Reconstruction of chaotic dynamics via a network of stochastic resonance neurons and its application to speech

Isao TOKUDA * , Tomokazu YANAI,

Department of Computer Science and Systems Engineering, Muroran Institute of Technology, Muroran, Hokkaido 050-8585, Japan

Kazuyuki AIHARA

Department of Mathematical Engineering and Information Physics, Faculty of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113, Japan; CREST, Japan Science and Technology Corporation (JST), 4-1-8 Hon-cho, Kawaguchi, Saitama 332-0012, Japan

* Corresponding Author:

Phone:+81-143-46-5477

FAX:+81-143-46-5499

E-MAIL: tokuda@csse.muroran-it.ac.jp

Abstract

As a new framework for understanding sensory mechanism of biological systems, a great deal of attention has been recently paid to *stochastic resonance*. The stochastic resonance explains properties of sensory neurons that accurately detect weak input stimuli by using a small amount of internal noise. In particular, Collins *et al.* reported that a network of stochastic resonance neurons gives rise to robust sensory function for detecting a variety of complex input signals. In this study, we investigate effectiveness of such stochastic resonance neural networks to chaotic input signals. By using the Rössler equations, we analyze the network capability of detecting chaotic dynamics. We also apply the stochastic resonance network systems to speech signals and examine plausibility of the stochastic resonance neural network as a possible model for the human auditory system.

Keywords - Stochastic Resonance, Neural Network, Chaos, Speech, Auditory System

1. Introduction

Recently, a great deal of attention has been paid to *stochastic resonance* [1] as a potential model for sensory neurons in real biological systems [2, 3, 4]. Stochastic resonance well elucidates function of the sensory neurons in the sense that it enables neurons to detect weak input stimuli by using a small amount of internal noise. From this viewpoint, stochastic resonance and its sensory function have been studied for a variety of neuron models such as the FitzHugh-Nagumo neuron model and the Hodgkin-Huxley neuron model [5]. Although periodic signals have been mainly considered for input stimuli, Collins *et al.* extended their idea to aperiodic input signals and demonstrated a neuronal capability to detect weak aperiodic input stimuli [6]. In particular, they showed that a network of stochastic resonance neurons gives rise to a sensory function which is much more robust than a single neuron in the sense that the network does not require any careful tuning of the optimal noise intensity [7]. Their results therefore imply that the network structure might be of significant importance for studying a sensory function in real neural systems.

On the basis of the Collins work [7], this paper applies the network of stochastic resonance neurons as a sensory model to detect chaotic input. We study how the dynamical structure of chaos is encoded into the firing rate of the neural network. The present study is of potential importance because chaotic dynamics have been recently discovered in a variety of biological systems [8, 9] and it is an important open problem to consider how the dynamical characteristics of chaos are encoded in neuronal information processing systems. Although there are several research works which discuss problems for encoding chaotic dynamics into single neuron models [10, 11, 12], as far as we know, encoding chaos into neural network models has not yet been thoroughly investigated.

By using the Rössler equations as a typical example of deterministic chaos, we study response characteristics of a network of stochastic resonance neurons to chaotic input stimuli. Our main focus is on the study of whether the geometrical structure of chaotic dynamics can be encoded into delay-coordinate of the firing rate of the neural network. By using the nonlinear prediction technique, accuracy of the geometrical encoding of chaos is evaluated. Dependence of the accuracy of the geometrical encoding on the neural network parameters such as the noise intensity, the number of the neural elements, and the time constant parameter is also investigated. As a possible application, we further apply the neural network model to reconstruct speech signal of a normal phonation of vowel `/a/` and examine plausibility of the neural network as a real physiological model for human auditory system.

The present paper is organized as follows: In section 2, we introduce a network of noisy FitzHugh-Nagumo neurons as a stochastic resonance network model. In section 3, we study response characteristics of the neural network to chaotic input generated from the Rössler equations. We investigate the network capability of reconstructing chaotic dynamics by varying the noise intensity, the network size, and the time constant parameter. In section 4, we apply the network model to speech signals and study how many neurons are required to reconstruct speech signals which are perceived clear enough to human ear. The final section is devoted for conclusions and discussions of the present study.

