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Author(s)	Cincotti, Alessandro; Iida, Hiroyuki
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Description	

OUTCOME UNCERTAINTY AND INTERESTEDNESS IN GAME-PLAYING: A CASE STUDY USING SYNCHRONIZED HEX

ALESSANDRO CINCOTTI*

Research Unit for Computers and Games
Japan Advanced Institute of Science and Technology
1-1 Asahidai, Nomi, Ishikawa, 923-1292, Japan
cincotti@jaist.ac.jp

HIROYUKI IIDA

Research Unit for Computers and Games
Japan Advanced Institute of Science and Technology
1-1 Asahidai, Nomi, Ishikawa, 923-1292, Japan
iida@jaist.ac.jp

PRESTO, Japan Science and Technology Agency, Japan

Outcome uncertainty is a key-factor to measure the interestedness of a game because people are not attracted by the game where the outcome is easily predictable. To create new variants of classical games is quite easy but to refine solved games, in order to make them much more fascinating, is challenging. We introduce two simple techniques (synchronism and stochastic elements) to refine the game of Hex. Experimental results show an increment of the outcome uncertainty defined as entropic function.

Keywords: Outcome Uncertainty; Interestedness; Synchronized Hex.

1. Introduction

The relation between games and uncertainty has been recently discussed and analyzed in the sense that uncertainty epitomizes a game [1]. Uncertainty in game-playing can be characterized in terms of (i) winning strategy, (ii) game outcome, and (iii) the game-theoretical value.

(i) This kind of uncertainty concerns the players because it is strictly connected to their difficulties of selecting the best moves in order to win the game.
(ii) Typically, people enjoy a game while trying to guess the final result. Moreover, the prediction of the audience is much more objective compared to the players' prevision because the audience is not directly involved in the game and no psychological aspects affect its judgment.
(iii) Formally, the game-theoretical value represents the outcome of a game under ideal play, i.e., when both players apply their best strategy. Therefore, this

*Corresponding author.

type of uncertainty is related to the nature of the game itself and generally game theorists are interested in this specific aspect of the game.

From the perspective of enjoying a game the second kind of uncertainty is essential because it emphasizes the importance of the process which reveals the outcome of the match gradually during its progress [2]. Undoubtedly, to be able to predict the outcome of a game after few moves from the initial position makes the game unexciting. In contrast, a fascinating game should always be unpredictable until the last moves of the game.

Because of recent advancements in computer technology, some of the traditional two-player complete information games have been solved and their theoretical values have become known [3,4]. Clearly, people tend to lose interest in a solved game because there is not more any kind of uncertainty about the outcome. For example, in Tic-Tac-Toe under perfect play a game always ends in a draw. In which way is it possible to refine a solved game to make it still stimulating and enjoyable?

Morphling [5] is a computer program for designing and playing whole class of combinatorial 2-person games. Such program is able to measure automatically the interestedness of a game while evaluating some criteria as average length of game, drawing quota, balanced chances or advantage for a certain player, etc.

In this paper, we measure the interestedness of a game using the concept of late chance or outcome uncertainty and introduce two simple techniques: synchronism and stochastic elements. The idea of synchronized games have been introduced in [6]. In such games both players move contemporaneously, therefore it does not exist any *unfair* advantage due to the turn to move. We use the game of Hex [7,8] as a case study because in this game, as known, the first player has always a winning strategy and for small board's size it has been completely solved [9], i.e., we know exactly the winning strategy.

1.1. *Outline*

In section 2 we explain the idea of synchronized games and introduce some new variants of the game of Hex. Moreover, we define the players' model that we use in our self-play experiments.

In section 3 we use an entropic function to measure the outcome uncertainty during the different phases of a game. This method was previously introduced in [10] and applied to the game of Chess to analyze the outcome uncertainty when players with different strength face off.

In section 4 and 5 we present respectively our experimental results and discussion. In particular, we focus on the outcome uncertainty in the final phase of games in order to understand whether new versions of Hex are much more exciting than the original version.

2. Synchronized Hex

The basic idea behind synchronized games is to abolish the concept of turns; both players make their moves at the same time until the end of game, therefore no player has an unfair initial advantage. Below we describe the modifications introduced in the game of Hex.

