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Description	

A Matched Filter Approximation for SC/MMSE Iterative Equalizers

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Abstract-This paper proposes a new iterative ISI equalization algorithm that offers low computational complexity: order L^2 with channel memory length L . The proposed algorithm is an extension of Reynolds and Wang's SC/MMSE (Soft Canceller followed by MMSE filter) equalizer: approximations are used properly to reduce the computational complexity. It is shown that the approximations used in the proposed algorithm do not cause any serious performance degradation compared to the conventional trellis-based iterative equalization algorithms.

I. Introduction

A key issue towards mobile multimedia communications is to create technologies for broadband signal transmission that can support high quality services. Reducing the effects of the severe Inter-symbol interference (ISI) inherent within broadband mobile communications requires a technological breakthrough. Iterative equalization [1], which is based on the Turbo decoding concept, is known as an excellent technique for reducing ISI effects. The maximum a posteriori probability (MAP) algorithm and its derivatives, such as Log-MAP and Max-Log-MAP as well as Soft Output Viterbi Algorithm (SOVA) [2], can be used as the Soft-Input/Soft-Output (SISO) algorithm needed for iterative equalization. However, their computational complexities increase exponentially with channel memory length L since they use a trellis diagram of the channel. Reynolds and Wang recently proposed a computationally efficient iterative equalization algorithm for severe ISI channels [3], which was derived from an iterative multiuser detector for CDMA systems [4]. Reference [3]'s iterative equalizer consists of a soft canceller (SC) followed by a linear adaptive filter whose taps are determined adaptively based on the minimum mean square error (MMSE) criterion, and hence is referred to as SC/MMSE in this paper for convenience. SC/MMSE's computational complexity is of the order of L^3 since it requires matrix inversion. Although [3] suggests the recursive use of the matrix inversion lemma, by which the computational complexity of each recursion can be reduced to the order of L^2 , total complexity is still of the order of L^3 .

This paper proposes a new version of SC/MMSE that offers further reduced computational complexity by eliminating the need for matrix inversion. For the first iteration, the filter taps are determined adaptively using the training sequence transmitted for channel estimation. For the 2nd and following iterations, the MMSE filter is replaced by a

matched filter matched to the channel. This approximation significantly reduces the complexity without causing any serious performance degradation. Computational complexity of the proposed SC/MMSE algorithm is of the order of L^2 . Our expectation is that at low iteration numbers our proposed algorithms not achieve the same performance as that with the trellis-based iterative equalization algorithms, but the performance difference be insignificant as the iteration process proceeds.

II. Principle of SC/MMSE Iterative Equalizer

Fig.1 shows a block diagram of the SC/MMSE iterative equalizer. Its conceptual basis is to replicate the ISI components by using the LLR of the coded bits, fed back from the channel decoder, and to subtract the soft replica of the ISI components from the received composite signal vector (this process is referred to as "soft cancellation"). Adaptive linear filtering then takes place to remove the interference residuals; taps of the linear filter are determined adaptively so as to minimize the mean square error (MSE) between the filter output and the signal point corresponding to the coded symbol. The LLR of the filter output is then calculated. After de-interleaving, the calculated LLR values of the filter output are brought to the channel decoder as extrinsic information. SISO decoding for the channel code used, is performed by the channel decoder. The process described above is repeated in an iterative manner. The key point of this scheme is that it offers much lower computational complexity than iterative equalizers using a trellis diagram of the channel.

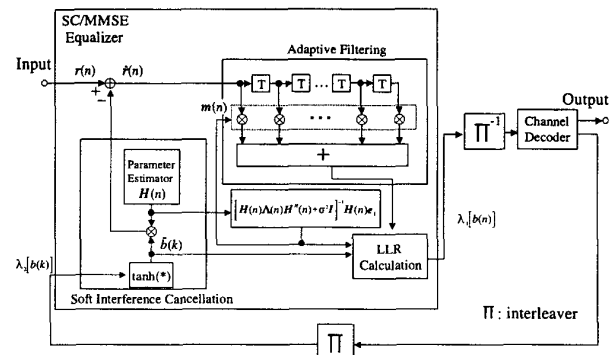


Fig.1. Block diagram of original SC/MMSE Iterative Equalizer

Assuming that there are M antenna diversity branches, the adaptive MMSE filter has ML taps. Vector $\mathbf{m}(n)$ corresponding to the filter taps is given by