2. A network of FitzHugh-Nagumo neurons

Let us consider a network of FitzHugh-Nagumo (FHN) neurons [13, 14] described as follows (see also Fig. 1):

$$\tau\epsilon\dot{v}_i = -v_i(v_i - 0.5)(v_i - 1) - w_i + S(t) + \xi_i(t), \quad (1)$$

$$\tau\dot{w}_i = v_i - w_i - e \qquad (i = 1, .., K). \quad (2)$$

The variables v_i and w_i stand for the dynamical states of the i-th neuron, ξ_i represents independent *Gaussian* white noise satisfying $E[\xi_i(t)] = 0$ and $E[\xi_i(t)\xi_j(s)] = 2D\delta(t-s)\delta(i-j)$, ($E[\cdot]$: ensemble average), and K represents the number of the neurons in the network. e and ϵ represents system parameters of the FHN model which are fixed as $\{e, \epsilon\} = \{0.15, 0.005\}$. The time constant parameter τ that controls the time scale of the neural dynamics is set to be $\tau = 0.01$. We consider that the neurons of the network are uniform and commonly receive a weak *subthreshold* input $S(t)$. By *subthreshold*, we mean that each neuron never fires without any noise. In the following numerical experiments, dynamics of the stochastic differential equations (1) and (2) are simulated by integrating the equations with a first-order approximate algorithm [15] with an integration step of $\Delta t = 5{\cdot}10^{-5}$.

When the i-th neuron potential $v_i(t)$ crosses a threshold value of $v_{th} = 0.7$, we define that the i-th neuron fires. Then the firing rate $R_i(t)$ of the i-th neuron is computed by counting a number of the firing times within a duration of W. Activity of the FHN neural network is finally measured by the firing rate of the summing unit $R_\Sigma(t)$ (see Fig. 1) that sums up and averages the firing rates of all neural elements as $R_\Sigma(t) = \frac{1}{K}\sum_{i=1}^{K} R_i(t)$.

In Fig. 2, response characteristic of a single neuron ($K = 1$) to time-constant input stimuli S is shown. The firing rate R over a long-term duration $W = 500$ is drawn by increasing the input stimuli S, where the noise intensity is set as $D = 0$

and 10^{-8}. In case of no noise ($D = 0$), the FHN model has a stable equilibrium point corresponding to the resting state and never fires for a small input signal S. Hence, the firing rate is zero until $S < 0.11231$. As the input signal is increased over $S \approx 0.11231$, a limit cycle attractor is generated via a Hopf bifurcation of the equilibrium point and the neuron exhibits oscillatory dynamics. The firing rate jumps up at the bifurcation point and then monotonically increases as the input stimuli S is further increased. In contrast to the noise-free model, in the case that there is noise of $D = 10^{-8}$, the neuron fires also in the *subthreshold* region ($S < 0.11231$) and the firing rate monotonically and continuously increases as the subthreshold input S is increased. This corresponds to the stochastic resonance region, where the small noise enables neuron to detect a weak input stimuli. In order to study the function of this type of stochastic resonance noise, in the following sections, we consider only the *subthreshold* input signals.

3. Reconstructing chaotic dynamics

3. 1. Reconstructing the Rössler attractor

In this section, we study response characteristics of the network of FHN neurons activated by chaotic input stimuli. With respect to several related works [7, 12], the present work has the following different viewpoints. In [12], Castro and Sauer studied response characteristics of a single noisy FHN model to chaotic input and reported that stochastic resonance phenomenon has been observed. Collins *et al.*[6, 7], on the other hand, presented a theory of a network of stochastic resonance neurons and showed the advantage of the network model which realizes a robust detection of weak input stimuli without any careful tuning of the noise intensity. In this study, we focus on chaotic input signals rather than general aperiodic signals so that we can apply techniques of nonlinear deterministic prediction to evaluate response characteristics of the neural network.

By using the $x(t)$-variable of the following Rössler equations [16]:

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay, \\ \dot{z} &= bx + z(x - c), \end{aligned} \tag{3}$$

we study how the dynamical structure of the chaotic input can be encoded into the firing rate of the network of FHN neurons. The parameter values of the Rössler equations are fixed as $(a, b, c) = (0.36, 0.4, 4.5)$ and the input signal is set to be

$$S(t) = 0.05 + 0.06\frac{x(t)}{\max_t |x(t)|} \tag{4}$$

Figures 3 (b) and (c) show dynamical structure of the chaotic input signal reconstructed by using a delay coordinate of the mean firing rate $R_\Sigma(t)$ of the FHN network as

