A Fuzzy Set Based Approach to Generalized Landscape Theory of Aggregation

Shigemasa SUGANUMA

Japan Advanced Institute of Science and Technology
1-1 Asahidai, Tatsunokuchi, Ishikawa 923-1292, JAPAN

Academy of Mathematics and Systems Sciences
Chinese Academy of Sciences, Beijing 100080, P.R. CHINA

Van-Nam HUYNH† and Yoshiteru NAKAMORI

Japan Advanced Institute of Science and Technology
1-1 Asahidai, Tatsunokuchi, Ishikawa 923-1292, JAPAN

† *On leave from University of Quinhon, VIETNAM*

{huynh,nakamori}@jaist.ac.jp

Shouyang WANG

Academy of Mathematics and Systems Sciences
Chinese Academy of Sciences, Beijing 100080, P.R. CHINA

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Abstract In this paper, we propose a fuzzy-set-theoretic based extension to the landscape theory of aggregation introduced by Axelrod and Bennett in 1993. The significance of landscape theory is that it can provide a deeper understanding of a wide variety of important aggregation processes in politics, economics, and society. To illustrate efficiency of the proposal, we make a simulation with the proposed framework for the international alignment of the Second World War in Europe. It is shown that the obtained results are essentially comparable to those given by the original theory. Consequently, the fuzzy-set-theoretic based extension of landscape theory can allow us to analyse a wide variety of aggregation processes that have previously been considered in a more flexible manner.

Keywords Landscape Theory, Aggregation, Alliance Analysis, Fuzzy Configuration, Genetic Algorithm

§1 Introduction

A formal theory of aggregation called the landscape theory was proposed by Axelrod and Bennett in 1993. "Aggregation" means the organization of elements of a system in patterns that tend to put highly compatible elements together and less compatible elements apart [A93)]. The landscape theory aims at predicting how aggregation will lead to alignments among actors (such as nations, business firms, etc.), whose leaders are myopic in their assessments and incremental in their actions. Typically, the theory mimics the idea of an abstract landscape that has been widely used in physical and natural sciences and, more recently, in artificial intelligence to characterise the dynamics of systems. Key concepts from these disciplines have been used in the landscape theory of aggregation. It has been shown that the landscape theory can be used to analyse a wide variety of important aggregation processes in political, economic, and social problems [A95),A97)]. Particularly, the landscape theory has been supported by the results of the international alignment of the Second World War in Europe [A93)], and the coalition formation in standard-setting alliances in the case of UNIX operating system [A95)]. It should be noticed that aggregation has been studied without landscapes as a descriptive problem in statistics with the most commonly used technique being cluster analysis [K90)]. However, cluster analysis has been considered as the process of finding groups in data that have been mainly of the form of tuples, while this is not the case for actors in the landscape theory. Furthermore, unlike the landscape theory, cluster analysis is not based on a dynamic theory of behaviour and can not be used to make any predictions.

In this paper, we propose a fuzzy-set-theoretic based extension to the landscape theory called fuzzy landscape theory. In particular, the notion of fuzzy partitions of a set of actors will be introduced into conventional landscape theory instead of crisp ones. Formally, the idea is similar to that of fuzzy clustering [R69)]. The motivation to develop a fuzzy version of landscape theory is that it can allow us to analyse a variety of important aggregation processes in political, economic, and social problems in a more flexible manner. For example, in international politics, the leaders of nations always want to find flexible policies in settlement of conflicts with others (such as a border dispute or a matter of ethnic problems). At the same time, there may be also some nations that play an intercessionary role in an attempt to facilitate an internationally peaceful status. Moreover, the consequences of such situations are not confined to the tensely international arena but also have consequences on business life [O77],[C00],[G92]. Even in situations where we need a crisp aggregation, fuzzification of the landscape theory also allows us to use membership values in deciding the core and boundary actors of alliances, thereby providing more useful information for dealing with boundary actors.

The rest of this paper is organized as follows. Section 2 briefly recall basic notions of landscape theory. After mathematically restating the landscape theory in terms of an optimization problem with constraints, a framework of fuzzy landscape theory is introduced in Section 3. In Section 4, we develop a GA-based algorithm for finding near-optimised solutions corresponding to predicted fuzzy configurations. Moreover, simulation results for the problem of the international alignment of the Second World War in Europe with the proposed algorithm are also presented. Finally, some concluding remarks and further work will be presented in Section 5.

