2003 4th IEEE Workshop on Signal Processing
Advances in Wireless Communications

TURBO EQUALIZATION OF MULTILEVEL CODED QAM

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ABSTRACT

A turbo-equalization technique is presented for transmissions, where QAM constellations are constructed via block-partitioning as linear combinations of rotated, separately channel-coded BPSK components. The multilevel encoded QAM can be decoded with multistage decoding level-by-level. Iterative equalization and multistage decoding is performed via soft interference cancellation and MMSE equalization followed by soft-in-soft-out channel decoders. Due to binary channel codes both the equalizer and decoder can operate with binary likelihood values without requiring symbol/bit conversion. The resulting scheme is suitable for robust transmission when transmitter has little or no knowledge of channel quality.

1. INTRODUCTION

Turbo equalization [1] has in the recent years been studied to realize high performance equalizers for intersymbol interference mitigation. Via iterative processing a turbo equalizer is able to mitigate channel distortion while avoiding the complexity of globally optimum equalizers. The optimal turbo equalizer utilizes soft-input, soft-output MAP processing both in equalization and channel decoding. One of the proposed sub-optimal approximations replaces the optimal MAP equalizer by interference cancellation followed by MMSE filtering [2] [3] [4]. The algorithm was originally proposed in [5] for turbo detection of coded DS-CDMA systems.

Higher order constellations are a straightforward way to increase the spectral efficiency of transmission. With the introduction of suitable coding methods in the form of trellis codes and bit-interleaved coded modulation they have become interesting techniques to provide high spectral efficiency. The application of turbo equalization to higher order constellations, however, remains as somewhat a complex task. When utilizing binary channel codes the decoder requires the mapping of symbol likelihoods provided by the equalizer module into binary bit likelihoods [6] [7].

Multilevel coding [8][9] has been proposed as a method of constructing higher order constellations by combining several independently encoded bit streams, levels, into a single symbol stream. A multilevel-coded transmission is decoded with a multistage decoder where each code level is decoded utilizing knowledge of already decoded levels. By suitable selection of constellation, mapping and code rates many design objectives can be fulfilled with relatively simple encoder and decoder structures.

In wireless channels one of the most interesting capabilities of multilevel coded signals is the ability to construct soft-degrading schemes, where the transmission fills the available channel capacity. Levels whose sum capacity is below channel capacity can be, in principle, received successfully and the rest received in error. Such behavior is highly desired when the transmitter has little or no knowledge of the channel state, e. g. in wireless channels utilizing bursty transmissions or broadcasting networks. In multilevel coding schemes the constellation partitioning plays a significant role in the construction of the transmission. The partitioning provides the parallel coded streams with an equivalent channel [9] with a certain capacity that the utilised channel code must match. Block partitioning [9] is a method for minimizing the variance of minimum distances between equivalent channels, resulting in identical optimal code rates for the streams. When a robust scheme is constructed via block partitioning, the parallel streams provide "capacity steps" with which to fill the available capacity. An additional benefit is the fact that the same channel code can be utilised for all streams. The combined equalization and decoding of multilevel coded transmission is studied in [10], where the authors utilise decision feedback detection along with multilevel decoding.

The purpose of this paper is to present a technique for the turbo equalization block-partitioned multilevel codes, thereby constructing a simple robust transmission method which can be equalized and decoded with binary likelihood operations. The utilised turbo equalizer also performs the

This work has been financially supported by Elektrobit, Finnish Air Force, Nokia and Tekes, the National Technology Agency of Finland.

separation of multilevel coded streams so that no explicit multistage decoding is required. This paper is organized as follows. In Section 2, the transmission method is described with the turbo equalization algorithm presented in Section 3. The performance limits of the scheme are discussed in Section 4, and numerical simulation results are presented in Section 5. The paper is concluded with a summary.