$$\{R_\Sigma(t), R_\Sigma(t-\theta), .., R_\Sigma(t-(d-1)\theta)\}, \tag{5}$$

where d and θ stand for the reconstruction dimension and the time lag, respectively [17, 18]. Compared to the original chaotic dynamics of Fig. 3 (a), dynamical characteristics of the Rössler attractor with “stretching” and “folding” are well reproduced in Figs. 3 (b) and (c). Although the reconstructed dynamics by a network of 20 neurons looks rather noisy in Fig. 3 (b), smoother nonlinear dynamics can be reconstructed by a network of 1000 neurons in Fig. 3 (c). This is due to the effect of the network structure which realizes a reliable and accurate detection of a weak input stimuli by using an ensemble average of the firing rates of many neural elements.

3. 2. Nonlinear prediction

In this subsection, we quantify the accuracy of reconstructing chaotic dynamics by stochastic resonance neural networks. Although Collins *et al.* [7] used correlation coefficient for measuring the accuracy of the signal detection, we exploit nonlinear prediction error NPE [19, 20] for evaluating the accuracy of the geometric reconstruction of chaotic dynamics as follows.

First, we compute time series of the averaged firing rate of the neural network

$$\{u(n) = R_\Sigma(n\Delta) : n = 0, 1, .., N\} \tag{6}$$

with a sampling rate of $\Delta = 0.1$. Then, the time series $\{u(n)\}$ are divided into first and second halves. From the first half data, a nonlinear predictor $\widetilde{f} : R^d \to R^d$ which approximates the data dynamics as $\boldsymbol{u}(n+1) \approx \widetilde{f}(\boldsymbol{u}(n))$ is constructed, using the delay coordinate $\boldsymbol{u}(n) = \{u(n), u(n-\theta), .., u(n-(d-1)\theta)\}$. For the nonlinear predictor $\widetilde{f}$, the Sugihara-May's local linear predictor [20] is exploited. Then, for the latter half data, nonlinear prediction is carried out. The forecasting procedure is that, for a give initial state $\boldsymbol{u}(n)$, the p-step further state $\boldsymbol{u}(n+p)$ is predicted as $\widetilde{\boldsymbol{u}}(n+p) = \widetilde{f}^p(\boldsymbol{u}(n))$ using the p-iterate of the predictor $\widetilde{f}$. The NPE is finally computed as the following normalized root-mean-square error:

$$NPE = \frac{\sqrt{\frac{2}{N}\sum_{n=N/2}^{N}\{u(n) - \widetilde{u}(n)\}^2}}{\sqrt{\frac{2}{N}\sum_{n=N/2}^{N}\{u(n) - E[u]\}^2}}. \tag{7}$$

In our analysis, the reconstruction dimension d, the time lag θ, the prediction step p, and the number of the data N are set as $(d, \theta, p, N) = (3, 15, 2, 5000)$.

Figure 4 shows nonlinear prediction curve computed for the FHN network with an increasing noise intensity from $D = 2.5{\cdot}10^{-10}$ to $5.12{\cdot}10^{-7}$. The four prediction curves correspond to the networks with different sizes of $K = 1, 10, 100$, and 1000. First, we focus on the single neuron's response curve ($K = 1$). In a small noise region, a very large prediction error is initially observed. As the noise intensity is slightly increased, the prediction error is decreased significantly and the response curve produces a single sharp minimum at $D \approx 10^{-8}$. As the noise intensity is further increased, the prediction error is increased eventually. This is a typical characteristic of *stochastic resonance*, which is known to exhibit small effective noise for detecting weak input signals.

Next, we see the resonance characteristic of a network of FHN neurons ($K = 10, 100, 1000$). We recognize a basically similar prediction curve with the single neuron model ($K = 1$) in Fig. 4. The significant difference, however, is the the range of the effective noise that enables accurate reconstruction of chaotic dynamics. Namely, as the number of the neurons is increased, the effective noise range is widen largely and robust reconstruction of chaotic dynamics is realized in a wide noise area. As has been pointed out by Collins *et al.* [7], this is due to the advantage of the network structure that reliably estimates the strength of the input stimuli by using an ensemble averaged firing rate of many neurons. In a statistical sense, it is reasonable that the estimation becomes more accurate as the number of the component neurons is increased.

