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Description	

# Downlink beamforming for frequency-duplex systems in frequency-selective fading

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**Abstract** – Downlink beamforming is analyzed for broadband frequency-duplex systems where frequency-selective fading dominates. In order to compare different beamforming strategies, system level simulations are carried out. It is assumed that at the receiver of the mobile station, path diversity can be exploited. The performance gain from maximal ratio combining of the signal components is evaluated with a new approach that combines system level simulations with an analytical calculation of the probability density function of the signal-to-interference power ratio (SIR) at the output of the combiner.

## I INTRODUCTION

The capacity of cellular mobile radio systems can be increased significantly with smart antennas. The following two concepts can be utilized: First, smart antennas enable the concept of space division multiple access (SDMA). This means that transmission from a base station to several mobiles at the same time and carrier frequency is possible. Second, cochannel interference in cellular systems can be reduced so that the frequency reuse is improved. The purpose of uplink beamforming is to receive as much power as possible from the desired user and as little power as possible from any undesired user. The purpose of downlink beamforming is to transmit as much power as possible to the desired mobile and as little power as possible to any undesired mobile.

For systems with time division duplex (TDD) almost the same channel conditions exist for uplink and downlink if the transmission frame is short compared with the maximum Doppler frequency. Therefore, beamforming is optimized instantaneously corresponding to the temporal fluctuations of the channel and is identical for uplink and downlink. In case of systems with frequency division duplex (FDD) the beamforming problem is more complex since the small scale fading processes for uplink and downlink are different. But, if the frequency separation is not too large, the uplink and downlink waves exhibit the same directional dependence. Therefore, downlink beamforming has to be carried out in an average sense based on the uplink channel measurements.

The present paper discusses the optimization of downlink beamforming for systems with FDD. It answers the question whether beamforming should be optimized in such a way that the transmitted power is mainly sent into the direction of the path with minimum average

attenuation or whether it is better to distribute the power to several relevant desired paths. The advantage from concentrating the power to the main path is that the interference power to other mobiles may be reduced due to a higher degree of freedom for beamforming. In case of power distribution on several paths, advantage from higher path diversity may be obtained.

A new approach is presented that can be used to estimate the gain from receivers which combine the signals from different paths. The approach can be applied for RAKE receivers in CDMA systems as well as for receivers for TDMA systems with adaptive equalizers.

## II CELLULAR SIMULATION MODEL

A cellular system model is considered which consists of two sections: In the central area the signal to interference power ratio (SIR) distribution in reference cells is evaluated. Downlink beamforming is also performed in the outer ring, but the SIR is not calculated. The objective of the present paper is to compare different beamforming concepts. In order to reduce the computational effort, circular arrays at the base stations are employed. Each antenna element exhibits an omnidirectional antenna pattern.

The cellular model with an example of the random

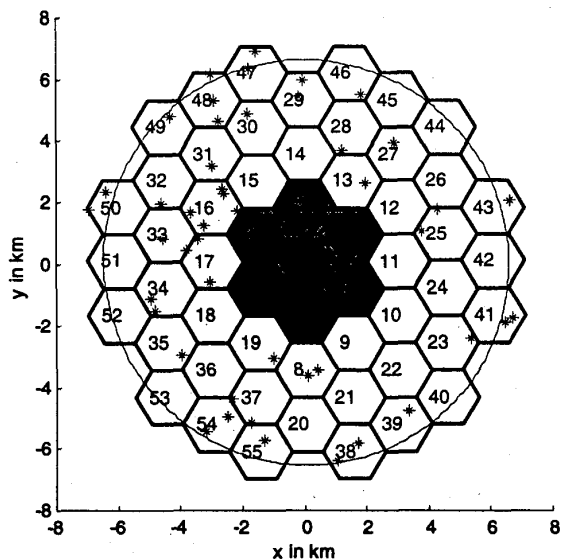


Fig.1: Locations of cells and mobiles.

distribution of mobiles (constant probability density function) is shown in Fig. 1.