2. BLOCK-PARTITIONED MULTILEVEL CODED QAM

For multilevel coding, the transmitted bit stream is serial-to-parallel converted to M streams, M indicating the number of bits per symbol in the QAM constellation. Each stream is encoded with a channel code, which is here assumed to be a convolutional code. The encoded bits are interleaved with a stream-specific random interleaver and BPSK-modulated to produce transmitted symbols vectors of length N

$$\mathbf{b} = \left[\mathbf{b}^{\mathrm{T}}(1), \ldots, \mathbf{b}^{\mathrm{T}}(n), \ldots, \mathbf{b}^{\mathrm{T}}(N)\right]^{\mathrm{T}} \quad (1)$$

$$\mathbf{b}(n) = \left[b_1(n), \ldots, b_m(n), \ldots, b_M(n)\right]^{\mathrm{T}} \quad (2)$$

and after being multiplied with a stream-specific complex weighting factor z_m summed into a QAM symbols Ξ. The complex weighting factor z_m is real for m even, imaginary for m odd. Each transmitted QAM symbol $\Xi(n)$ is then given as $\Xi(n) = \mathbf{z}\mathbf{b}(n) = \mathbf{z}[b_1(n), \ldots, b_M(n)]^{\mathrm{T}}$, and the whole transmitted symbol vector is given as

$$\Xi = \mathbf{Z}\mathbf{b}, \quad (3)$$

where $\mathbf{Z}$ is the block diagonal matrix of the mapping row vectors $\mathbf{z} = [z_1, \ldots, z_m, \ldots, z_M]$. To demonstrate the block-partitioned linear mapping, Figure 1 demonstrates how the mapping vector $z = [1, j, 1/2, 1/2j]$ is utilised to construct a block-partitioned 16-QAM: the partitioning consists of two superpositioned 4-QAM (four BPSK) constellations. In this paper these 4-QAM constellations are caller layers, to separate them from coding levels, since each 4-QAM constellation is treated as an I/Q pair in the transmission and reception. The layers are ordered from the largest constellation to smallest, with numbering $1 \ldots M/2$.

When the symbols Ξ are transmitted across a time-dispersive channel and received with multiple antennas, the received signal, embedded in complex Gaussian noise with variance σ_0^2, is given as

$$\mathbf{r} = \mathbf{H}\Xi + \mathbf{n}. \quad (4)$$

The multipath channel matrix $\mathbf{H}$ with L channel taps and J receiver antennas is given as

$$\mathbf{H} = \left[\mathbf{H}(1), \ldots, \mathbf{H}(n), \ldots, \mathbf{H}(N)\right], \quad (5)$$

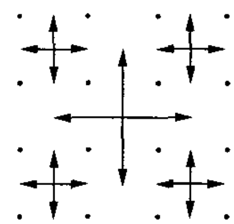

Fig. 1. Block-partitioned 16-QAM.

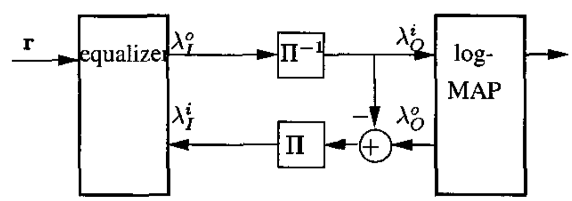

Fig. 2. Turbo equalizer

where $\mathbf{H}(n) = \left[\mathbf{0}^{\mathrm{T}}_{(n-1)*J}, \mathbf{h}^{\mathrm{T}}(n), \mathbf{0}^{\mathrm{T}}_{(N-n+1)*J}\right]^{\mathrm{T}}$ incorporates the channel response

$$\mathbf{h}(n) = \left[\mathbf{h}_1^{\mathrm{T}}(n), \ldots, \mathbf{h}_l^{\mathrm{T}}(n), \ldots, \mathbf{h}_L^{\mathrm{T}}(n)\right]^{\mathrm{T}} \quad (6)$$

$$\mathbf{h}_l(n) = \left[h_{l,1}(n), \ldots, h_{l,j}(n), \ldots, h_{l,J}(n)\right]^{\mathrm{T}} \quad (7)$$

and the transmission time of the symbol n. The channel is sampled once per symbol. Combining Equations (4) and (3) the linear system model for transmitted BPSK symbols is given as

$$\mathbf{r} = \mathbf{H}\mathbf{Z}\mathbf{b} + \mathbf{n}, \quad (8)$$

where the modulation has been integrated into the linear system model.