§2 Basics of Landscape Theory

Landscape theory begins with a set of n *actors* (for example, nations), denoted by $\mathcal{A} = \{A_1, \ldots, A_n\}$. Each actor A_i has its own *size*, $s_i > 0$, that reflects the important of that actor to others. For example, the size of a nation might be measured by demographic, industrial, or military factors, or a combination of these depending on what is taken to be important in a particular application [A93]. In addition, each pair of actors A_i and A_j has a *propensity*, denoted by p_{ij}, that is a measure of how willing the two actors are to be in the same coalition together. For example, in the language of international alignments, the propensity number is positive and large if the two nations get along well together and negative if they have many sources of potential conflict. Further, if one country has a source of conflict with another then the second country typically has the same source of conflict with the first. Thus, the theory assumes that propensity is symmetric, that is $p_{ij} = p_{ji}$.

By a *configuration* we mean a partition of the set of actors. That is, each actor is placed into one and only one group. Given a configuration X, we then define the *distance* between any two actors A_i and A_j within X, denoted by $d_{ij}(X)$. For example, assume that $X = \{X_1, \dots, X_m\}$ is a configuration, the measure of distance can be defined as follows

$$d_{ij}(X) = \begin{cases} 0 & \text{if } A_i, A_j \in X_k, \text{ for some } k \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

Using distance and propensity, the so-called *frustration* of an actor A_i is defined as the measurement of how poorly or well a given configuration satisfies the propensities of a given actor to be near or far from each other. Formally, the frustration of an actor A_i in a configuration X is defined as follows

$$F_i(X) = \sum_{i \neq j} s_j p_{ij} d_{ij}(X) \tag{2}$$

where s_j is the size of A_j, p_{ij} is the propensity of A_i to be close to A_j, and $d_{ij}(X)$ is the distance between A_i and A_j in X.

The *energy* E of a configuration X is now defined as the weighted sum of the frustrations of each actor in the configuration, where weights are just the sizes of the actors. More exactly, the energy of a configuration X is

$$E(X) = \sum_i s_i F_i(X) \tag{3}$$

Equivalently, the following equation defines the energy of a configuration in terms of size of the actors, their propensities, and their distances in the configuration:

$$E(X) = \sum_{i,j} s_i s_j p_{ij} d_{ij}(X) \tag{4}$$

where the summation is over all ordered pairs of distinct actors. The predicted configurations are then based on the attempts of actors to minimise their frustrations based on their pairwise propensities to align with some actors and oppose others. These attempts lead to a local minimum in the energy landscape of the entire system.

§3 An Extension of Landscape Theory

In this section, we propose a framework of fuzzy landscape theory based on the notion of fuzzy partitions. It should be also noticed that in landscape theory, the measure of distance in a configuration can be defined differently according to various applied situations. For instance, Kijima [K01b)] and Iriuchijima

[100]) have extended the landscape theory by introducting different measures of distance accompanied with corresponding application situations. In the present paper we extend the theory by relaxing the notion of membership grades in the definition of a configuration. To this end, we first mathematically restate landscape theory in terms of an optimization problem with constraints.

Assume that we are given the followings

- $\mathcal{A} = \{A_1, \ldots, A_n\}$ – a set of n actors, and each A_i has its size s_i, for $i = 1, \ldots, n$,
- $[p_{ij}]_{n\times n}$ – the symmetric matrix of propensities,
- $X = \{X_1, \ldots, X_m\}$ – a configuration, where $1 < m < n$.

For any $A_i \in \mathcal{A}, X_k \in X$, let us denote

$$u_{ik} := \mu_{X_k}(A_i) = \begin{cases} 1, & \text{if } A_i \in X_k \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where μ_{X_k} denotes the characteristic function of X_k.