3. 3. Effect of network size and time constant parameter

In this subsection, we study dependence of the network capability of reconstructing chaotic dynamics on the network size K and the time constant parameter τ. By using the same chaotic input (4) from the Rössler equations (3), nonlinear predictability of the chaotic dynamics reconstructed by the network of FHN neurons is computed by changing the network size and the time constant parameter within the range of $(K, \tau) \in [10, 200] \times [0.01, 0.1]$. As is shown in Fig. 5, the prediction error is decreased not only by increasing the network size K but also by decreasing the time constant parameter τ. The reason why the nonlinear predictability is improved by decreasing the time constant parameter can be explained as follows. As the time constant parameter is decreased, dynamics of the FHN neuron becomes faster. Compared to the accelerated neural dynamics, temporal change in the input signal becomes relatively slower. As a consequence, neuronal capability of estimating the strength of slow-varying input stimuli is improved and accurate reconstruction of chaotic dynamics is realized.

Suppose we have noisy reconstruction of chaotic dynamics by the present network model. Then, how can we improve the reconstruction capability? As we have seen, one way is to set the time constant value small enough so that the temporal resolution of a single neuron becomes higher. The other way is to increase the network size so that more reliable estimation of the input stimuli is realized by using an ensemble averaged firing rate of many neurons. From a physiological viewpoint, it is not realistic to control the network capability by changing the time constant parameter, because biological neurons have their own physical properties are not easy to adjust although not impossible [21]. It is physiologically more plausible to control the network size in order to improve the network capability for reconstructing chaotic dynamics.

In Fig. 6, a required number of neurons to reconstruct chaotic dynamics with an accuracy of $NPE < 0.1$ is drawn by changing the time constant parameter. As the time constant parameter is increased, the required number of neurons is increased in an exponential manner. In physiological modeling of robust sensory systems, this figure presents an abstract idea for how many neurons are required for the network model.

4. Application to speech

In this section, we apply the network of FHN neurons to speech signals. By this experiment, we examine the neural network as a possible model for the human auditory system.

As a sample speech signal, normal phonation of a vowel `/a/` (`mausy003.ad`) in the standard ATR (Advanced Telecommunications Research Institute International) data-base is exploited. The waveform structure of the speech signal:

$$\{x(t) : 0 < t < 200[msec]\} \tag{8}$$

is shown in Fig. 7 (a). The subject is a male speaker who has no evidence for laryngeal pathology. The speech signal is low-pass filtered with a cut-off frequency of 8 kHz and digitized with a sampling rate of 20 kHz and with 16 bit resolution. The recording condition and the speech quality are good enough in the sense that natural vocal sound can be reproduced by D/A conversion of the speech signal.

By using the speech signal, the input signal for the neural network is set to be

$$S(t) = 0.1 + 0.01\frac{x(t)}{\max_t |x(t)|} \tag{9}$$

so that the speech is consider as a *subthreshold* input signal. The parameter values of the FHN neuron are fixed as $(D, W, \tau) = (5.7 \cdot 10^{-11}, 5 \cdot 10^{-5}, 0.001)$. Under these parameter values, the firing rate of a single neuron activated by constant input of $S = 0.1$ becomes 98.8 Hz. According to ref. [22], an average frequency in the active state of the sensory neurons in the human auditory system is about 100Hz. Hence, this parameter setting may well correspond to the situation of the real sensory neurons.

Figures 7 (b) and (c) show time series of a firing rate $R_\Sigma(t)$ of a network of FHN neurons with $K = 10^4$ and 10^6. Although the network of 10^4 neurons generates rather noisy dynamics, the firing rate $R_\Sigma(t)$ of the network of 10^4 neurons produces qualitatively similar time-waveform structure with the original speech signal of Fig. 7 (a). Figure 8 shows nonlinear prediction errors computed for speech signals reconstructed by the FHN neural networks with $K \in [10^4, 10^6]$. We see that as the number of the neurons is increased the inverse of the prediction error ($1/NPE$) is also increased and accuracy of the speech reconstruction is improved.

We also examine the sound quality of the reconstructed speech signals on the basis of the psychoacoustic experiment. As the method of the psychoacoustic experiment, the paired-comparison test [23] is adopted. For four speech signals reconstructed by the network with $K = 2{\cdot}10^4, 10^5, 5{\cdot}10^5$, and 10^6, "clarity" of the speech sound is evaluated as follows.

