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Description	

Optimal Rate Error Protection for Coded M-ary FSK FFH/CDMA Mobile Radio Channels

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ABSTRACT - Forward link capacity of fast frequency hopping code division multiple access (FFH/CDMA) cellular mobile communication systems is analyzed. M-ary FSK with non-coherent detection is assumed. Coding is used for both random address sequencing and error correction. Selection diversity reception is also applied. The channel cutoff rate of an interference-limited coded FFH/CDMA channel with a hexagonal cell layout is analyzed for both synchronous and asynchronous cases. Theoretical capacity limit is calculated from the channel cutoff rate. The optimal code rate that maximizes the user capacity is then determined. Reed Solomon (RS) codes are then applied as a practical approach. It is shown that the optimal RS code rate is 0.8 for 256-ary FSK with 19 hops/symbol, which is almost the same as that derived from the channel cutoff rate.

1. Introduction

Since Goodman et al. presented a fast frequency hopping M-ary FSK code division multiple access (FFH/CDMA) system and applied it to mobile /personal radio communications in 1980 [1], it becomes a strong candidate for FFH/CDMA applications and several performance improvements have been made [2]-[4]. In the FFH/CDMA scheme, the same frequency band is shared by many users, and the transmitted symbol is extracted from the detection matrix which indicates the detected frequencies. The signal transmission performance of this scheme depends on two probabilities: the probability p_C that the desired signal energy is detected on the correct row and the average insertion probability p_I that an false signal energy is detected in a frequency slot due to interference.

Ref[5] applied this FFH/CDMA scheme to a cellular mobile radio system with 3 cells. However, if sufficiently large transmitter power is applied so that the probability of false energy detection due to noise is made negligibly small, the major cause of performance degradation is interference (such channels are referred to as "interference-limited channels".) In this environment, interference from cells beyond the two adjacent cells can be received, and this can also degrade communication quality.

One powerful strategy is to use coding for error protection. Several practical codes suitable for FFH/CDMA channels have been proposed [4],[6] and their decoding performances have been analyzed taking into account the desired signal deletions and interference insertions. However, no performance analysis has been presented for general coding schemes.
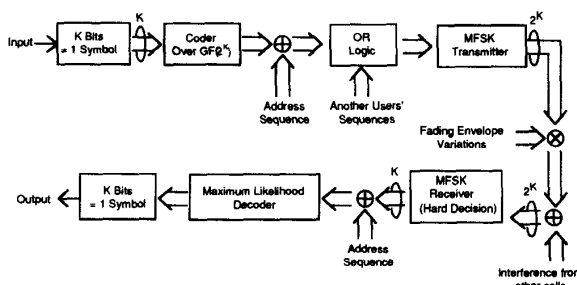

Fig.1 System Model

This paper theoretically analyzes the forward link capacity of a FFH/CDMA cellular mobile communication system that employs coding and selection diversity. Theoretical capacity limit is calculated from the channel cutoff rate R_0 of the interference-limited channel with a hexagonal cell layout comprised of 12 cells. The optimal code rate and the threshold level for energy detection that maximize the forward link capacity are then determined. It is shown that diversity reception is effective to reduce the probability that interference signal powers transmitted from the non-adjacent 9 cells are detected as spurious insertion. Decoding performance of the Reed Solomon (RS) codes with error-and-erasure correction [9] is also analyzed.

2. System Model

Fig. 1 shows the system model used in this analysis. The input bit stream to be transmitted is stored in a K-bit shift register and segmented into K-bit symbols. The K-bit symbol sequence is then encoded into the coded symbol sequence of an (n,k) code defined over the Galois field of $GF(2^K)$. The coded symbol with symbol duration T is then divided in time into L chips, each of which has a chip duration of T/L, and a random address sequence with rate L/T is added to the chip sequence with modulo-2^K. 2^K frequencies are available for the forward link transmission, and one, corresponding to a chip is transmitted over the chip interval. Prior to transmission, the chip sequence is fed to an OR logic circuit to allow other users' chips to hit the same frequency slot in the same cell. These actions are duplicated in transmitter in each cell.

