\bar{b}_1, ..., \bar{b}_N]^T$ and $\bar{\mathbf{b}}_{\mathcal{G}_h} = [\bar{b}_{a_1}, ..., \bar{b}_{a_{G_h}}]^T$ with $\bar{b}_i = \mathrm{E}\{b_i\}$ are vectors of soft estimates of the elements in $\mathbf{b}$ and $\mathbf{b}_{\mathcal{G}_h} = [b_{a_1}, ..., b_{a_{G_h}}]^T$, respectively, and $\mathbf{H}_{\mathcal{G}_h} = [\mathbf{h}_{a_1}, ..., \mathbf{h}_{a_{G_h}}]$ denotes the channel for the hth group with $\mathbf{h}_{a_l}$ being the a_lth column of $\mathbf{H}$. For further suppression of residual interference components, $\tilde{\mathbf{y}}_{\mathcal{G}_h}$ is filtered with each group's MMSE filter, whose weighting matrix $\mathbf{W}_{\mathcal{G}_h}$ is designed such that the hth group users' signals $\mathbf{b}_{\mathcal{G}_h}$ are jointly detected. More explicitly, it satisfies the minimization criterion

$$\arg \min_{\{\mathbf{W},\mathbf{A}\}} \mathrm{E}\left\{\left\|\mathbf{W}^H_{\mathcal{G}_h}\tilde{\mathbf{y}}_{\mathcal{G}_h} - \mathbf{A}^H_{\mathcal{G}_h}\mathbf{b}_{\mathcal{G}_h}\right\|^2\right\}, \tag{4}$$

where a constraint $diag\{\mathbf{A}_{\mathcal{G}_h}\} = \mathbf{e}_{G_h}$ for matrix $\mathbf{A}_{\mathcal{G}_h}$ has to be introduced to avoid the trivial solution[2] [3]. Due to the lack of space, the derivation of the filter weighting matrix $\mathbf{W}_{\mathcal{G}_h}$ satisfying Eqn. (4) is omitted in this letter. Instead, we only present the result in Appendix. The filter output, denoted as $\mathbf{z}_{\mathcal{G}_h} = \mathbf{W}^H_{\mathcal{G}_h}\tilde{\mathbf{y}}_{\mathcal{G}_h}$, is assumed to be a Gaussian process $p(\mathbf{z}_{\mathcal{G}_h}|\mathbf{b}_{\mathcal{G}_h}) \sim \mathcal{N}_c(\mathbf{\Phi}_{\mathcal{G}_h}\mathbf{b}_{\mathcal{G}_h}, \mathbf{\Theta}_{\mathcal{G}_h})$ having a mean $\mathbf{\Phi}_{\mathcal{G}_h}\mathbf{b}_{\mathcal{G}_h}$ with $\mathbf{\Phi}_{\mathcal{G}_h} = \mathbf{W}^H_{\mathcal{G}_h}\mathbf{H}_{\mathcal{G}_h}$ and a covariance $\mathbf{\Theta}_{\mathcal{G}_h} = \mathbf{W}^H_{\mathcal{G}_h}\mathbf{\Sigma}\mathbf{W}_{\mathcal{G}_h} + \mathbf{\Phi}_{\mathcal{G}_h}\mathbf{D}^{-1}_{\mathcal{G}_h}\mathbf{\Phi}^H_{\mathcal{G}_h}$, where the matrices $\mathbf{\Sigma}$ and $\mathbf{D}_{\mathcal{G}_h}$ are defined in Eqn. (10) and Eqn. (9), respectively, in Appendix. Using the Gaussian approximation of the filter output, the group-wise MAP (symbol) detection can be performed by considering 2^{G_h} possible hypotheses on $\mathbf{b}_{\mathcal{G}_h}$. Thus, the MAP decision rule of the extrinsic LLR for the bit b_{a_n} corresponding to the nth element of $\mathbf{b}_{\mathcal{G}_h}$ is given by

$$L[b_{a_n}] = \log\frac{\sum_{\mathbf{d}\in\mathcal{A}^{<h>}_{n,+1}} p(\mathbf{z}_{\mathcal{G}_h}|\mathbf{d})Pr(\mathbf{d})}{\sum_{\mathbf{d}\in\mathcal{A}^{<h>}_{n,-1}} p(\mathbf{z}_{\mathcal{G}_h}|\mathbf{d})Pr(\mathbf{d})}, \tag{5}$$