Among the four speech signals, pick up a pair of two signals A and B and synthesize the sounds one by one. Subjects are requested to judge which one is clearer than the other after listening to two signals in each paired-comparison test. For every pair selected from the four speech signals, the paired-comparison test is carried out. On the basis of the Thurstone's method [23], "clarity" of the four speech signals is finally evaluated. The subjects were composed of ten males and ten females, where none of them were experts of the psychoacoustic experiments.

In Fig. 8, "clarity" measures of the four speech signals are shown. As the number of the neurons is increased from $K = 2{\cdot}10^4$ to $5{\cdot}10^5$, we see that "clarity" of the speech is improved significantly. Not so much difference, on the other hand, is recognized between $K = 5{\cdot}10^5$ and 10^6. This implies that a network of more than $5{\cdot}10^5$ neurons is capable of reconstructing speech signals which are clear enough to be perceived as the original speech signal. The present result therefore suggests that in order to apply stochastic resonance neural networks to auditory systems at least $5{\cdot}10^5$ sensory neurons are required.

We also note that there is a good agreement between the inverse of the nonlinear prediction error ($1/NPE$, dotted line) and the "clarity" measure (solid line) in Fig. 8. This implies a possibility of using the nonlinear predictability as the measure for quantifying the "clarity" and the dynamical characteristic of the speech signals. Recently, there are growing research interests in the nonlinear analysis of speech signals and many researchers consider possible application of nonlinear dynamical statistics for characterizing speech signals [24, 25, 26, 27]. The present results encourage further applications of the nonlinear statistics such as nonlinear predictability for evaluating speech signals.

5. Conclusions and discussions

In this paper, we studied a network of FHN neurons for detecting chaotic dynamics in weak input stimuli. We investigated whether the geometrical structure of chaotic dynamics can be reconstructed in a delay-coordinate space of the averaged firing rate of the neural network, where the reconstruction accuracy is evaluated by nonlinear prediction errors.

By using the Rössler equations, first, we studied dependence of the network capability of reconstructing chaotic dynamics on the noise intensity. Our analyses showed stochastic resonance property that gives rise to an intermediate noise level for efficient reconstruction of chaotic dynamics. We also showed that more robust and accurate reconstruction of chaos can be realized by increasing the network size and by decreasing the time constant parameter. From a physiological viewpoint, in order to improve the network capability, it is not realistic to change the time constant parameter because real neurons have their own physical properties. It may be biologically more plausible to increase the network size for realizing a robust sensory mechanism.

On the basis of the experiments with the Rössler equations, we further applied the network of FHN neurons for reconstructing speech signals and considered the stochastic resonance neural network as a possible model for the human auditory system. By using the neural elements with a dynamical frequency of about 100Hz, we have confirmed that the speech signal which is perceived to be “clear” enough for human ear can be reconstructed by using a network of more than $5 \cdot 10^5$ neurons. This implies a possibility that the human auditory system uses this type of noisy neural networks for detecting speech signals. As a possible engineering application, the present result also encourages to use the stochastic resonance network model for a sound amplifier device to aid hearing disabilities [28].

We note that the present investigation is only preliminary in the sense that no detailed physiological structure has been considered for the human auditory system. The real auditory system has more complex and systematic organization such as the spectral decomposition of basilar membrane [22]. Also, neural elements may not be as uniform as we have considered in the present study. Namely, individual neurons may have different frequency ranges and the noise intensities can be different in the brain area. Hence, it is an important future problem to study a network of non-uniform neuron models. More realistic auditory models should be constructed to discuss plausibility of the stochastic resonance neural network for the real auditory systems.

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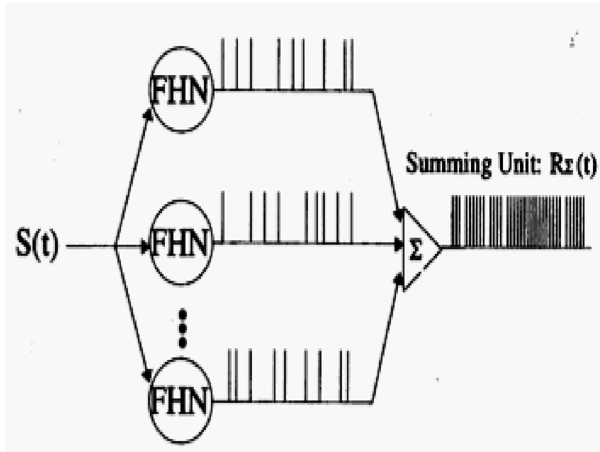

Figure 1: Schematic illustration of a network of FHN neurons with *Gaussian* white noise. Each neuron receives a same input stimuli $S(t)$. The network activity is measured by the firing rate of the summing unit $R_{\Sigma}(t)$ that computes an averaged firing rate of all the neural elements.