$$\mathbf{m}(n) = \left[\mathbf{H}(n)\mathbf{\Lambda}(n)\mathbf{H}^H(n) + \sigma^2\mathbf{I} \right]^{-1} \mathbf{H}(n)\mathbf{e}_L. \quad (1)$$

which is an MMSE solution to the minimization problem

$$\mathbf{m}(n) = \arg \min_{\mathbf{m}(n)} E \left\{ \left\| b(n) - \mathbf{m}^H(n)\tilde{\mathbf{r}}(n) \right\|^2 \right\}, \quad (2)$$

where n is the symbol timing index. \mathbf{e}_L is the $2L-1$ vector whose elements are all zero except for the L -th element (which is 1), and $\mathbf{H}(n)$ given by

$$\mathbf{H}(n) = \begin{bmatrix} \tilde{h}(n) & 0 & \cdots & 0 \\ 0 & \tilde{h}(n) & & \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{h}(n) \end{bmatrix}, \quad (3)$$

with

$$\tilde{h}(n) = \begin{bmatrix} \tilde{h}_0(n; 0) & \cdots & \tilde{h}_0(n; L-1) \\ \vdots & \ddots & \vdots \\ \tilde{h}_{M-1}(n; 0) & \cdots & \tilde{h}_{M-1}(n; L-1) \end{bmatrix}. \quad (4)$$

is the space-time channel matrix [3] whose dimensionality is $ML \times (2L-1)$. The $(2L-1) \times (2L-1)$ diagonal matrix $\mathbf{\Lambda}(n)$ is the covariance matrix of the soft canceller output vector, given by

$$\mathbf{\Lambda}(n) = \text{diag}(1-\tilde{b}^2(n+(L-1)), \cdots, 1-\tilde{b}^2(n+1), 1, 1-\tilde{b}^2(n-1), \cdots, 1-\tilde{b}^2(n-(L-1))), \quad (5)$$

where $\tilde{b}(n)$ is a soft estimate of the coded bit $b(n)$ is given by

$$\tilde{b}(k) = \tanh \left[\frac{\lambda_2[b(k)]}{2} \right], \quad n-(L-1) \leq k \leq n+(L-1). \quad (6)$$

$\lambda_2[b(n)]$ is the extrinsic information provided by the channel decoder. $\tilde{\mathbf{r}}(n)$ is the soft canceller output given by

$$\tilde{\mathbf{r}}(n) = \mathbf{r}(n) - \mathbf{H}(n)\tilde{\mathbf{b}}(n), \quad (7)$$

where $\mathbf{r}(n)$ is the received signal vector with

$$\tilde{\mathbf{b}}(n) = \left[\tilde{b}(n+(L-1)) \cdots \tilde{b}(n+1) \ 0 \ \tilde{b}(n-1) \cdots \tilde{b}(n-(L-1)) \right]^T. \quad (8)$$

Obviously, calculating of $\mathbf{m}(n)$ requires matrix inversion, which incurs order L^3 complexity.

III. Approximations

Fig.2 shows a block diagram of the proposed SC/MMSE iterative equalizer. The algorithm proposed in this paper aims to eliminate the matrix inversion needed to calculate the MMSE filter taps. It is obvious that $\mathbf{\Lambda}(n) = \mathbf{I}$ for the first iteration since no extrinsic information is provided by the channel decoder. Thus, the MMSE filter taps can be determined adaptively by using the training sequence transmitted to estimate the channel matrix $\mathbf{H}(n)$. In the 2^{nd} and later iterations, $\mathbf{\Lambda}(n) \neq \mathbf{I}$, so this technique cannot be used. However, if the soft estimates of the coded bits, obtained by using the LLR, are perfect, which is more likely to happen at longer iteration numbers, diagonal matrix $\mathbf{\Lambda}(n)$ becomes

$$\mathbf{\Lambda}(n) = \tilde{\mathbf{\Lambda}} = \text{diag} \{ 0, 0, \cdots, 0, 1, 0, \cdots, 0, 0 \}. \quad (9)$$