The transmitted signal suffers from Rayleigh fading. We assumed that fading is non-frequency selective because a large value of K makes it possible to reduce the chip rate sufficiently
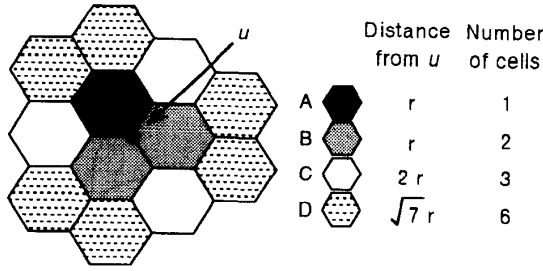
Fig.2 Hexagonal Cell Configuration

		Distance from u	Number of cells
A		r	1
B		r	2
C		$2r$	3
D		$\sqrt{7}\,r$	6

Fig.3 pD,N vs. Normalized SNR

compared to the channel delay spread, while keeping the input information bit rate constant.

In the receiver, the received power of each of the 2^K frequency slots is detected by an energy detector. Hard decision is used for energy detection. Both the transmitter and receiver know the predetermined address sequence, and the address sequence is then added with modulo-2^K to the detection matrix which indicates received detected frequencies. The detection matrix is the input to the decoder, and the transmitted symbol is estimated.

Fig.2 shows a hexagonal cell layout where the reference user "u" is assumed to be in the cell marked as "A". Because of the interference-limited channel, all the received signal powers are assumed to be sufficiently large compared with the noise power. The location of the reference user u is fixed at cell A's edge where the distance from the transmitter is r. This user u receives the desired signal with the lowest strength, whereas the composite interference signal strength from other cells is strongest. Thus, it is reasonable to define the system capacity as the number of the users such that the communication quality in this worst area satisfies a specific requirement. The transmitters in other cells marked as "B","C" and "D" are separated from user u by r, 2r and √7r, respectively. There are 2 cells classified as category B, 3 as C and 6 as D.

The received signal transmitted from the base station separated by distance d has the average signal-to-noise power ratio (SNR) of

$$\Gamma_d = (d/r)^{-\alpha} \cdot \Gamma_0, \qquad (1)$$

where Γ_0 is the average SNR of the signal transmitted from the base station of the cell A (distance d=r) , and α is the propagation constant. The average received SNR of the composite signal comprised of several signals, which are independent narrow band complex Gaussean processes, can be obtained as the sum of the average SNR's of each signal. If there are $a(=0$ or 1) cells in cell category A, $b(=0\sim2)$ cells in B, c ($=0\sim3$) cells in C and d ($=0\sim6$) cells in D, which are transmitting at the same frequency, the average SNR of the received composite signal in this frequency slot is expressed as

$$\Gamma_{a,b,c,d} = (a + b + c/16 + d/49) \cdot \Gamma_0 \qquad (2)$$

when $\alpha=4$.

3. Channel Cutoff Rate

The channel cutoff rate R_0 represents one of the upper bounds of the information bit rate. With bit rate R ($\leq R_0$)
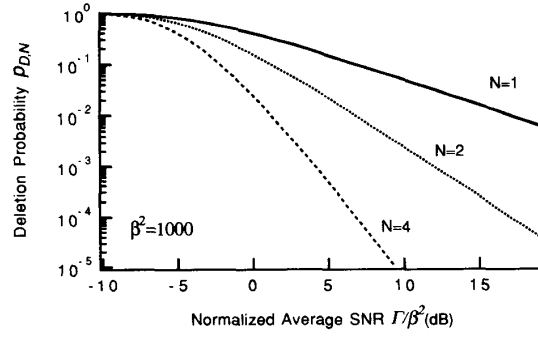
information bits /channel symbol, the input information can be transmitted with the specific requirement on the BER of BER$\leq e^{-n(R_0-R)}$ where n is the code length (communication with R$\leq R_0$ is referred to as "reliable communication").

R_0 is determined from the average deletion probability $p_{D,N}$ that with N branch selection diversity reception, the transmitted frequency is not detected. $p_{D,N}$ is expressed as [7]

$$p_{D,N} = \int_0^\infty \{1 - Q(\sqrt{2\gamma},\,\beta^2)\} \cdot \frac{N}{\Gamma} e^{-\gamma/\Gamma} \{1 - e^{1-\gamma/\Gamma}\}^{N-1}\, d\gamma$$

$$= N \cdot \sum_{i=0}^{N-1} \frac{(-1)^i}{i+1} \binom{N-1}{i} \left\{1 - \exp\left(\frac{-\beta^2}{2}\frac{i+1}{\Gamma+i+1}\right)\right\} \qquad (3)$$

where β is the threshold level normalized with respect to the receiver noise power [7]. Γ is assumed to be sufficiently large so that deletion probability $p_{D,N}(\Gamma)$ can be set at any value arbitrarily by using an appropriate value of the threshold level β. Thus, $p_{D,N}(\Gamma)$ is expressed as