It can be easily seen that the measure of distance defined by (1) can be restated in terms of membership grades as follows:

$$d_{ij}(X) = \frac{1}{2}\sum_{k=1}^{m}(u_{ik} - u_{jk})^2 \tag{6}$$

Substituting (6) into (4) yields the following expression of the energy of a configuration

$$E(X) = \frac{1}{2}\sum_{i=1}^{n}\sum_{i\neq j=1}^{n} s_i s_j p_{ij} \sum_{k=1}^{m}(u_{ik} - u_{jk})^2 \tag{7}$$

As mentioned in the previous section, the problem is to find out predicted configurations which are local minima in the energy landscape of the entire system. Mathematically, the problem can be formulated in terms of an optimization problem that is very similar to the optimization task to be solved in objective function based clustering as follows:

$$\min \; E(X) = \frac{1}{2}\sum_{i=1}^{n}\sum_{i\neq j=1}^{n} s_i s_j p_{ij} \sum_{k=1}^{m}(u_{ik} - u_{jk})^2 \tag{8}$$

subject to

$$\begin{cases} 1 < m < n \\ u_{ik} \in \{0,1\}, \text{ for all } i \in \{1,\ldots,n\} \textit{ and } k \in \{1,\ldots,m\} \\ \sum_{i=1}^{n} u_{ik} > 0, \text{ for all } k \in \{1,\ldots,m\} \\ \sum_{k=1}^{m} u_{ik} = 1, \text{ for all } i \in \{1,\ldots,n\} \end{cases} \tag{9}$$

At this point we can see that the purpose of landscape theory is somehow similar to that of clustering based on objective functions with the well-known k-means algorithms. Normally, the parameter m may be small and could be determined easily in most practical situations. For simplicity, in this study we assume that m is given and fixed a priori.

With the above formulation, we now propose an extension of landscape theory by relaxing the notion of membership grades of actors in a configuration. Namely, we use the notion of a fuzzy configuration instead of a crisp one as defined below.

By a fuzzy configuration we mean a fuzzy partition of the set of actors. Formally, a fuzzy m-partition of $\mathcal{A}$ is a set $X = \{X_1, \ldots, X_m\}$ such that each X_k, for $k = 1, \ldots, m$, is a fuzzy set of $\mathcal{A}$ with its membership function denoted by μ_{X_k}, and $\sum_{k=1}^{m} u_{ik} = 1$, where $u_{ik} \stackrel{def}{=} \mu_{X_k}(A_i)$, for any $A_i \in \mathcal{A}$.

Similarly, the energy of a fuzzy configuration is also defined in terms of size of the actors, their propensities, and their distances in the configuration by the equation (7) as above.

Under such a formalization, the problem in fuzzy landscape theory is of finding out fuzzy configurations that minimise the objective function (7) subject to the constraints as follows

$$\begin{cases} u_{ik} \in [0,1], \text{ for all } i \in \{1,\ldots,n\}, k \in \{1,\ldots,m\} \\ \sum_{i=1}^{n} u_{ik} > 0 \text{ for all } k \in \{1,\ldots,m\} \\ \sum_{k=1}^{m} u_{ik} = 1 \text{ for all } i \in \{1,\ldots,n\} \end{cases} \tag{10}$$

In other words, let us denote by

$$\mathcal{U} = \{[u_{ik}]_{n\times m} | u_{ik} \in [0,1] \wedge \sum_{k=1}^{m} u_{ik} = 1 \wedge \sum_{i=1}^{n} u_{ik} > 0, \forall i, k\}$$

the set of all fuzzy m-configurations of $\mathcal{A}$. We will have to look for fuzzy configurations in $\mathcal{U}$ in order to reduce the energy of the entire system as much as possible

In the next section we will develop a GA-based algorithm for finding near-optimised solutions corresponding to predicted fuzzy configurations. Then the simulations with the proposed algorithm for the problem of the international alignment of the Second World War in Europe [A93)] in comparison with those given by original landscape theory will be presented.

§4 Simulation in International Alignments

4.1 Algorithm

Due to complexity of a multidimensional nonlinear optimization problem as stated in the preceding section, it would be inefficient to get locally optimal solutions in an analytical manner. However, we overcome this deficiency by developing an GA-based algorithm for luckily finding a near-optimal solution corresponding to a predicted fuzzy configuration. The algorithm proposed for solving the above optimization problem is described as follows.