where $\mathcal{A}^{<h>}_{n,\pm1} = \{\mathbf{d} \in \{+1,-1\}^{G_h\times 1}\,|d(n) = \pm 1\}$, and $p(\mathbf{z}_{\mathcal{G}_h}|\mathbf{d}) \sim \exp\left(-\left(\mathbf{z}_{\mathcal{G}_h} - \mathbf{\Phi}_{\mathcal{G}_h}\mathbf{d}\right)^H \mathbf{\Theta}^{-1}_{\mathcal{G}_h}\left(\mathbf{z}_{\mathcal{G}_h} - \mathbf{\Phi}_{\mathcal{G}_h}\mathbf{d}\right)\right)$. The bit extrinsic LLRs $L[b_n]$, $n = 1, ..., N$ are calculated for the signals in each group and each sub-carrier over the entire frame. Note that the computational complexity for the MAP part of the detector is at an exponential order of the group size, which may dominate the required computational effort. On the contrary, for the small size groups, the most computationally complex part is due to the inversion of the matrix given by Eqn. (10). Thus, the overall complexity is given by $O(\max\{2^{G_{max}}, M^3\})$, where $G_{max} = \max_{h=1,..,H}\{G_h\}$.

User grouping algorithm based on correlation

We now propose a correlation-based grouping algorithm, given the maximum group size G_{max} as well as the total number of groups H as design parameters. The algorithm allocates the signals from users that have a high pairwise receive correlation into one group, and hence, it reduces noise enhancement due to the MMSE interference suppression of highly correlated user signals. For the algorithm description, let us define by ρ_{kl} the *pairwise* normalized correlation between the two users' channels $\mathbf{h}_k$ and $\mathbf{h}_l$, as

$$\rho_{kl} = \frac{\mathrm{E}_q\{\mathbf{h}^H_k\mathbf{h}_l\}}{\mathrm{E}_q\{\|\mathbf{h}_k\|\}\mathrm{E}_q\{\|\mathbf{h}_l\|\}} \tag{6}$$

with $\mathrm{E}_q\{\cdot\}$ denoting the expectation operator taken over all sub-carriers q $(q = 0, ..., Q-1)$.

All possible user groupings can be represented by using an N-state trellis diagram (each state corresponds to one user) having G_{max} stages, each indexed by υ_k, and branch metrics $\eta^k_{n,n'} \triangleq \{\upsilon_k = n, \upsilon_{k+1} = n'\} = |\rho_{n,n'}|$, $n \neq n'$. A path in the trellis entering the state $\upsilon_k = n$ is denoted by $\mathbf{p}_{k,n} = (p_1, ..., p_k)$, where $p_k = n$, and $\mathbf{p}_{k,n} \in \mathcal{P}_k$ with $\mathcal{P}_k$ representing the set of paths at stage k. Each path $\mathbf{p}_{k,n}$ has associated with it a path metric $m_{k,n}$, $m_{k,n} \in \mathcal{M}_k$ with $\mathcal{M}_k$ being the set of metrics at stage k. Based on the trellis, the Viterbi algorithm finds H paths with high total path metrics, which is summarized as follows:

Grouping algorithm (Viterbi trellis search)

1. *Let* $h = 1$. *Initialize the set* $\mathcal{N}_h = \{1, ..., N\}$.
2. *Let* $k = 1$. *Initialize* $\mathcal{P}_k = \{\boldsymbol{p}_{k,n}\}$ *with* $\boldsymbol{p}_{k,n} = (n)$, $\forall n \in \mathcal{N}_h$ *and* $\mathcal{M}_k = \{m_{k,n}\}$ *with* $m_{k,n} = 0$.
3. *If* $|\mathcal{N}_h| - k \leq H - h$, *go to Step 4. Otherwise, increment* k *with one. For each state* n', *select along all state transitions* $n \rightarrow n'$, $n, n' \in \mathcal{N}_h$, $n' \notin \boldsymbol{p}_{k-1,n}$, $\boldsymbol{p}_{k-1,n} \in \mathcal{P}_{k-1}$, *the highest metric*
$$m_{k,n'} = \max_n\left\{m_{k-1,n} + \eta^k_{n,n'}\right\},\; m_{k-1,n} \in \mathcal{M}_{k-1},$$
$$\widetilde{n} = \underset{n}{\mathrm{argmax}}\left\{m_{k-1,n} + \eta^k_{n,n'}\right\},$$
and extend the candidate path $\boldsymbol{p}_{k,n'} = (\boldsymbol{p}_{k-1,\tilde{n}}, n')$, *which is added to* $\mathcal{P}_k$ *with the corresponding updated metric* $m_{k,n'}$, $m_{k,n'} \rightarrow \mathcal{M}_k$. *If* $k < G_{max}$, *repeat Step 3.*
4. *Find the highest total metric* $m_{k,n^*} = \arg\max_{m_{k,n}\in\mathcal{M}_k}\{m_{k,n}\}$, *and assign the corresponding path* $\boldsymbol{p}_{k,n^*}$ *to* $\mathcal{G}_h$, $\mathcal{G}_h = \boldsymbol{p}_{k,n^*}$, $\boldsymbol{p}_{k,n^*} \in \mathcal{P}_k$.
5. *Increment* h *with one. Produce a new set* $\mathcal{N}_h$, $\mathcal{N}_h = \mathcal{N}_{h-1} - \mathcal{G}_{h-1}$. *If* $|\mathcal{N}_h| \neq 0$, *go to Step 2. Otherwise, output the grouping* $\{\mathcal{G}_1, ..., \mathcal{G}_H\}$.