$$p_{D,N} \approx N \cdot \sum_{i=0}^{N-1} \frac{(-1)^i}{i+1} \binom{N-1}{i} \left\{1 - (1-p_{D0})^{(i+1)\frac{\Gamma_0}{\Gamma}}\right\}, \qquad (4)$$

where $p_{D0} = p_{D,1}$ for $\Gamma=\Gamma_0$. Fig.3 shows $p_{D,N}$ value versus Γ/β^2 with diversity order N as a parameter. The reduction in the $p_{D,N}$ value is inversely in proportion to the $(\Gamma/\beta^2)^N$.

Then, for the reference user u, the two probabilities of p_I and p_C can be obtained using Eq.s (2) and (4) as

$$p_I = \sum_{a=0}^{1} \sum_{b=0}^{2} \sum_{c=0}^{3} \sum_{d=0}^{6} \binom{1}{a}\binom{2}{b}\binom{3}{c}\binom{6}{d} \cdot p_u^{\prime a}\,\overline{p_u}^{\prime 1-a}\,p_u^{b+c+d}\,\overline{p_u}^{11-b-c-d}\,p_{D,N} \qquad (5)$$

with $\Gamma=\Gamma_{a,b,c,d}$ and

$$p_C = \sum_{b=0}^{2} \sum_{c=0}^{3} \sum_{d=0}^{6} \binom{2}{b}\binom{3}{c}\binom{6}{d} \cdot p_u^{b+c+d}\,\overline{p_u}^{11-b-c-d}\,p_{D,N} \qquad (6)$$

with $\Gamma=\Gamma_{1,b,c,d}$, where $\overline{p_u} = (1-p_u)$, $\binom{A}{B}$ is the binomial coefficient and M is the number of users in each cell. p_u is the probability that a frequency corresponding to a spurious row in the detection matrix of user u is transmitted from a transmitter, as is given by

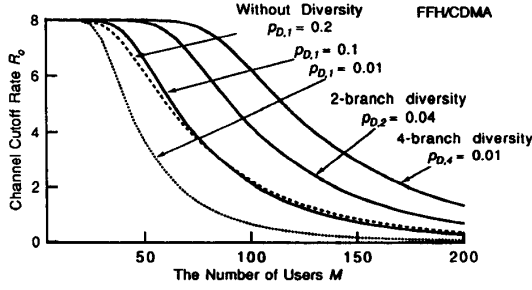$$p_u = 1 - (1 - 2^{-K})^M, \qquad (7a)$$

and

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Fig.4 Channel Cutoff Rate

$$p_u' = 1 - (1 - 2^{-K})^{M-1}. \tag{7b}$$

The received de-hopped frequencies are represented in the detection matrix. If no deletion occurs, the matrix has a full row corresponding to the transmitted 2^K-ary FSK frequency. Therefore, it is reasonable that the number of the entries in the row corresponding to the symbol of the detection matrix is used as the metric. The Chernoff bound of the metric difference $D(\lambda)$ can be represented as [8]

$$D(\lambda) = \sum_{u=0}^{L} e^{-\lambda u} \binom{L}{u} p_C{}^u (1-p_C)^{L-u}$$

$$\times \sum_{v=0}^{L} e^{\lambda v} \binom{L}{v} p_I{}^v (1-p_I)^{L-v}. \tag{8}$$

Since the FFH/CDMA channel is a symmetric channel, R_0 is given by

$$R_0 = K - log_2 \left\{ 1 + (2^K - 1) D \right\}, \tag{9}$$

where

$$D = \min_{\lambda \geq 0} D(\lambda). \tag{10}$$

Fig. 4 shows the calculated channel cutoff rate R_0 versus the number M of users per cell for L=19 and K=8 with $p_{D,N}$ as a parameter. The minimum value of $D(\lambda)$ with respect to λ was determined numerically. It is found from Fig. 4 that R_0 approaches its maximum value of 8 (=K) as M decreases. In an environment with relatively large deletion probability, i.e., the desired signal deletion dominates the transmission quality, a larger p_D value decreases the p_C value, and this results in a smaller Ro value. On the other hand, when interference signal insertion dominates the transmission quality, a smaller p_D value increases the pI value, and this also decreases the R_0 value. Therefore, if the p_I value can be made small while keeping the p_C value constant, R_0 can be further increased. The p_I and p_C values are mainly differentiated from the received power difference between the desired and interference signals. Hence, as the $p_{D,N}$ curve versus Γ becomes steeper, the difference between the p_I and p_C values increases, and this results in larger R_0 values. As described by Fig.3, this threshold effect can be achieved with diversity reception.
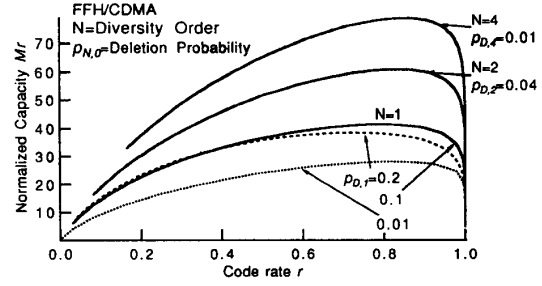