### III PROPAGATION MODEL

In other investigations about downlink beamforming, deterministic propagation scenarios have been used [2,4] or models which do not take into account the statistic nature of all parameters (e.g. the number of paths) [1,3]. In the present paper a completely statistic propagation model between each base station and each mobile is used which takes into account the following aspects:

- The number of propagation paths is random and exhibits a binomial distribution.
- In order to model shadowing between base stations and mobile stations, log-normal fading for the attenuation is assumed.
- Also a log-normal fading is used to describe the fading of the reflected paths with respect to the direct path.
- Furthermore, an additional constant attenuation and an additional attenuation proportional to the excess delay is taken into account for the delayed paths.
- The average attenuation is described by the COST-Hata model [5]. For small distances, a break point limits the attenuation to a certain minimum value.
- The excess delays of reflected paths are exponentially distributed. The described model yields an exponential power delay profile.
- The directions of arrival (DoA) exhibit a Gaussian or Laplacian distribution (standard deviation = several tens of degrees).

Small scale fading processes are included analytically (see section V).

### IV BEAMFORMING ALGORITHM

A covariance matrix based beamforming is performed [1,3]. The algorithm maximizes the received average signal power  $P_S$  at the desired mobile

$$P_S = \mathbf{w}^H \mathbf{R}_S \mathbf{w} \quad (1)$$

while keeping the sum of the total interference power  $P_I$  at all undesired mobiles

$$P_I = \mathbf{w}^H \mathbf{R}_I \mathbf{w} \quad (2)$$

constant. Here,  $\mathbf{w}$  denotes the array weighting vector which has to be optimized.  $\mathbf{R}_S$  is the covariance matrix for the desired mobile:

$$\mathbf{R}_S = \sum_{i=1}^{N_{P,0}} \mathbf{s}(\theta_i) \mathbf{s}(\theta_i)^H \cdot P_{\text{down}} \cdot \langle a_{\text{down},i} \rangle. \quad (3)$$

$\mathbf{s}(\theta_i)$  is the steering vector for the DoA  $\theta_i$ . Summation is over the  $N_{P,0}$  paths to the desired mobile.  $P_{\text{down}}$  denotes the downlink power which is assumed to be the same for all mobiles and  $\langle a_{\text{down},i} \rangle$  denotes the average power transfer factor of the desired downlink path  $i$ . By

including all desired paths or only the main path into  $\mathbf{R}_S$ , the type of beamforming can be controlled.  $\mathbf{R}_I$  is the covariance matrix for the undesired mobiles:

$$\mathbf{R}_I = \sum_{j=1}^{N_{\text{mob}}} \sum_{i=1}^{N_{P,j}} \mathbf{s}(\theta_{ij}) \mathbf{s}(\theta_{ij})^H \cdot P_{\text{down}} \cdot \langle a_{\text{down},ij} \rangle. \quad (4)$$

The summation in case of  $\mathbf{R}_I$  is over all  $N_{P,j}$  paths of all undesired mobiles.  $\theta_{ij}$  and  $\langle a_{\text{down},ij} \rangle$  are the DoA and average power transfer factor of the path  $i$  to the undesired mobile  $j$ .

Because of the abundance of the path components received by the base station, it is very likely that  $\mathbf{R}_I$  becomes non-singular. Equations (1) and (2) result in the generalized eigenvalue problem [1,3]:

$$\mathbf{R}_S \mathbf{w} = \lambda \cdot \mathbf{R}_I \mathbf{w}. \quad (5)$$

The optimum weight vector  $\mathbf{w}_{\text{opt}}$  is the eigenvector associated with the largest eigenvalue.