3. TURBO EQUALIZATION

3.1. For one code level

In this section, the equalizer proposed in [2] is applied into the equalization of the signal presented in Section 2. The equalizer, outlined in Figure 2, consists of a soft-in-soft-out equalizer module and a channel decoder module. The equalizer module calculates binary extrinsic likelihood information of a symbol by utilizing the linear model (4). The symbol likelihood computation is based on the Gaussian approximation [5] and calculated for one binary symbol in layer m as

$$\lambda_I^o(n) = \frac{2\Re\{x_m(n)\}}{1 - \mu_m(n)}, \quad (9)$$

where the metrics $x_m(n)$ and $\mu_m(n)$ are calculated as

$$
\begin{aligned}
x_m(n) &= z_m^* \mathbf{h}^{\mathrm{H}} \Theta^{-1} \left(\mathbf{r} - \tilde{\mathbf{r}} + \mathbf{h} z_m \tilde{b}_m(n) \right) && (10) \\
\mu_m(n) &= z_m^* \mathbf{h}^{\mathrm{H}} \Theta^{-1} \mathbf{h} z_m && (11) \\
\Theta &= \mathbf{H} \mathbf{Z} \Lambda \mathbf{Z}^{\mathrm{H}} \mathbf{H}^{\mathrm{H}} + I \sigma_0^2 && (12) \\
&\quad + \mathbf{h} z_m (1 - \tilde{b}_m^2(n)) z_m^* \mathbf{h}^{\mathrm{H}} && \\
\Lambda &= \mathrm{diag}\{1 - \tilde{b}^2(n - L + 1), \ldots && (13) \\
&\quad \ldots, 1 - \tilde{b}^2(n + L - 1)\} && \\
\tilde{\mathbf{r}} &= \mathbf{H} \mathbf{Z} \tilde{\mathbf{b}}. && (14)
\end{aligned}
$$

Equation (10) presents the core of the equalization algorithm: interference cancellation with the received signal replica $\tilde{\mathbf{r}}$ followed by MMSE filtering, where the MMSE taps are defined for the interference cancelled signal. Oversampling of the received signal or multiple receiver antennas are required to increase the rank of the channel matrix. The algorithm is performed utilizing an equalization window of $2L - 1$ symbols, but the indexing is dropped from Equations (9)-(14) for clarity. The soft feedback is provided by the channel decoder, whose a-posteriori likelihoods are converted into MMSE estimates of transmitted binary symbols as

$$\tilde{b}(n) = \tanh\left(\frac{\lambda_I^i(n)}{2}\right). \qquad (15)$$

The channel decoder uses the de-interleaved equalizer module outputs λ_I^o and calculates a-posteriori likelihoods of channel symbols. Extrinsic information is calculated by subtracting the decoder input likelihood from the a-posteriori likelihood at the decoder output

$$\lambda_I^i = \lambda_O^o - \lambda_O^i. \qquad (16)$$

The equalization is then performed iteratively for a maximum allowed number of iterations unless a stopping condition has been set. Optimally, the ISI removal is perfect, and the equalizer module performs effectively maximal ratio combining and likelihood generation.

3.2. Layer Separation

A crucial observation in understanding the algorithm is that multilevel codes are designed to be decoded level-by-level with knowledge only from already decoded levels. In the case of QAM, the decoding can also be performed layer at a time, since I and Q branches can be decoded in parallel [11]. With the scheme proposed here lower layers can be decoded without prior knowledge from higher layers and the turbo equalizer can be initialized by performing the equalizer algorithm only for the lowest layer, incorporating higher layers into the decoding iteration after an appropriate number of iterations. When a layer is not incorporated into equalizer calculation, λ_I^i for the corresponding streams are set to

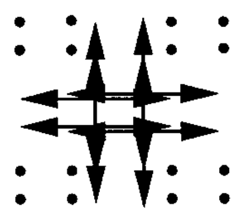

Fig. 3. 16-QAM after almost perfect first layer cancellation.

zero. In such case, the equalizer algorithm treats the layer as interference and the calculated likelihood information is adjusted accordingly. After each layer has been decoded, the interference cancellation shrinks the interfering constellation size of the decoded layer. Figure 3 demonstrates this for 16-QAM where the first layer has been cancelled almost totally and the second layer constellations have moved on top of each other. The 16 possible constellation points for the second layer are almost overlapping.