2.1. *Rules*

In synchronized Hex, if both players want to play in the same board's cell then a conflict occurs. How can we resolve this kind of conflicts?
One simple idea is to permit both players to play in the same cell; this choice has a crucial consequence because in this way we introduce the possibility to get a draw which is impossible in the normal game of Hex. Another simple idea is to flip a coin to decide which player can move when a conflict comes out. This second possibility is more drastic compared to the first one but it remains still impossible to get a draw. Actually, it is possible to consider the two solutions previously described as two particular cases of a more general methodology.
Let $X = \{p, (1-p)/2, (1-p)/2\}$ be a probabilistic distribution where $0 \leq p \leq 1$ is the probability that, in case of conflict, both players will move in the same cell, and $(1-p)/2$ is the probability that only the black player will make his/her move. The probability that only the white player will make his/her move must be the same of the black player to maintain a fair game. When $X = \{1, 0, 0\}$ we always permit both players to move in the same cell instead when $X = \{0, 1/2, 1/2\}$ we always flip a coin to decide which player is allowed to move in the conflictual cell.
Now we have a flexible way to resolve a conflict and we can describe the modification introduced in the game of synchronized Hex.

- LV0 rule: It is the normal way to play.
- LV1 rule: At each step, both players execute their move contemporaneously, if a conflict occurs then it will be resolved according to the probabilistic distribution X fixed before the game starts.
- LV2 rule: Both players have to announce at the same time the next two moves and execute them. If the second move of one player is equal to the first move of the other player, this second move is not allowed to be executed. If the first (second) move of one player is equal to the first (second) move of the other player, then the conflict will be resolved according to the probabilistic distribution X fixed before the game starts.

2.2. *Players' model*

Introducing stochastic elements in synchronized games (e.g., to flip a coin) to resolve conflicts has a remarkable effect on the game strategy. In the game of backgammon it is still possible to apply the minimax algorithm introducing probabilistic nodes in the decisional tree and calculating the expectation value of each option [11]. In synchronized games, this model does not work because even if we know the probability by which we resolve the conflict, we have no idea about the probability to have a conflict considering that it depends on the opponent's move. Therefore, for our experiments we define computer players which play using two different heuristics:

- P1 (Attack). This player finds the shortest path (i.e, the path where it is necessary the minimum number of marks to complete the chain) between its two sides of the board and chooses one (LV0 and LV1) or two (LV2) cells belonging to this path in a random way. If there exists more than one minimum path it just chooses one randomly.
- P2 (Attack and Defense). This player is stronger than the previous one because it chooses one cell (or two cells in LV2) in its shortest path which is (are) involved in the shortest path (if there exists more than one path it just chooses one randomly) of its opponent.

3. Outcome Uncertainty

According to the information theory [12], a measure of uncertainty for a probability distribution $P = (p_1, p_2, \ldots, p_n)$ is defined as

$$H(P) = -\sum_{i=1}^{n} p_i \ln p_i \tag{3.1}$$

Analogously, we can define the outcome uncertainty of a game position G as an entropic function:

$$U(G) = -\sum_{i=1}^{k} p_i \ln p_i \tag{3.2}$$

where

- k is the number of possible outcomes (1 White wins, 2 Black wins, and 3 the game ends in a draw). Therefore, the max uncertainty will be either $\ln(2)$ or $\ln(3)$ because it depends on the possibility to get a draw.
- p_i is the probability that the outcome of the game G will be i.

In order to calculate the probability p_i, we execute 1000 self-play experiments beginning from the game position G and we calculate

$$p_1 = N_W/N \tag{3.3}$$

$$p_2 = N_B/N \tag{3.4}$$

$$p_3 = N_D/N \tag{3.5}$$

where N_W, N_B, and N_D are respectively the number of times White wins, Black wins, and the game ends in a draw. In this way, it is possible to calculate the outcome uncertainty from the beginning (empty board) to the end of the game. We observe that the players P1 and P2 are not deterministic therefore we have executed 1000 games to calculate the average value of the outcome uncertainty. It follows that for a 5 by 5 board's size we have evaluated about $D * 10^6$ positions where D is the average length of the game. If we increase the dimension of the board then we have to increase the number of self-play experiments as well as the number of games to calculate the average value of the outcome uncertainty and all the process becomes time-consuming.

4. Experimental Results

Experimental results show the average value of the outcome uncertainty during the course of a typical game, i.e., from the beginning to the average length of the game. We measure the length of the game in number of turns where every turn is represented by:

- one move for the black player and one move for the white player in LV0,
- one move for the black player and one move for the white player in LV1 (Of course, in this case the two moves occur simultaneously),
- two moves for the black player and two moves for the white players in LV2 that occur according to the LV2 rule previously described.

Figure 1 shows the results for $P1$ vs. $P1$ on a 5 by 5 board's size where p is the probability that players move in the same cell when a conflict occurs. We recall that for those games which cannot finish in a draw, i.e., LV0, LV1 ($p = 0$), and LV2 ($p = 0$), the max value of uncertainty is $\ln(2)$. Instead, for those games where it is possible to get a draw, i.e., LV1 ($p = 1/2$), LV1 ($p = 1$), LV2 ($p = 1/2$), LV2 ($p = 1$), the max value of uncertainty is $\ln(3)$. The result concerning $P1$ vs. $P2$ and $P2$ vs. $P2$ are shown respectively in Figure 2 and 3.

