

Title	Theoretical Analysis and Model Construction for Optimal R&D Investment Control
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Citation	年次学術大会講演要旨集, 15: 390-393
Issue Date	2000-10-21
Type	Conference Paper
Text version	publisher
URL	http://hdl.handle.net/10119/5882
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Description	一般論文

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Abstract

A hypothetical view is postulated on the basis of the observation of structural change in Japan's techno-economic behavior. In order to construct a virtuous cycle trajectory between R&D investment, technology stock and economic growth, R&D investment decision making has become a crucial issue. An optimal R&D investment model based on the optimal control theory postulated by Pontryagin is constructed.

1. Introduction

R&D is a key determinant of long-run productivity and consumer welfare. According to the observation of structural change in Japan's techno-economic behavior, there is a fear to vicious cycle between R&D investment, technology stock and economic growth. Therefore, R&D investment decision making has become a crucial issue. Furthermore, this decision is difficult because of the complex interrelationships among governing factors of industry R&D. A number of studies have analyzed R&D contribution to growth¹. However, there are hardly satisfactory in identifying optimal R&D investment trajectory.

In this paper, section 2 empirically examines the hypothetical view of the fear to vicious cycle between R&D and growth. Section 3 constructs an optimal R&D investment control model and provides the analytic solution of the model. Section 4 briefly summarizes the conclusions.

2. Examination of the Fear to Vicious Cycle between R&D and Growth

Currently, the stagnation of technology development has become a crucial structural problem common to many advanced economies [2]. Similarly, Japan has been suffering from a collapse of its long lasting "virtuous cycle" between technology development and economic growth [6]. The structural stagnation of Japanese industry's R&D activities can be demonstrated by trends in change rate of R&D intensity in major sectors of its manufacturing industry (MA: manufacturing average; FD: food; PM: primary metals; CH: chemicals; and EM: electrical machinery) over the period 1975-1996 as shown in Fig. 1.

Another noteworthy trends in the Japanese manufacturing industry's techno-economic behavior

under increasing technology spillover [7] can be observed in the rise and fall of marginal productivity of technology, stagnation of technology substitution for scarce resources and stagnation of assimilation capacity (*AC*: the ability to utilize spillover technology).

Fig. 2 illustrates trends in marginal productivity of technology (*MPT*) in three of Japan's leading manufacturing industries (EM, CH and PM) over the period 1960-1997 [8]. We note that *MPT* is sensitive to economic circumstances in respective period.

Fig. 3 illustrates trends in the elasticity of technology substitution for labor (*TSL*) in Japan's manufacturing industry (MA) and the same three leading sectors (EM, CH and PM) over the period 1981-1997 [8]. Fig. 3 demonstrates that *TSL* started to decrease in the 1980s and continued to decrease in the 1990s.

Fig. 4 illustrates trends in assimilation capacity for leading sectors (EM, CH and PM) over the period 1981-1995 [7]. We note that *AC* in EM and PM increased before the bubble economy in 1987. However, this changed to a dramatic decrease starting from the period of the bubble economy. While assimilation capacity of CH continues to decline from 1983.

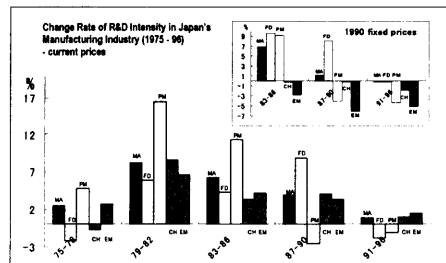


Fig. 1 Trends in Change Rate of R&D Intensity in Japan's Manufacturing Industry (1975-1996) - %

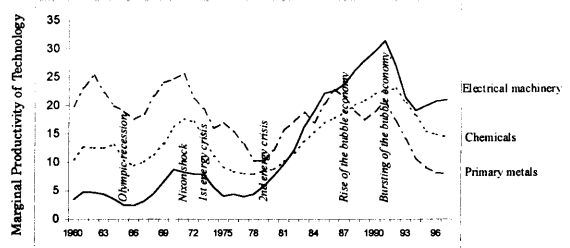


Fig. 2 Trends in *MPT* in Japan's Major Manufacturing Industries (1960-1997) - %

¹ See details of relevant existing works in [9].