When in the traditional multistage decoding each decoding stage feeds information to the next stage, here the equalizer is utilised for forwarding a-priori information between the decoders, which can operate in parallel.

4. PERFORMANCE LIMITS

In the hypothetical case of perfect symbol likelihood feedback and channel state information each symbol stream can be totally separated at the receiver. This is due to the fact that all ISI components and undesired symbols of the constellation Ξ have been removed from the received signal. The receiver performance corresponds then to the case of maximal-ratio combining the multiple channel paths without intersymbol interference. Given layer m amplitude is weighted by $|z_m|$, the received energy per symbol $E_{\mathrm{S},m}$ for the layer in the case of perfect feedback is

$$E_{\mathrm{S},m} = \frac{\gamma E_{\mathrm{S,tot}} \left|z_m\right|^2}{\sum_{m=1}^{M} \left|z_m\right|^2}, \qquad (17)$$

where $E_{\mathrm{S,tot}}$ is the total symbol energy and γ is the total squared channel response. Equation (17) shows, that block-partitioning allocates transmitted power to the layers unevenly via the definition of $\mathbf{z}$. In the case of symmetric QAM, the difference in $E_{\mathrm{S},m}$ between layers is 6dB. When designing the transmission for robustness the inter-layer difference together with the characteristics of the channel code define the operating point of each layer. Given all levels utilise the same channel code, the operating point is defined by the partitioning. A 64-QAM scheme would then tolerate channel state uncertainty in the order of ±6dB on the transmitter side while still guaranteeing that at least one layer is transmitted succesfully and one third of the maximal instantaneous throughput is achieved.

In reality the noise-limited case described above is unrealistic as a transmission design method. Whenever a layer is unreliably received, cancellation is also imperfect. Imperfectly cancelled layers remain as interference in the received signal and limit the performance of other layers. The joint convergence behavior of the layers defines the effective SINR after equalization. Numerical simulation results of the behavior are provided in Section 5.

5. NUMERICAL RESULTS

Simulations were performed to test the performance of the scheme. A 10 path Rayleigh fading channel with uniform average tap profile and two receive antennas were used. The channel taps were assumed to be uncorrelated and static over the transmission period, but changing frame-by-frame. The channel state is assumed known at the receiver. The channel code is the rate one-half convolutional code with generator polynomials (5,7). The receiver performs one processing iteration for each layer before including the next layer into processing.

To test the receiver algorithm a 16-QAM system containing two layers was simulated. The resulting bit-error-rate performance, averaged over the I/Q branches of a layer, is presented in Figure 4 as a function of the average E_b/N_0 per antenna. The two curve groups, solid and dashed lines, correspond to the reached BER at each receiver iteration of the first and second layers, respectively. The figure shows how the scheme provides a certain reliability for the layers at different E_b/N_0. If equal error protection of transmitted data is desired data is de-multiplexed into parallel streams and multiplexed at the receiver, and the provided reliability is the mean of the streams' reliabilities. The mean BER of the simulated 16-QAM system is shown in Figure 5.

When evaluating the scheme for transmission over channels of unknown quality the expected performance in instantaneous channel conditions is more relevant than the average performance over channel realisations in determining how the scheme behaves in varying conditions. A 64-QAM scheme was simulated so that each random channel realisation was normalised and the received E_b/N_0 was set to a predetermined level. Such a setup allows us to view the received E_b/N_0 as a random operating point of the link. The set of performance curves in Figure 6 shows how the scheme performs in each operating point. The three curve groups - solid, dashed and dotted lines - correspond to first, second and third layers, respectively. If the required BER is set to 10^{-3}, layer 1 provides it at approximately 0.7dB. As the E_b/N_0 increases, layer 2 fulfills the requirement at 5.5dB and layer 3 at 9dB. The E_b/N_0 step between layers is not fully consistent with the transmission layer difference