5. Discussion

What is the desiderable relation between the outcome uncertainty and interestedness? We believe that an enjoyable game should be characterized by a fast decrement of uncertainty only in the final phase of the game. In this way, players play under great tension and the outcome of the game is uncertain until the very end of the game.

Table 1 shows the average branching factor, average game length (as number of turns), search-space complexity, and the average percentage of outcome uncertainty solved in the $D-1$-th turn, i.e., one turn before to reach the average length of the game for $P1$ vs. $P1$ on a 5 by 5 board's size. The values of U greater than the value obtained using LV0 are indicated in boldface.

Table 2 and 3 show the same data respectively for $P1$ vs. $P2$ and $P2$ vs. $P2$. We can make the following observations:

(i) Using LV2 rule produces a huge increment of the average branching factor and a decrement of the average length of the game. As consequence, we have an increment of the complexity compared to LV0.

(ii) The average percentage of outcome uncertainty solved U increases as p decreases. This is true for both rules LV1 and LV2. When p decreases, the probability to get a draw decreases and games become more unpredictable.

(iii) LV2 dominates LV1, i.e., the value of U using LV2 is always greater than the value of U using LV1 when we use the same probability p. As consequence, the game of synchronized Hex using LV2 ($p = 0$) rule results the most fascinating version.

(iv) The previous observations are true even for $P1$ vs. $P2$ and $P2$ vs. $P2$ but we note that only in a few cases it is possible to get a value of U greater than LV0. It means that to refine a game, when players have different skills or when players are both expert, is much more difficult than to refine a game for beginner players.

6. Conclusions and Future Works

A case study using the game of synchronized Hex and some possible variants was presented to study the relation between the outcome uncertainty defined as entropic function and the interestedness of the game. We believe that the outcome uncertainty is a key-factor to valuate the interestedness of the game. Moreover, experimental results show that we are able, using together synchronism and stochastic elements, to produce an increment of the outcome uncertainty. It follows that these two techniques represent a possible approach to refine solved games.
Future works concern the following points:

- To introduce synchronism and stochastic elements in more complex games such as Checkers, Chess, Shogi, etc.
- To define a mathematical model in order to measure the interestedness in game-playing.

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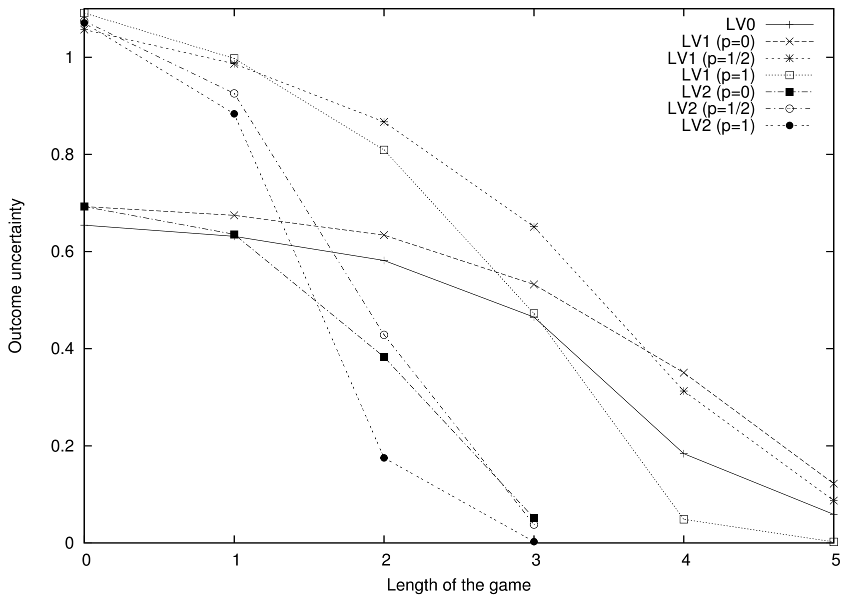

Fig. 1. Outcome uncertainty as a function of the game length (P1 vs. P1).