Fig.5 Normalized Capacity

4. Normalized User Capacity

The number of the users can be increased by using a low rate code with large error correction capability. However, low rate codes increase the transmission bandwidth. Because of this tradeoff, there exists an optimum code rate that maximizes the radio link capacity normalized by the code rate. The channel cutoff rate R_0 is expressed as a function of the number of the users M. Therefore, with coding whose rate is r=R/K, if M satisfies

$$M \leq R_0{}^{-1}(r K), \tag{11}$$

reliable communication is possible for all M users via the coded FFH/CDMA channel having a bandwidth 1/r times larger than the uncoded FFH/CDMA channel. Fig. 5 shows the number of the per-cell users normalized by the code rate, Mr, versus code rate r for L=19 and K=8 with p_D and N as parameters.

It is found from Fig. 5 that Mr is maximized at a code rate of around 0.8 for N=1,2 and 4. Also there exists an optimal $p_{D,N}$ value which maximizes Mr because of the tradeoff described in Section 3. The optimal $p_{D,N}$ values are found to be around 0.1 for N=1, 0.04 for N=2 and 0.01 for N=4. Without diversity reception, 42 users can achieve reliable communication with r=0.8 and $p_{D,1}$=0.1, using the same bandwidth as that of an uncoded system. With these optimal $p_{D,N}$ values, 60 users can be accommodated with 2-branch diversity, and 78 users with 4-branch diversity.

5. Asynchronous Case

In the previous sections all the received signals transmitted from different cells are assumed to have the identical chip timing . This assumption is quite unrealistic because each transmitted signal has different propagation delay even if all base station's transmission timing is synchronized. The channel cutoff rate for such an asynchronous system is derived in this section. Let the average SNR for a certain interference signal be denoted by $\mu\Gamma_0$, where Γ_0 is as defined in Eq. (1). Since chip timing of this interference signal is different from that of the desired signal as depicted in Fig. 6, the received interference power is divided into two chip time slots in the desired signal chip sequence. Therefore, the received signal SNR is increased by $\epsilon\mu\Gamma_0$ if the preceding chip of interference signal is transmitted on the frequency slot. Thus, insertion probability p_I' for the asynchronous system is expressed as
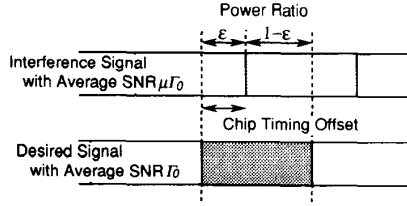
Fig.6 Asynchronous Case

$$p_I' = p_u \overline{p_u}(1-p_D((1+\varepsilon\mu)\Gamma))$$
$$+ \overline{p_u}p_u(1-p_D([1+(1-\varepsilon)\mu]\Gamma))$$
$$+ p_u{}^2(1- p_D((1+\mu)\Gamma) . \qquad (12)$$

The worst communication quality occurs at the maximum p_I' values. The p_I' value is maximized when $\varepsilon=0.5$. Thus, we can calculate the channel cutoff rate for this worst case. The normalized link capacity versus code rate for the asynchronous case is plotted in Fig.7. Without diversity reception, 31 users can be accommodated, which is about 74% of that in the synchronous case. 2-branch diversity reception achieves the normalized user capacity of 43 users/cell. This is slightly greater than that in synchronous case without diversity, and about 72% of that in the synchronous case with 2-branch diversity.