The resulting power gain factor of the antenna array

$$g(\theta) = |\mathbf{w}_{\text{opt}}^H \mathbf{s}(\theta)|^2 \quad (6)$$

shows very deep minima which are quite unrealistic (see Fig. 2). A more realistic behavior is obtained by assuming that the power distributions of the incident waves are continuous rather than discrete. In general, for each wave an individual angular distribution has to be taken into account. The received power is obtained from integration of the product of antenna gain and angular distribution of the power transfer factors:

$$P = P_{\text{down}} \int_{-\pi}^{\pi} g(\theta) \cdot \sum_i \langle a_{\text{down},i} \rangle \cdot D_i(\theta - \theta_i) d\theta \quad (7)$$

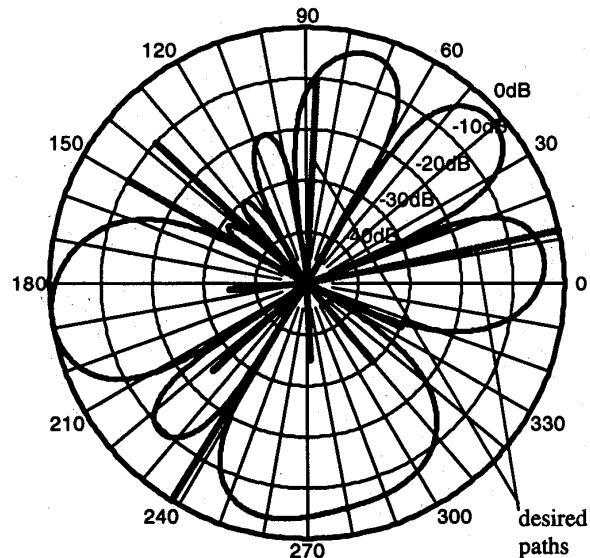


Fig.2: Example for the optimized antenna gain  $g(\theta)$  for two desired and several undesired paths. Bars show the normalized power transfer factors of the individual paths.

where  $D_i(\theta - \theta_i)$  denotes the power distribution of a single path  $i$  with respect to the average DoA  $\theta_i$ . For simplicity it is assumed that all the waves exhibit the same power distribution with respect to their average DoA:  $D_i(\theta) = D(\theta)$ . In this case, the received power results to:

$$P = P_{\text{down}} \int_{-\pi}^{\pi} g(\theta) \cdot \sum_i \langle a_{\text{down},i} \rangle \cdot D(\theta - \theta_i) d\theta$$

$$= P_{\text{down}} \sum_i \langle a_{\text{down},i} \rangle \cdot g_P(\theta_i) \quad (8)$$

where  $g_P(\theta_i)$  denotes the path gain which is obtained from convolution of the antenna gain  $g(\theta)$  with the angular power distribution  $D(\theta)$ :

$$g_P(\theta_i) = \int_{-\pi}^{\pi} g(\theta) \cdot D(\theta - \theta_i) d\theta \quad (9)$$

The path gain  $g_P(\theta_i)$  describes the power transfer factor of the adaptive antenna with respect to macropaths and is therefore in general different from the antenna gain factor  $g(\theta)$ . For the following simulations, a Laplacian power distribution (standard deviation  $\sigma_\theta < 1^\circ$ ) is assumed. Figure 3 shows a corresponding path diagram for a standard deviation  $\sigma_\theta = 0.5^\circ$ . The following two cases have been compared numerically:

- The angular distribution of the individual paths is correctly taken into account for the calculation of the covariance matrices.
- In a simplified analysis for the calculation of the covariance matrices, paths are described only by discrete waves without any angular spread.