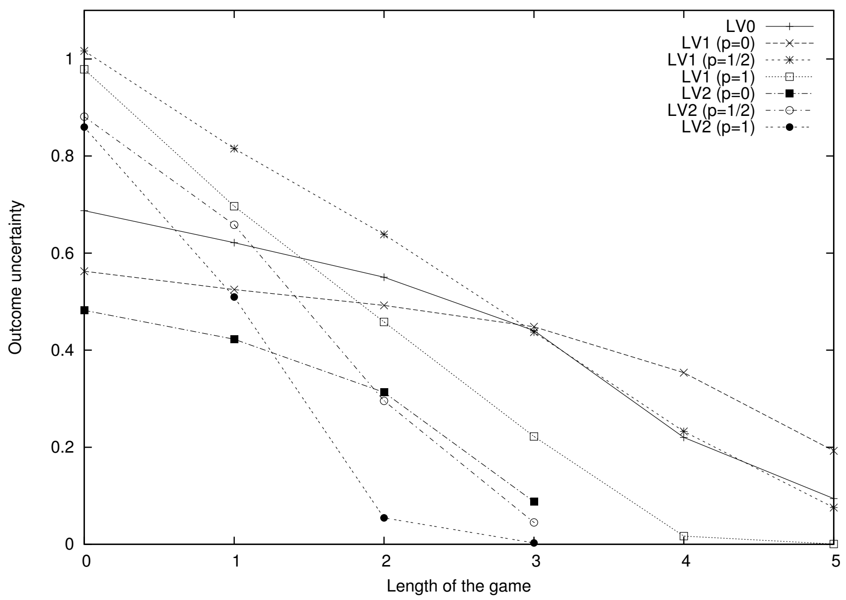

Fig. 2. Outcome uncertainty as a function of the game length (P1 vs. P2).

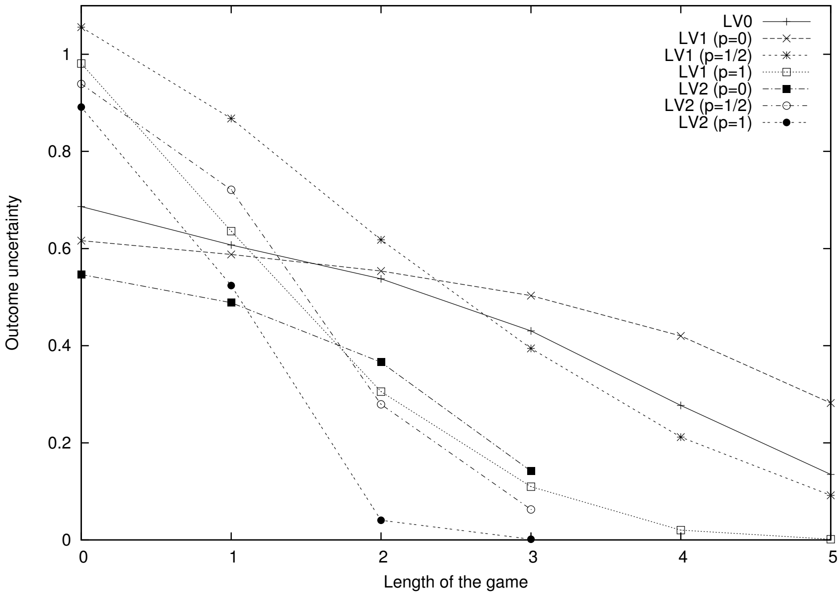

Fig. 3. Outcome uncertainty as a function of the game length (P2 vs. P2).

Table 1. Comparison of Hex variants (P1 vs P1).

	B	D	B^D	U(%)
LV0	20.5	5.3	1.1e+7	18.0
LV1 ($p = 0$)	20.7	5.4	1.3e+7	**33.0**
LV1 ($p = 0.5$)	20.8	5.2	8.2e+6	**20.5**
LV1 ($p = 1$)	21.0	5.0	5.1e+6	4.2
LV2 ($p = 0$)	430.9	3.1	1.5e+8	**47.8**
LV2 ($p = 0.5$)	434.1	3.0	1.1e+8	**35.5**
LV2 ($p = 1$)	438.4	3.0	8.9e+7	15.6

Table 2. Comparison of Hex variants (P1 vs P2).

	B	D	B^D	U(%)
LV0	20.3	5.6	1.9e+7	18.2
LV1 ($p = 0$)	20.4	5.9	6.6e+7	**23.2**
LV1 ($p = 0.5$)	21.0	5.3	1.0e+7	14.3
LV1 ($p = 1$)	21.2	5.1	5.1e+6	1.5
LV2 ($p = 0$)	426.6	3.2	3.5e+8	**32.5**
LV2 ($p = 0.5$)	437.0	3.1	1.5e+8	**22.8**
LV2 ($p = 1$)	443.1	3.0	8.6e+7	4.6

Table 3. Comparison of Hex variants (P2 vs P2).

	B	D	B^D	$U(\%)$
LV0	19.9	5.8	4.0e+7	20.5
LV1 ($p = 0$)	20.1	6.4	2.6e+8	19.9
LV1 ($p = 0.5$)	20.9	5.4	1.4e+7	10.9
LV1 ($p = 1$)	21.2	5.1	6.0e+6	1.7
LV2 ($p = 0$)	417.1	3.3	7.6e+8	**32.3**
LV2 ($p = 0.5$)	433.8	3.1	2.1e+8	19.7
LV2 ($p = 1$)	443.4	3.0	1.0e+8	3.5