6. Decoding Performance with RS Codes

In this section, we evaluate the link capacity of the FFH/CDMA channel with Reed Solomon (RS) codes defined over Galois Field of $GF(2^K)$ as a practical coding scheme. The bit error rate (BER) of a RS coded FFH/CDMA channel was theoretically calculated in Ref [9], given the probabilities of p_I and p_C. We used this calculation method to evaluate link capacity. Fig. 8 shows the number of users ,whose BER is less than or equal to 10^{-3}, normalized by code rate for the synchronous case. The optimal $p_{D,N}$ values were used for each N values. The optimal code rate is around 0.8. This code rate is identical with that determined from the channel cutoff rate. When 2-branch diversity reception is used, 53 users can be accommodated, which is 88% of the capacity limit.
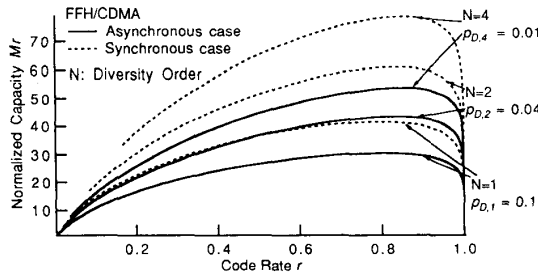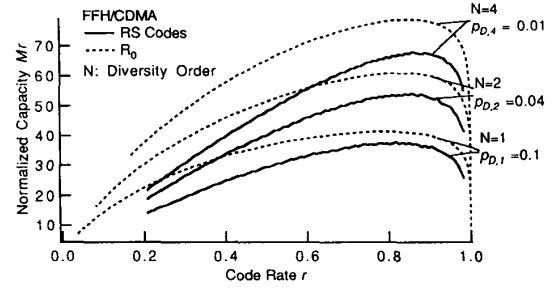

Fig.7 Normalized Capacity in Asynchronous Case

Fig.8 Normalized Capacity with RS Codes

6. Conclusions

The channel cutoff rate of the coded FFH/CDMA channel was calculated for a hexagonal cell layout comprised of 12 cells. The optimal code rate and threshold level that maximize the forward link capacity were determined. The optimal code rate for 256-ary FSK with 19 hops/symbol was shown to be around 0.8. It has been shown that diversity reception significantly increases forward link capacity. In the synchronous case, 41% more users can be accommodated with 2-branch diversity, and 90% more users with 4-branch diversity. In the asynchronous case, 43% more users can be accommodated with 2-branch diversity, and 76% more with 4-branch diversity. The forward link capacity achieved with the RS codes was also evaluated as a practical coding scheme. It was shown that the optimal code rate of the RS codes is almost the same as that derived from the channel cutoff rate. For 256-ary FSK with 19 hops /symbol, the optimal RS(255,201) code with code rate of 0.8 can achieve 90 % of the capacity limit without diversity reception.

References
[1] D.J.Goodman, P.S.Henry and V. K. Prabhu, "Frequency-Hopped Multilevel FSK for Mobile Radio", Bell Syst. Tech. J., Vol. 59, pp. 1257-1275, Sept. 1980.
[2] U.Timor, "Multistage Decoding of Frequency- Hopped FSK Systems", Bell Syst. Tech. J., Vol. 60, pp.471-483, Apr. 1981.
[3] T.J.Healy, "Coding and Decoding for Code Division Multiple User Communication System", IEEE Trans. Com. Vol. COM-33, No.4, pp.310-316, Apr. 1985.
[4] S.W.Kim and W. Stark, "Optimum Reed-Solomon Codes for Frequency-Hopping Spread-Spectrum Multiple-Access Communication System", IEEE Trans. Com. Vol. COM-37, No.2, pp.138-144, Feb. 1989.
[5] R.Viswanathan and Someshwar C.Gupta, "Adjacent Cell Interference in FH-MFSK Cellular Mobile Radio System", IEEE Trans. VT., Vol. VT-32, No. 2, pp.191-200, May 1983.
[6] A. J. Viterbi, "A Processing Satellite Transponder for Multiple Access by Low-Rate Mobile Users", Dig. Satellite Commun. Conf. Montreal, P. Q., Canada, pp.166-174, Oct. 23-25, 1978
[7] M.Schwartz, W.R.Bennett and S.Stein, "Communication Systems and Techniques", McGraw-Hill, pp.396-440, 1966
[8] T. Kawahara and T. Matsumoto, "User Capacity Limit of a Coded FFH/CDMA Mobile Radio Channel", IEE Elec. Letts., Vol.27, No.21, pp.1918-1919, 1991
[9] T. Kawahara and T. Matsumoto, "Optimum Rate Reed Solomon Codes for FFH/CDMA Mobile Radios", IEE Elec. Letts., Vol.27, No.21, pp.2066-2067, 1991