The above algorithm adopts an incremental procedure to form the H groups, at which after each iteration the number of states used for the Viterbi search is reduced. As a result, the user grouping $\{\mathcal{G}_1, ..., \mathcal{G}_H\}$ maximizing the correlation sum

$$\mathcal{G}_h = \underset{\mathbf{x}\in\mathcal{P}_{\mathcal{G}_h}}{\mathrm{argmax}} \sum_{x_i\in\mathbf{x}} |\rho_{x_i,x_{i+1}}|, h = 1, .., H \tag{7}$$

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[2] Notice that when each user is regarded as one group by itself, i.e., $H = N$, the matrix $\mathbf{A}_{\mathcal{G}_h}$ reduces to a scalar and the optimization criteria in Eqn. (4) becomes equivalent to that for the SC-MMSE detector in [1].

with $\mathcal{P}_{G_h}$ the set containing the retained paths at stage G_h which are the permutations of G_h elements from $\mathcal{N}_h$, $\mathcal{N}_h = \mathcal{N}_{h-1} - \mathcal{G}_{h-1}$ $(h = 2, .., H)$ and $\mathcal{N}_1 = \{1, ..., N\}$, is found.

IV. EXIT CHART ANALYSIS AND NUMERICAL RESULTS

The convergence property of the proposed hybrid turbo receiver was analyzed using EXIT charts [4]. We considered an OFDM system in a 15-path ($L = 15$) Rayleigh fading environment having $N = 8$ active users, $M = 8$ uncorrelated receive antennas ($\mathbf{R} = \mathbf{I}_8$), and $Q = 64$ sub-carriers with a 16-symbol cyclic prefix. For the EXIT analysis, the rate-1/2 convolutional code with generator polynomials $[7, 5]_{oct}$ and the Log-Map algorithm were used. A scenario was assumed where three of the eight users' channels are significantly spatially correlated and the remaining users' channels are close to orthogonal, so that the off-diagonal elements of the transmit correlation matrix $\mathbf{S}$ are given by: $s_{ij} = 0.95$, if $i, j \in \{1, 2, 5\}$, $i \neq j$, and $s_{ij} = 0$, otherwise. Fig. 1 illustrates the EXIT chart for each user of the SC-MMSE ($(H; G_{max}) = (8; 1)$) and the Hy SC-MMSE-MAP detector with $(H; G_{max}) = (6; 3)$. Note that the detectors' two-dimensional EXIT curves were obtained from the multi-dimensional EXIT surfaces by the projection technique[3] [5]. As observed in Fig. 1, the SC-MMSE detector does not provide reasonable mutual informations (MIs) for the correlated user signals needed for convergence of the turbo receiver. In contrast, the Hy SC-MMSE-MAP detector provides significantly higher MI transfer characteristics for all three highly correlated user signals which stems from the group selection that allocates those users into one MAP group. This indicates that the Hy SC-MMSE-MAP detector improves the convergence threshold of the turbo receiver, and hence, it can achieve better performance in the presence of high spatial correlation.

In addition to the EXIT analysis, simulations were carried out to evaluate the BER performance of the proposed detector. The effect of the group size on the BER performance of the Hy SC-MMSE-MAP detector after 6 turbo iterations is shown in Fig. 2. For comparison, the performance of the SC-MMSE detector in a spatially uncorrelated Rayleigh fading channel is shown as well, and is referred as SC-MMSE (ref). For the case of $(H; G_{max}) = (6; 3)$, the detector can almost achieve the same performance as the SC-MMSE detector in an uncorrelated Rayleigh fading channel.

V. CONCLUSION

In this paper, we have proposed for OFDM MU-SIMO systems a novel hybrid turbo group-wise signal detection technique, Hy SC-MMSE-MAP, that offers a design flexibility in terms of complexity in computation and robustness against channels' spatial correlation. It has been shown that Hy SC-MMSE-MAP can achieve the same performance as the SC-MMSE detector in uncorrelated Rayleigh fading channels, even when strong correlation among the user signals exists.