$\tilde{\mathbf{\Lambda}}$ is no longer a function of symbol timing index n , and $\mathbf{H}(n)\tilde{\mathbf{\Lambda}}\mathbf{H}^H(n)$ becomes a rank-one matrix. Hence, the MMSE filter taps can be obtained as

$$\mathbf{m}(n) = \left[\mathbf{h}(n)\mathbf{h}^H(n) + \sigma^2\mathbf{I} \right]^{-1} \mathbf{h}(n), \quad (10)$$

where $\mathbf{h}(n)$ is the L -th column vector of the space-time channel matrix $\mathbf{H}(n)$, expressed as

$$\mathbf{h}(n) = [\tilde{h}_0(n; L-1) \cdots \tilde{h}_{M-1}(n; L-1) \cdots \tilde{h}_0(n; 0) \cdots \tilde{h}_{M-1}(n; 0)]^T. \quad (11)$$

By using the matrix inversion lemma [5], $\mathbf{m}(n)$ becomes

$$\begin{aligned} \mathbf{m}(n) &= \left[\mathbf{h}(n)\mathbf{h}^H(n) + \sigma^2\mathbf{I} \right]^{-1} \mathbf{h}(n) \\ &= \left[\frac{1}{\sigma^2}\mathbf{I} - \frac{1}{\sigma^4} \left(1 + \frac{1}{\sigma^2}\mathbf{h}^H(n)\mathbf{h}(n) \right)^{-1} \right] \mathbf{h}(n)\mathbf{h}^H(n)\mathbf{h}(n) \\ &= \alpha\mathbf{h}(n). \end{aligned} \quad (12)$$

where $\alpha = \frac{1}{\mathbf{h}^H(n)\mathbf{h}(n) + \sigma^2}$. If the channel variation due to fading is slow enough compared to the frame length, the symbol timing index n is no longer needed, and hence $\mathbf{m}(n) \equiv \mathbf{m}$ and $\mathbf{h}(n) \equiv \mathbf{h}$. It is obvious from (12) that the MMSE filter $\mathbf{m}^H\tilde{\mathbf{r}}(n)$ is equivalent to the matched filter matched to the channel. This is quite reasonable given a sufficient number of iterations, since almost all ISI components can be eliminated by the soft canceller, and the role of the MMSE filter at this iteration stage is merely to maximize the signal energy, which can be done by the matched filter.

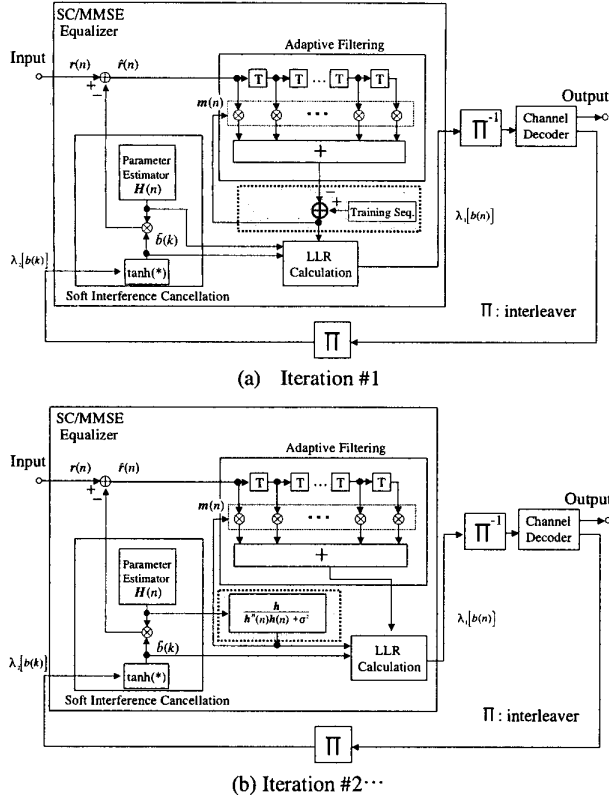


Fig.2. Block diagram of proposed SC/MMSE Iterative Equalizer

We reach the key point of the proposed SC/MMSE: the matched filter $\alpha \mathbf{h}^H$ is used instead of (1) even from the 2nd iteration. This approximation significantly reduces the computational complexity since matrix inversion is no longer needed. Furthermore, the tap vector \mathbf{m} does not have to be updated at each iteration. This approximation reduces the computational complexity of SC/MMSE to the order of L^2 . Surprisingly, this approximation does not cause any serious performance degradation as shown in Section V.

IV. Noise Power Estimation

Equation (1) assumes that the SC/MMSE iterative equalizer has knowledge about the space-time channel matrix $\mathbf{H}(n)$ and the noise variance σ^2 . However, in practice, the equalizer has to estimate them.