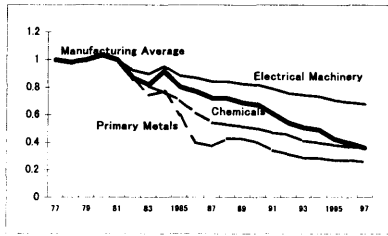


Fig. 3 Trends in TSL in Japan's Major Manufacturing Industries (1981-1997): Index: 1981 = 1

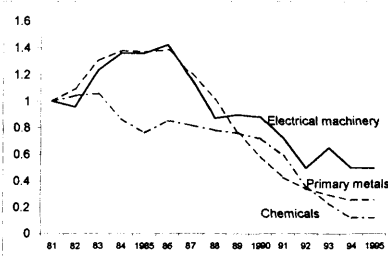


Fig. 4 Trends in Assimilation Capacity of Japan's Major Manufacturing Industries (1981-1995) - Index: 1981 = 1

By combining with some empirical analyses, these observations suggest the following hypothetical view:

- (i) R&D intensity (r/y or r/V), the marginal productivity of technology (MPT), technology substitution for labor (TSL), and assimilation capacity (AC) correlate with each other constructing a comprehensive subtle system as illustrated in Fig. 5.
- (ii) This system has both possibilities leading to virtuous or vicious spin cycle between R&D and growth.
- (iii) R&D intensity (r/y) plays a trigger role deciding this trajectory.
- (iv) Due to its stagnation, empirical analyses demonstrate a strong fear of vicious spin cycle between R&D and growth.
- (v) Therefore, in order to avoid this fear, optimal R&D intensity (r/y) control has become critical.

3. Construction and Solution of the Optimal R&D Investment Control

3.1 The System Model

To construct the dynamic model of manufacturing and R&D investment, the following variables are used: $y = y(t)$: manufacturing production; t : time trend; \dot{y}/y : change rate of production where $\dot{y} = dy/dt$; $T = T(t)$: technology knowledge stock (accumulated R&D investment r); $\dot{T} \approx r = r(t)$: change in technology knowledge stock (approximated by R&D investment); r/y : R&D intensity; $X (=L, K, M, E)$: production factors

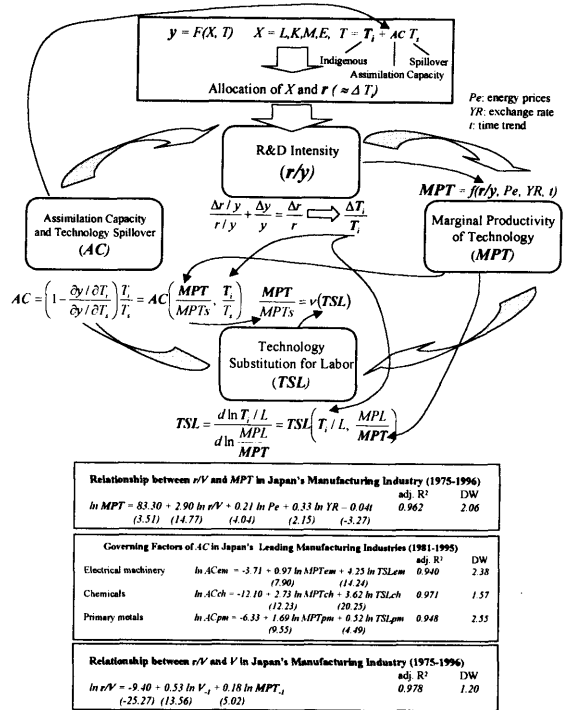


Fig. 5 Schematic Representation of the Relation between R&D Intensity, MPT , TSL and AC

(labor, capital, materials and energy), which involve both factors for manufacturing and R&D; and $X_T (=L_T, K_T, M_T, E_T)$: factors input directing to R&D.

The classical production function is used to construct dynamics as follows:

$$y = F(t, (L - L_T), (K - K_T), (M - M_T), (E - E_T), T) \quad (1)$$

Assume that the functional dependence between the L_T, K_T, M_T, E_T and the accumulated R&D investment T is given by function of substitution type:

$$T = T(L_T, K_T, M_T, E_T) = \min\{h_1(L_T), h_2(K_T), h_3(M_T), h_4(E_T)\} \quad (2)$$

and the inverse relations exist

$$\begin{aligned} L_T &= L_T(T) = h_1^{-1}(T), & K_T &= K_T(T) = h_2^{-1}(T) \\ M_T &= M_T(T) = h_3^{-1}(T), & E_T &= E_T(T) = h_4^{-1}(T) \end{aligned} \quad (3)^2$$

Differentiating (1) by time t and taking into account (3),

$$\frac{\dot{y}}{y} \approx \frac{\partial F}{\partial t} \frac{1}{y} + \sum \frac{\partial F}{\partial X} \frac{X}{y} \frac{\dot{X}}{X} - \sum \frac{\partial F}{\partial X} \frac{\partial X_T}{\partial T} \frac{\dot{T}}{y} + \frac{\partial F}{\partial T} \frac{\dot{T}}{y} \quad (4)$$

Here we can approximately treat $\dot{T} \approx r$. In line with the previous approach [5], technology knowledge stock in time t , T_t can be measured as follow:

$$T_t = r_{t-m} + (1-\rho) T_{t-1} \quad (5)$$

$$T_0 = r_{1-m} / (\theta + \rho) \quad (6)$$

² The rationality of the existence of this reverse relation has been checked by using the empirical analysis in Japan's manufacturing industry [9].

where r_{t-m} : R&D expenditure in time $t-m$; m : time lag between R&D and commercialization; ρ : rate of obsolescence of technology; and θ : increase rate of r .