Simulations have shown that there is only a small

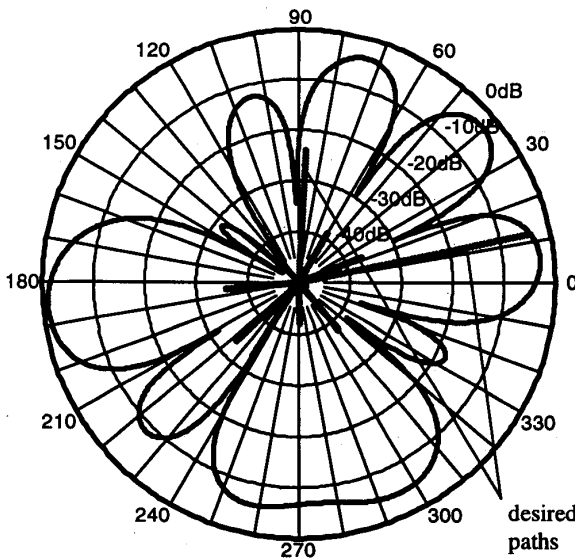


Fig.3: Example for the optimized path gain  $g_P(\theta)$  for two desired and several undesired paths. Bars show the normalized received power of individual paths.

difference in the resulting path diagrams. Especially for the strong paths, the difference can be neglected if the RMS angular spread is smaller than  $1^\circ$ .

For the simulations it is assumed that average (with respect to small scale fading) path attenuations and average DoAs are known at the base stations. In reality, these parameters can be estimated from the uplink snapshot vectors by using the sub-space method. However, instead of estimating path attenuations and DoAs,  $\mathbf{R}_S$  and  $\mathbf{R}_I$  themselves can be estimated directly from the snapshot vectors if some knowledge about the transmitted signal sequence is available. Estimation errors will cause a degradation of the performance.

## V SIMULATION METHODOLOGY

The objective of the investigations is the calculation of the cumulative probability density function of the SIR. Compared with an omnidirectional antenna, the performance gain for different beamforming algorithms is calculated.

The simulations are based on snapshots. No movement of the mobiles is taken into account. In a first step, average powers for the individual paths are calculated ignoring the small scale fading processes. In a second step, small scale fading processes and diversity combining are considered.

First, the interfering signals shall be considered. In the downlink, at each mobile a large number of interfering signals arrives from all undesired base stations. It is assumed that all interfering signals exhibit Rayleigh fading. Since these signals are statistically independent and because of their large number, the total interference power  $N_0$  will fluctuate only slightly around its average value  $\langle N_0 \rangle$  which is large compared with the average power of a single path. But numerical evaluations show that this is not really true: The significant number of paths

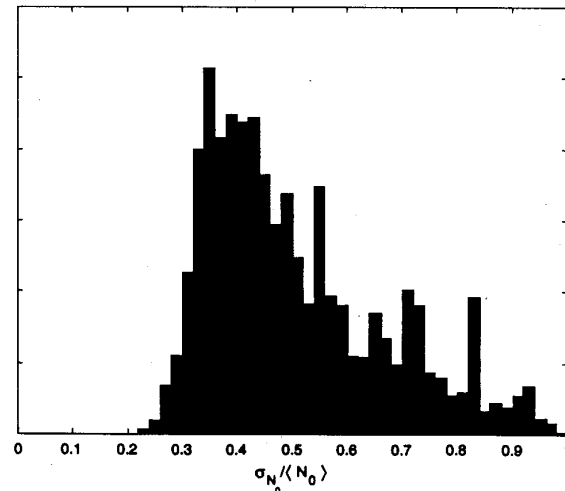


Fig.4: Histogram of the standard deviation  $\sigma_{N_0}$  of the total interference power  $N_0$  normalized by the average interference power  $\langle N_0 \rangle$ .

is not very large and it is not very unlikely that a small number paths dominates. In Fig. 4 the standard deviation of the interfering power normalized by its mean is displayed. Obviously, the standard deviation  $\sigma_{N_0}$  is not

by far smaller than the average value  $\langle N_0 \rangle$ . But nevertheless, for simplicity, in the following it will be assumed that the average interference power is constant and does not depend on small scale fading processes.

For a small delay difference between the paths, no path diversity can be exploited. For large delay differences between the different paths, the paths can be resolved and path diversity gain can be obtained at the mobile stations. For CDMA systems, combining can be realized with a RAKE receiver. In case of TDMA systems, adaptive equalization also combines the contributions of different paths in an optimized way. For simplicity and in order to determine the performance limitation, a maximal ratio combining (MRC) of the individual paths at the mobile stations is assumed. The performance gain from maximal ratio combining can be calculated analytically. For the following it is assumed that all paths can be resolved and combined with MRC.