The objective of the channel estimation is to obtain estimates $\hat{\mathbf{h}}$ of the parameter vector $\mathbf{h}(n)$ of the channels between the transmitter and each of the M antennas used. The estimation optimality is, in the least square (LS) sense, can be given as

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}(n)} E \left\{ \left\| \mathbf{r}(n) - \mathbf{h}^H(n) \cdot \mathbf{u}(n) \right\|^2 \right\}, \quad (13)$$

where $\mathbf{u}(n)$ is the training sequence heading the information sequence to be transmitted. This optimization problem can be adaptively solved by using some adaptive algorithm. This paper proposes the use of the mean square error (MSE) as variance estimate $\hat{\sigma}^2$ after the convergence of some adaptive algorithm for channel estimation

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{n=0}^{N-1} \left\| \mathbf{r}(n) - \hat{\mathbf{h}}^H \cdot \mathbf{u}(n) \right\|^2, \quad (14)$$

where N is the training sequence length.

V. Simulation Results

The performance of the proposed algorithm was evaluated through a series of computer simulations. Table.1 summarizes the simulation parameters. Fig.3 shows the BER performances of SOVA and the original SC/MMSE iterative equalizers proposed by Reynolds and Wang. We assume a frequency selective Rayleigh fading channel with normalized Doppler frequency $f_D T_S = 1/12000$. The training and information sequences are 128 and 512 symbols long, respectively. The recursive least square (RLS) algorithm was used to estimate the channel matrix, and SOVA was used in the SISO decoder. BER performance with the maximum likelihood sequence estimator (MLSE) followed by a hard decision Viterbi decoder is also shown in Fig.3 (as indicated by MLSE-VA). It is found that both SOVA and SC/MMSE offer improved BER performance over MLSE-VA. With SOVA the performance difference between the 1st and 2nd iterations is very small. As shown in Fig.3, SC/MMSE's BER is worse at the first iteration than that of SOVA, but the performance difference becomes very small after two-to-three iterations. Fig.4 shows the BER performances of SOVA and the proposed SC/MMSE algorithm with the matched filter approximation. At the first iteration, the proposed SC/MMSE offers worse performance than the original SC/MMSE. However, after three iterations, the proposed SC/MMSE matches the performance of the original SC/MMSE, and thus that of SOVA.

Table 1
Simulation parameters

Modulation	BPSK
Information symbol	512
Training symbol	128
Channel coding	Convolutional code (R=1/2, K=3)
Interleaver	Random
Channel model	Rayleigh fading (Normalized Doppler frequency: $f_D T_S = 1/12000$)
Channel estimation	RLS algorithm (Forgetting factor: 0.99)
SISO Decoder	SOVA

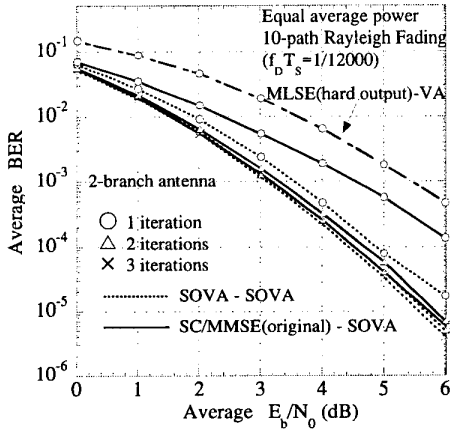


Fig.3. Comparison of SOVA and original SC/MMSE

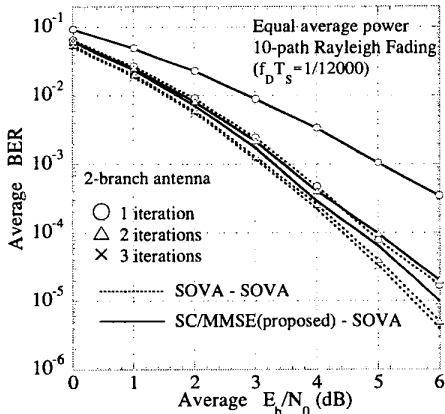


Fig.4. Average BER performances of proposed SC/MMSE

Fig.5 shows the BER performance of the proposed SC/MMSE equalizer with and without noise power estimation. Without noise power estimation, the exact value of σ^2 was assumed to be known. It is found that the proposed SC/MMSE with noise power estimation achieves exactly the same performance as when the noise power was known.