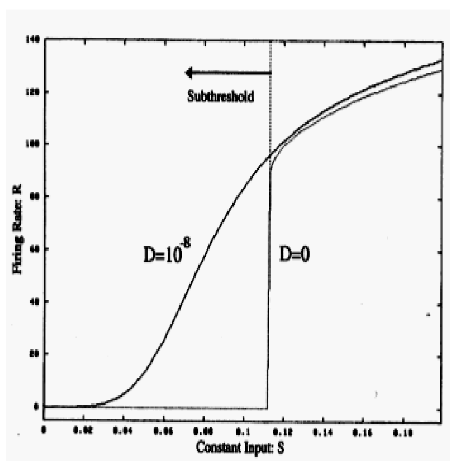

Figure 2: Response characteristic of a single neuron to a time-constant input S. The firing rate R over a long-term duration $W = 500$ is drawn by increasing the input stimuli from $S = 0$ to 0.2. The noise intensity is set to be $D = 0$ and 10^{-8}.

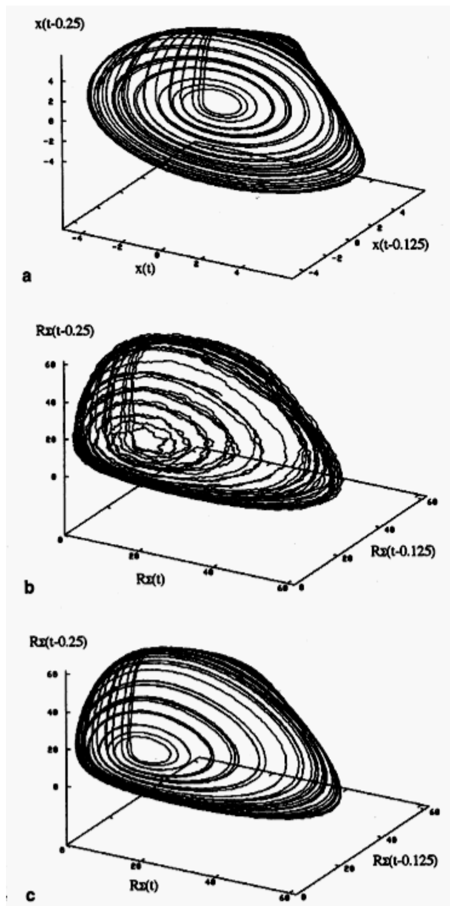

Figure 3: (a): 3-dimensional delay-coordinate reconstruction $\{x(t), x(t-0.125), x(t-0.25)\}$ of the Rössler attractor. (b),(c): 3-dimensional delay-coordinate $\{R_\Sigma(t), R_\Sigma(t-0.125), R_\Sigma(t-0.25)\}$ of the mean firing rate $R_\Sigma(t)$ of a network of FHN neurons with $K = 20$ and 1000, respectively.

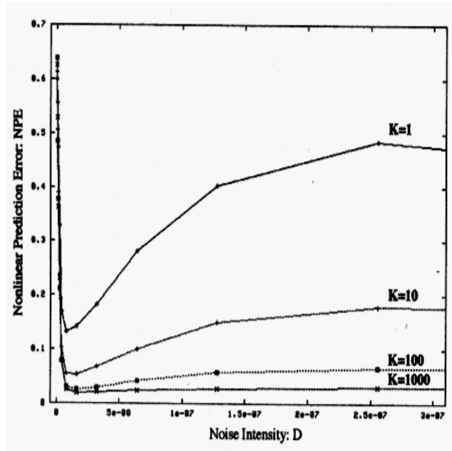

Figure 4: Nonlinear prediction curve computed for a network of FHN neurons. The noise intensity is increased from $D = 2.5{\cdot}10^{-10}$ to $5.12{\cdot}10^{-7}$ and the network size is varied as $K = 1, 10, 100$, and 1000.