Input: the set of sizes of actors $\{s_1, \ldots, s_n\}$, the propensity matrix $[p_{ij}]_{n\times n}$

Output: $U \in \mathcal{U}$ that minimises the energy function E.

1. Set $t = 0$. Initiate the first configuration U_0 randomly.
   Set a value for T, the number of loop steps at most.
2. Calculate the energy value for $U_0 = \left[u_{ik}^{(0)}\right]_{n\times m}$ by

$$E(U_0) = \frac{1}{2} \sum_{i=1}^{n} \sum_{i\neq j=1}^{n} s_i s_j p_{ij} \sum_{k=1}^{m} \left(u_{ik}^{(0)} - u_{jk}^{(0)}\right)^2$$

3. Set $t = t + 1$
4. Use GA to generate the next configuration U_t from the preceding configuration.
5. Caluculate the energy value $E(U_t)$.

6. While $\mid E(U_t) - E(U_{t-1}) \mid \geq \varepsilon$ and $t \leq T$ do
   if $E(U_t) < E(U_{t-1})$ Then go to Step 3.
   Else set $t := t - 1$ go to step 3.

Using the landscape theory, Axelrod and Bennett have illustrated with the international alignment problem of Europe in the years preceding the Second World War. In the next subsection, we use the fuzzy landscape theory developed in the last section to study the same alignment problem and make a comparison of the obtained results with those given by the landscape theory.

4.2 Simulation for the Alignment of the Second World War in Europe

In any application of landscape theory, the operationalization and testing require answers to the following four questions. 1. Who are the actors? 2. What are their sizes? 3. What are the propensities between every pair of actors? 4. What is the actual outcome? The answers to these question depend on the specific domain being investigated [A93)]. In this simulation, the actors are the seventeen European nations who were involved in major diplomatic action in the 1930's. The size of each nation is measured with the national capabilities index of the Correlates of the War project [S72)]. Propensities are estimated from the presence of ethnic conflict, the similarity of the religions of the populations, the existence of a border disagreement, the similarity of the types of governments, and the existence of a recent history of wars between two nations. All these data of the years from 1936 to 1939 taken from the Complexity of Cooperation Web Site (http://pscs.physics.lsa.umich.edu/Software/CC/CCAB.html) are used to test with the proposed algorithm.

According to a criterion specified by Axelred and Bennett [A93)], the actual alignment of the Second World War in Europe was Britain, France, the Soviet Union, Czechoslovakia, Denmark, Greece, Poland, and Yugoslavia versus Germany, Italy, Hungary, Estonia, Finland, Latvia, Lithuania, and Romania. Portugal, which has a defense agreement with Britain, was neutral. With this actual alignment, "the theory does very well in predicting the European alignment of the Second World War with data up to 1936, but does even better as late data are used. By 1938, the prediction is narrowed from two to one, and by 1939 data the single prediction becomes accurate for all but one of the seventeen countries." as [A93)] concluded.

By applying the framework developed in the preceding section and the proposed algorithm, we have obtained the results for predicting the alignment of the Second World War in Europe that are almost the same as those given by landscape theory. Especially, using the 1938 data the prediction presented by the proposed framework shows a better result in comparison to that given by original landscape theory. That is the prediction becomes accurate for all but one of the seventeen countries, namely Portugal, as the prediction based on the 1939 data. Tables 1–2 show the predicted results resulting from both landscape theory and fuzzy landscape theory based on 1938 and 1939 data. In the Tables we only presented the membership grades of nations to an alignment in the predicted fuzzy configuration. The membership grades of nations to the other alignment would be easily obtained by the third constraint in (10).

Table 1 The predicted result from landscape theory (a) and fuzzy landscape theory (b) of the Second World War based on 1938 data.