Because of the small scale fading, the signal power of each path is exponentially distributed. Since the interference power is assumed to be constant, also the signal-to-interference power ratio  $\gamma_i$  of each path is distributed exponentially:

$$f_{\gamma_i}(\gamma_i) = \frac{1}{\bar{\gamma}_i} e^{-\frac{\gamma_i}{\bar{\gamma}_i}} \quad (10)$$

where  $\bar{\gamma}_i$  denotes the average SIR of a single desired path. It is given by:

$$\bar{\gamma}_i = \frac{\sum_{j=1}^{N_{\text{mob}}} \sum_{i=1}^{N_{P,j}} \langle a_{\text{down},ij} \rangle \cdot g_P(\theta_{ij})}{\sum_{j=1}^{N_{\text{mob}}} \sum_{i=1}^{N_{P,j}} \langle a_{\text{down},ij} \rangle \cdot g_P(\theta_{ij})} \quad (11)$$

Assuming that the additive noise in each path is independent, the SIR after MRC results to [6]:

$$\gamma = \gamma_1 + \gamma_2 + \gamma_3 + \dots \quad (12)$$

Furthermore, it is assumed that the small scale fading of the individual desired paths is statistically independent. Since  $\gamma$  is the sum of the random variables  $\gamma_i$ , the resulting probability density function (PDF) is obtained from convolving the individual PDFs:

$$f_\gamma(\gamma) = f_{\gamma_1} * f_{\gamma_2} * f_{\gamma_3} * \dots \quad (13)$$

This means for the characteristic function

$$\Psi_\gamma(\Omega) = \int_{-\infty}^{\infty} f_\gamma(\gamma) \cdot e^{j\Omega\gamma} d\gamma \quad (14)$$

that the individual functions have to be multiplied:

$$\Psi_\gamma(\Omega) = \Psi_{\gamma_1}(\Omega) \cdot \Psi_{\gamma_2}(\Omega) \cdot \Psi_{\gamma_3}(\Omega) \cdot \dots \quad (15)$$

The PDFs (10) correspond to the characteristic functions:

$$\Psi_{\gamma_i}(\Omega) = \frac{1}{1 - j\Omega\bar{\gamma}_i} \quad (16)$$

After MRC the characteristic function of the overall SIR results to:

$$\Psi_\gamma(\Omega) = \prod_{i=1}^{N_{P,0}} \frac{1}{1 - j\Omega\bar{\gamma}_i} \quad (17)$$

Since in general the average values  $\bar{\gamma}_i$  are different, the back transform can be realized with a partial-fraction expansion:

$$\Psi_\gamma(\Omega) = \sum_{i=1}^{N_{P,0}} \frac{k_i}{1 - j\Omega\bar{\gamma}_i} \quad (18)$$

with

$$k_i = \prod_{\substack{j=1 \\ j \neq i}}^{N_{P,0}} \frac{\bar{\gamma}_i}{\bar{\gamma}_i - \bar{\gamma}_j} \quad (19)$$

The final result for the PDF of the overall SIR including path diversity is:

$$f_\gamma(\gamma) = \sum_{i=1}^{N_{P,0}} \frac{k_i}{\bar{\gamma}_i} e^{-\frac{\gamma}{\bar{\gamma}_i}} \quad (20)$$

In order to compare the different beamforming algorithms the PDF  $f_\gamma(\gamma)$  has to be averaged over all mobiles and possibly over several simulations where different locations for the mobiles and different radio channels are determined. Most information can be extracted from the averaged distribution function of the SIR:

$$F_\gamma(\gamma) = \int_0^\gamma \langle f_\gamma(u) \rangle du \quad (21)$$

The distribution function  $F_\gamma(\gamma)$  makes it possible to directly read the outage probability of the downlink transmission.