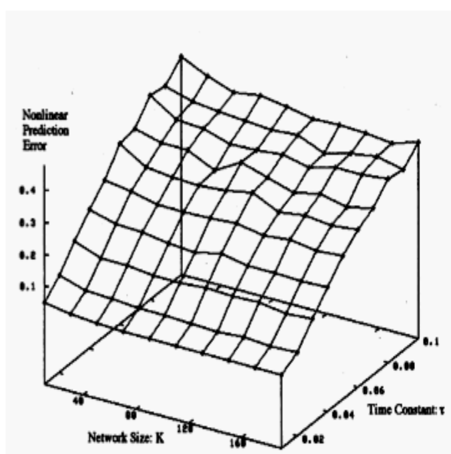

Figure 5: Nonlinear prediction error NPE computed by changing the network size K and the time constant parameter τ within a range of $(K, \tau) \in [10, 200] \times [0.01, 0.1]$.

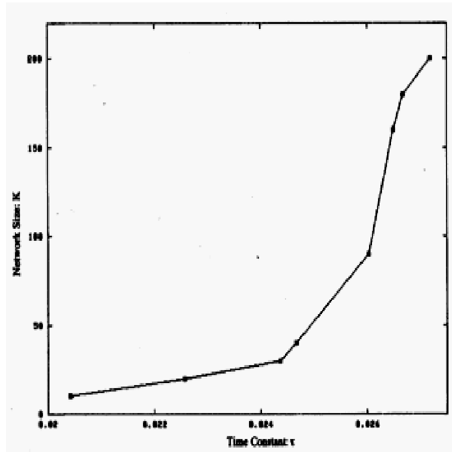

Figure 6: A required number of neurons to reconstruct chaotic dynamics with a prediction accuracy of $NPE < 0.1$. The time constant parameter is set to be $0.02 < \tau < 0.027$.

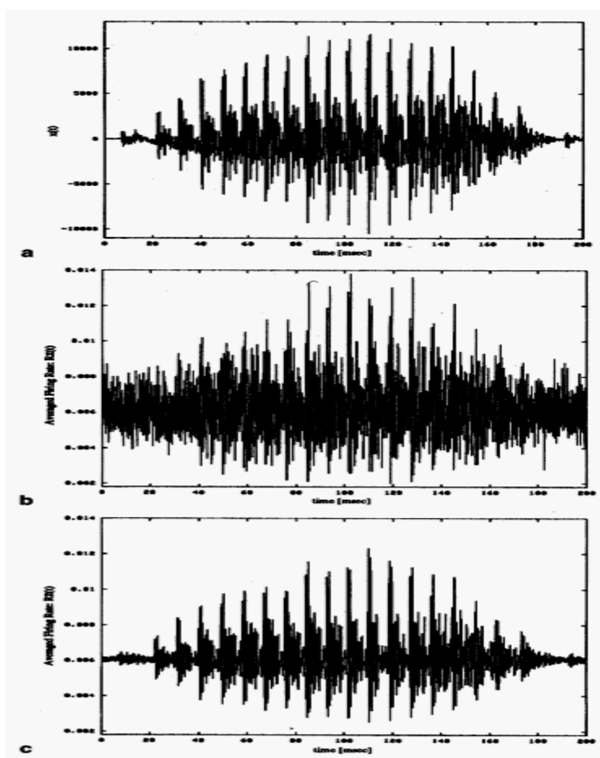

Figure 7: (a): Speech signal of a normal phonation of vowel /a/. (b),(c): Time series of the averaged firing rate $R_\Sigma(t)$ of network of FHN neurons activated by speech signal of Fig. 7 (a). The number of the neurons is set to be $K = 10^4$ for (b) and $K = 10^6$ for (c).

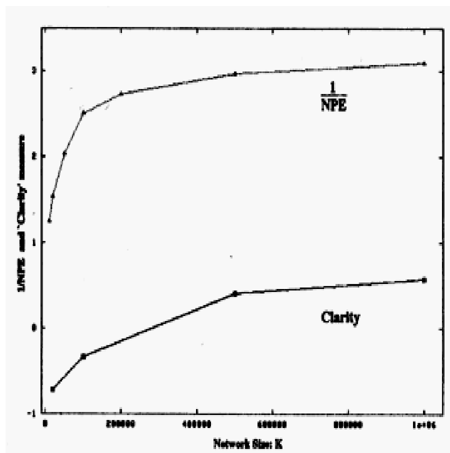

Figure 8: Simultaneous plots of the inverse of the nonlinear prediction error ($1/NPE$, dotted line) and the 'clarity' measure (solid line) computed for speech signals reconstructed by FHN neural network. The network size is varied from $K = 10^4$ to 10^6.