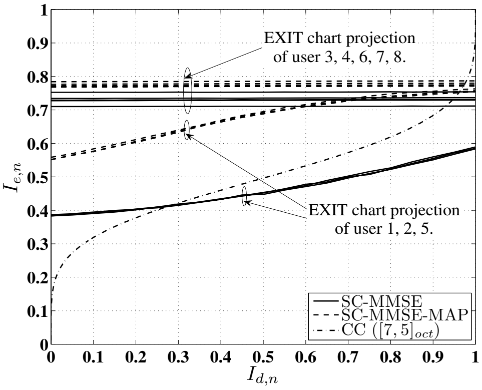

Fig. 1. EXIT chart projection for each user of the SC-MMSE and the Hy SC-MMSE-MAP detector with $(H; G_{max}) = (6; 3)$ for a random channel realization at 5 dB E_b/N_0 and $K = 2^{15}$.

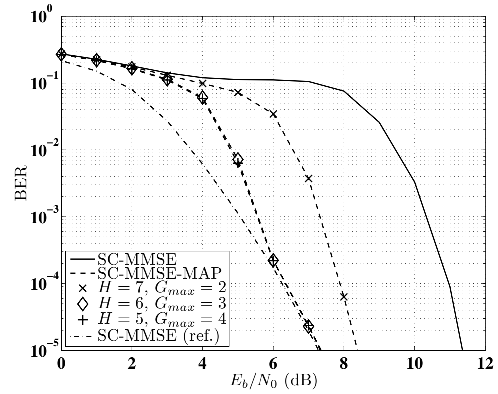

Fig. 2. BER performance of the SC-MMSE and the Hy SC-MMSE-MAP detector with $(H; G_{max}) = (7; 2), (6; 3), (5; 4)$ and $K = 4096$.

APPENDIX

The filter weighting matrix $\mathbf{W}_{\mathcal{G}_h}$ can be expressed as

$$\mathbf{W}_{\mathcal{G}_h} = \text{diag}\left\{\mathbf{I}_{G_h} + \mathbf{\Omega}_{\mathcal{G}_h}^H \mathbf{H}_{\mathcal{G}_h} \mathbf{\Xi}_{\mathcal{G}_h}\right\}^{-1} \mathbf{\Omega}_{\mathcal{G}_h} \mathbf{\Xi}_{\mathcal{G}_h},$$

where the following definitions have been used

$$\mathbf{\Xi}_{\mathcal{G}_h} = \left(\mathbf{D}_{\mathcal{G}_h} + \mathbf{\Omega}_{\mathcal{G}_h}^H \mathbf{H}_{\mathcal{G}_h}\right)^{-1} \mathbf{D}_{\mathcal{G}_h}, \tag{8}$$

$$\mathbf{D}_{\mathcal{G}_h} = \left(\text{diag}\left\{\bar{\mathbf{b}}_{\mathcal{G}_h}\right\}^2 - \mathbf{I}_{G_h}\right)^{-1} \tag{9}$$

with $\mathbf{\Omega}_{\mathcal{G}_h} = \mathbf{\Sigma}^{-1}\mathbf{H}_{\mathcal{G}_h}$ and $\mathbf{\Sigma}$ denoting the covariance of $\tilde{\mathbf{y}}$ in Eqn. (3), given by

$$\mathbf{\Sigma} = \left(\mathbf{H}\left(\mathbf{I}_N - \text{diag}\left\{\bar{\mathbf{b}}\right\}^2\right)\mathbf{H}^H + \sigma_0^2 \mathbf{I}_M\right). \tag{10}$$

REFERENCES

[1] T. Abe and T. Matsumoto, "Space-time turbo equalization in frequency-selective MIMO channels," *IEEE Trans. Veh. Technol.*, vol. 52, pp. 469-475, May 2003.

[2] H. Bölcskei *et al.*, "On the capacity of OFDM-based spatial multiplexing systems," *IEEE Trans. Commun.*, vol. 50, pp. 225-234, Feb. 2002.

[3] N. Veselinovic, "Iterative receivers for interference cancellation and suppression in wireless communications," Ph.D. thesis, University of Oulu, Dec. 2004.

[4] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Trans. Commun.*, vol. 49, pp. 1727-1737, Oct. 2001.

[5] F. Brännström *et al.*, "Convergence analysis and optimal scheduling for multiple concatenated codes," *IEEE Trans. Inf. Theory*, vol. 51, pp. 3354-3364, Sept. 2005.

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[3] A vertical step in the EXIT chart between the dashed-dotted curve and the nth solid (dashed) curve corresponds to an unspecified number of activations of all components of the turbo receiver except the nth decoder, until the mutual information $I_{e,n}$ (corresponding to the detector output of the nth user) has converged to a fixed value. A horizontal step between the nth solid (dashed) line and the dashed-dotted line (corresponding to the mutual information $I_{d,n}$) represents a single activation of the nth decoder.