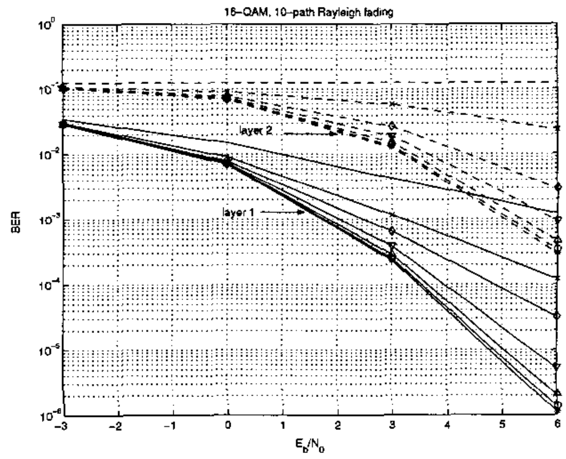

Fig. 4. 16-QAM bit-error rate.

(6dB). This is due to the fact that each layer degrades the effective SINR of the other layers at SNR regions where the decoder feedback for the layer is not reliable. In such regions the layer cannot be canceled effectively and remains as interference. This is demonstrated as inter-layer interference from high layers at low SNR. At high signal-to-noise ratios all layers decode well and little ISI or inter-layer interference remains – the scheme operates close to the limits given in Section 4. In general it can be noted that the receiver is able to follow the instantaneous channel quality by offering more throughput as the E_b/N_0 increases.

6. SUMMARY

A new turbo equalizer to detect and decode multilevel modulated, block-partitioned QAM constellations was presented. The algorithm does not require symbol-to-bit likelihood conversion at the receiver due to the linearity of the utilised constellation mapping, which enables the integration of the mapping into the equalizer. The transmission scheme can be utilised to provide throughput robustness in cases when channel quality information at the transmitter is incomplete or non-existent.

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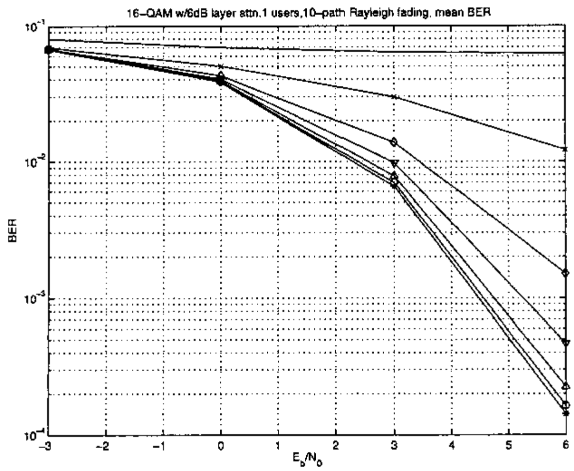

Fig. 5. 16-QAM mean bit-error rate over all streams.

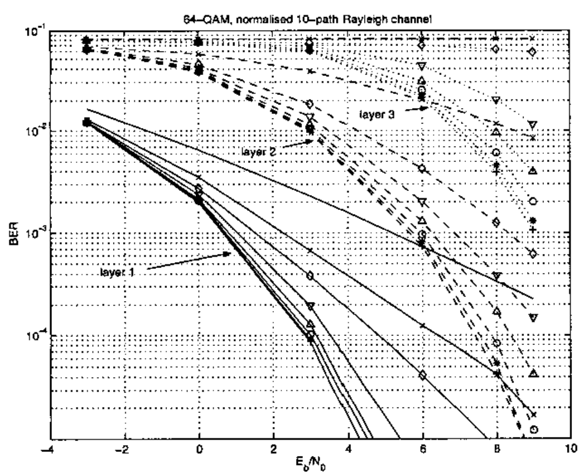

Fig. 6. 64-QAM bit-error rate - channel realisations normalised to provide pre-defined E_b/N_0.

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