(a)

Alignment 1	*Alignment* 2
Britain	Germany
France	Italy
Czech.	Hungary
Demmark	Romania
Greece	Poland
Soviet Union	Estonia
Yugoslavia	Latvia
	Lithuania
	Finland
	Portugal

(b)

Britain	0.9999	Germany	4.0E-05
France	0.99979	Italy	2.7E-04
Czech.	0.99982	Hungary	0.00451
Greece	0.99753	Romania	5.7E-04
Denmark	0.98905	Estonia	0.00212
Soviet U.	0.99998	Latvia	4.5E-04
Yugos.	0.99908	Lithuania	0.00239
Poland	0.99892	Finland	0.00681
		Portugal	0.00385

§5 Conclusion

In this paper we have mathematically reformalised the landscape theory so that the distance between any two actors can be expressed via their membership grades in a specific configuration. With this basic we naturally extended the theory to a generalized one by considering fuzzy configurations instead of crisp ones. Formally the idea is strongly similar to that of objective function based fuzzy clustering. Due to the complexity of a multidimensional nonlinear optimization problem, it would be inefficient to get locally optimal solutions

Table 2 The predicted result from landscape theory (a) and fuzzy landscape theory (b) of the Second World War based on 1939 data.

(a)

Alignment 1	*Alignment* 2
Britain	Germany
France	Italy
Czech.	Hungary
Demmark	Romania
Greece	Estonia
Soviet Union	Latvia
Yugoslavia	Lithuania
Poland	Finland
	Portugal

(b)

Britain	0.99939	Germany	3.17E-04
France	0.99993	Italy	0.001079
Czech.	0.99901	Hungary	0.003166
Denmark	0.99789	Romania	9.15E-04
Greece	0.99766	Estonia	0.001422
Soviet U.	0.99995	Latvia	0.003611
Yugos.	0.99998	Lithuania	0.001203
Poland	0.99886	Finland	4.11E-04
		Portugal	0.002207

in an analytical manner. However, we have developed an GA-based algorithm which fortuitously presents us with a solution corresponding to a predicted fuzzy configuration. The simulation using the real data in the problem of the international alignment of the Second World War on Europe show that the algorithm should be feasible.

It should be emphasized that landscape theory offers promise in applications to business alliance, parliamentary coalitions, friendship networks, social cleavages, and organizational structures [A97)]. We have made the simulation with the proposed framework for the problem of coalition formation in standard-setting alliances in the case of UNIX operating system. The obtained results are also comparable to those given in [A95)]. However, the restriction to symmetrical propensity in landscape theory becomes unsuitable in some particular applications [K01a, K01b)]. In the further work we are taking the assumption of asymmetrical propensity into account and to apply the proposed framework to the problem of business alliances, particularly in Automobile Industry partnership.

References

[A93] Axelrod, R. and Scott Bennett, D., "A landscape theory of aggregation," *British Journal of Political Science, 23*, 211–233, 1993.

[A95] Axelrod, R. et al., "Coalition formation in standard-setting alliances," *Management Science, 41*, 1493–1508, 1995.

[A97] Axelrod, R., *The Complexity of Cooperation*, Princeton University Press, Princeton, New Jersey, 1997.

[K90] Kaufman, L. and Rousseeuw, P.J.,*Finding Groups in Data - an Introduction to Cluster Analysis*, New York, Wiley, 1990.

[R69] Ruspini, E.H., "A new approach to clustering," *Information and Control, 15*, 22–32, 1969.

[O77] Owen, G., "Values of Games with a Priori Unions," in: *Essays in Mathematical Economics and Game Theory* (Hein, R. and Moeschlin, M. eds.), Springer-Varlag, New York, 76–88, 1977.

[C00] Crotts, J.C., Buhalis, D. and March, R., *Global Alliances in Tourism and Hospitality Management*, Haworth Press, 2000, pp. 1–10.

[G92] George, H.C., *The Human Group*, New Brunswick: Transaction Publishers, 1992.

[S72] Singer, D., Bremer S. and Stuckey, J., "Capability distribution, uncertaintry and major power war: 1920-1965," in *Peace, War and Numbers*, (Bruce Russett, Ed.), Beverly Hills, California, 1972, 19–48.

[K01a] Kijima, K., *Invitation to Drama Theory* (in Japanese), Ohmsha, 2001.

[K01b] Kijima, K., "Generalized landscape theory: Agent-based approach to alliance formations in civil aviation industry," *Journal of Systems Science and Complexity, 14*, 113–123, 2001.

[I00] Iriuchijima, M., *Generalization of landscape theory and its application,* Master Thesis, Tokyo Institute of Technology, Tokyo, 2000.