The calculation method for  $f_\gamma(\gamma)$  after (20) could also be used for calculation of the interference power, but numerical evaluations of  $k_i$  show that numerical problems occur if the number of paths is larger than 10.

## VI SIMULATION RESULTS

The objective of the following simulations is to compare the SIR distribution functions for different beamforming strategies. Simulations have shown that the system displayed in Fig. 1 with 55 cells is large enough to describe the interference in the reference cells correctly. Table 1 shows the main simulation parameters.

In order to obtain a data-independent distribution function, averaging has been carried out over 500 independent scenarios so that, in total, the SIR is averaged over more than 3500 mobiles.

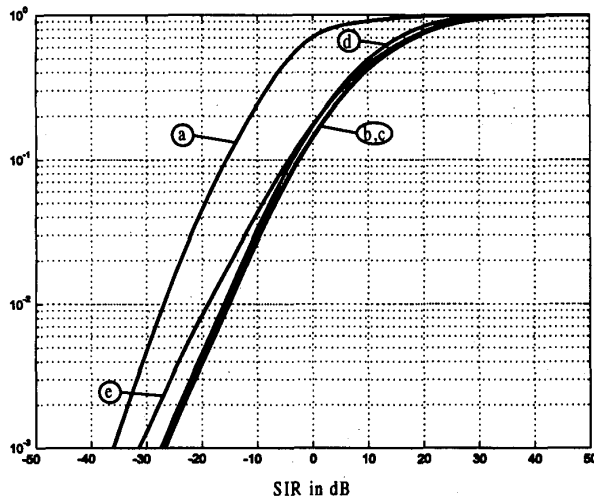


Fig.5: Probability distribution function of the signal-to-interference ratio (SIR) for different beamforming concepts.

Average number of mobiles per cell	1
Maximum number of mobiles per cell	5
Cell radius	1 km
Carrier frequency	2 GHz
Base station antenna height	30 m
Mobile station antenna height	1.7 m
Break point of average attenuation for small distances	100 m
Standard deviation of the large scale fading	8 dB
Average number of paths	3
Maximum number of paths	6
Standard deviation of fading of the delayed paths with respect to the direct path	6 dB
Average attenuation of the delayed paths with respect to the direct path	8 dB
Additional delay-dependent attenuation	4 dB/ $\mu$ s
Standard deviation of DoAs with respect to the direct path (Laplacian PDF)	20°
Number of elements of the antenna array	8
Radius of the array	0.12 m

Tab. 1: Main simulation parameters.

The simulation result is shown in Fig. 5. The following beamforming strategies are compared:

- no beamforming – omnidirectional base station antennas,
- optimized beamforming after (5) where all desired paths are included in  $R_s$ ,
- optimized beamforming after (5) where only the strongest desired path is included in  $R_s$ ,
- optimized beamforming after (5) where only the strongest desired path is included in  $R_s$ , all other desired paths are considered as interference and included in  $R_i$ ,
- same as c), but only the strongest path is exploited at the receiver – no diversity technique is utilized.

For cases a), b), c), and d) maximum ratio combining is carried out. Obviously, there is almost no performance difference between the different beamforming strategies b, c, and d, especially at low SIRs where the outage probability is determined. It can be noticed that an SIR gain of about 8 dB is obtained at low SIRs using smart antennas when compared with omnidirectional antennas. The MRC gain can be observed by comparing curves c) and e).

No significant difference for the SIR statistics was found when comparing Laplacian angular power distribution for  $0 < \sigma_\theta < 1^\circ$ .

## VII CONCLUSION

A simulation model of downlink beamforming for systems with FDD has been presented. A new simulation methodology is used that takes into account the gain from path diversity in a realistic way.

Furthermore, some simulation results have been shown which give some upper limits of the system performance due to the above described assumptions and simplifications.

In future investigations the effect of the random distribution of interference power caused by the small scale fading has to be studied. Furthermore, also the effect of power control has to be taken into account.

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