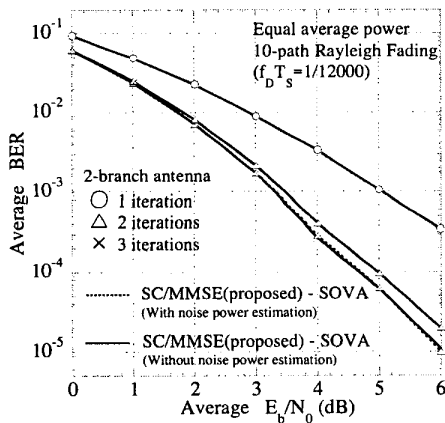


Fig.5. Average BER performances of proposed SC/MMSE with and without noise power estimation

Fig.6 shows the BER performance of the proposed SC/MMSE equalizer with the number of antenna diversity branches as a parameter. Correlation between antenna branches was assumed to be zero. After three iterations, the E_b/N_0 value required to achieve 10^{-4} BER is 1dB lower with 3-branch diversity than with 2-branch diversity. Obviously, however, BER performance is degraded in the presence of channel estimation error, and increasing the diversity order increases the number of channel parameters that need to be estimated. Hence, there should be a tradeoff between the diversity order and performance, given the fixed length of the training sequence.

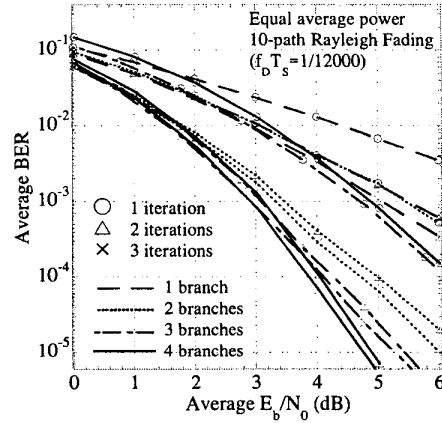


Fig.6. Average BER performance of the proposed SC/MMSE with the diversity branch number as a parameter

Fig.7 shows, for $M=2$ and $E_b/N_0=4$ dB, the BER performance of the proposed SC/MMSE equalizer versus fading correlation ρ between the antenna branches. It is found that increasing the ρ value degrades BER performance. The performance sensitivity of the proposed SC/MMSE is, however, comparable to that of SOVA.

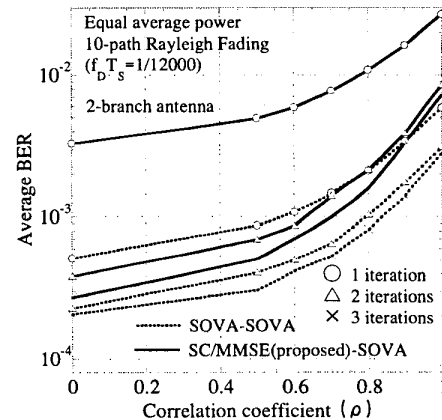


Fig.7. BER sensitivity to correlation coefficient between 2 antennas

Fig.8 shows, for $L=20$, the BER performances of the original and proposed SC/MMSE equalizers. An exponentially decaying path model was assumed for the 1st and the 20th received paths. After three iterations, the proposed SC/MMSE equalizer achieves almost the same performance as the original SC/MMSE.

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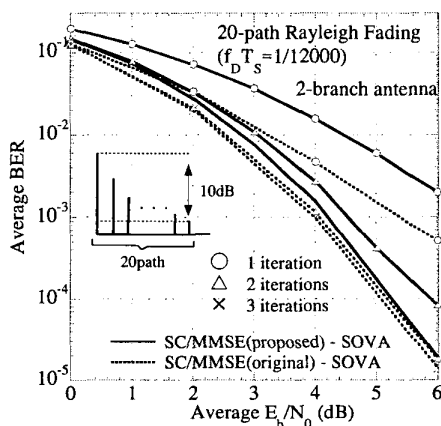


Fig.8. Comparison of original and proposed SC/MMSE (L=20)

VI. Conclusions

In this paper we introduced a new iterative ISI equalization algorithm, SC/MMSE with a matched filter approximation, that offers low computational complexity: order L^2 with channel memory length L . The BER performance of the proposed algorithm was evaluated through computer simulations. Results show that the proposed algorithm basically matches the performance of SOVA. Computer simulations also showed that the proposed SC/MMSE equalizer with noise power estimation achieves exactly the same performance as when the noise power is known. Therefore, the proposed algorithm can solve the complexity problem inherent in conventional trellis-based iterative equalizers.

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