Using $\dot{T} \approx r$ and rewrite (4) in the form of:

$$\frac{\dot{y}}{y} = f - p \frac{r}{y} + q \frac{r}{y} \quad (7)$$

where terms related to the production factors X ($=L, K, M, E$), effects of institutional change (e.g. learning and scale effects) are combined into function f

$$f = \frac{\partial F}{\partial t} \frac{1}{y} + \sum \frac{\partial F}{\partial X} \frac{X}{y} \frac{\dot{X}}{X} \quad (8)$$

decrease in manufacturing due to R&D spending X_T ($=L_T, K_T, M_T, E_T$) is collected into function p

$$p = p(t) = \sum \frac{\partial F}{\partial X} \frac{\partial X_T}{\partial T} \quad (9)$$

increase in manufacturing by technology knowledge stock is described by the marginal productivity of technology q

$$q = q(t) = \frac{\partial F}{\partial T} \quad (10)$$

the control parameter r stands for change in technology knowledge stock \dot{T} .

Collecting the terms $(r/y)p$, $(r/y)q$ which depend on the control parameter r into the net contribution by R&D intensity $(r/y)g$, the equation for the dynamic control process can be obtained as follow:

$$\frac{\dot{y}}{y} = f - g \frac{r}{y} \quad (11)$$

$$\text{where } g = g(t) = p(t) - q(t) > 0 \quad (12)$$

3.2 Utility of the System Trajectory

In order to formalize the goal for designing the control parameter $r = r(t)$ and indicate the profit of R&D investment in the long-run, the utility function represented by the present value of the consumption of the invented products³ with the discount coefficient η is considered.

$$U_t = \int_0^{\infty} e^{-\eta(s-t)} \ln D(s) ds \quad (13)$$

$$D = D(s) = \left(\int_0^n x^\alpha(j) dj \right)^{1/\alpha}, \quad n = n(s) \quad (14)$$

$$y = n \cdot x(j), \quad y = y(s), \quad n = n(s) \quad (15)$$

$$n = n(s) = br^\beta T^{\beta_2}, \quad r = r(s), \quad T = T(s) \quad (16)$$

where $D(s)$: demand function; s : running time; t : the initial time; j : current index of innovative goods; $x(j)$: consumption of brand j innovative goods; $n(s)$: number of available varieties at time s ; α : parameter of elasticity of substitution between any two innovative goods (ε , $\varepsilon = 1/(1-\alpha)$); and β_1, β_2 : elasticities of r and T to n .

Combining (14), (15) and (16), the following demand function can be obtained:

$$D(s) = \left[\int_0^n \left(\frac{y}{n} \right)^\alpha dj \right]^{1/\alpha} = \frac{y}{n} (n)^{1/\alpha} = y \cdot n^{\frac{1-\alpha}{\alpha}} \quad (17)$$

From equation (5) we can get following formula:

$$T_t - T_{t-1} = -\rho T_{t-1} + r_{t-m} \quad (18)$$

When t is long enough to satisfy $t > t-1 \gg m-1$,

$$\Delta T = -\rho T + r \quad (\Delta T: dT/dt) \quad (19)$$

Solve the differential equation (19),

$$T(t) = \frac{r_t}{\theta + \rho} + e^{-\rho(t-t_0)} T_0 (1 - e^{\theta(t_0-1+m)}) \quad (20)$$

where θ : the average increase rate of $r(t)$.

Under the condition $t_0 - 1 + m \approx 0$

$$T(t) \approx \frac{r_t}{\theta + \rho} \quad (21)$$

$$\therefore n = n(s) = br^\beta T^{\beta_2} = br^\beta \left(\frac{r}{\theta + \rho} \right)^{\beta_2} \quad (22)$$

$$\ln n = (\ln b - \beta_2 \ln(\theta + \rho)) + (\beta_1 + \beta_2) \ln r \quad (23)$$

Combining (13), (17) and (23), the following expression for the utility function can be obtained:

$$U_t = \int_0^{\infty} e^{-\eta(s-t)} \left(\ln y + \frac{1-\alpha}{\alpha} ((\ln b - \beta_2 \ln(\theta + \rho)) + (\beta_1 + \beta_2) \ln r) \right) ds \quad (24)$$

3.3 The Analytic Solution of the Model

The Pontryagin's maximum principle [3] is used to solve the classical optimal control problem constructed by dynamics (12) and utility function (24). The main elements in the analysis are Hamiltonian H and the adjoint variable ψ .

The Hamiltonian has the form

$$H(y, r, \psi) = \ln y + \frac{1-\alpha}{\alpha} ((\ln b - \beta_2 \ln(\theta + \rho)) + (\beta_1 + \beta_2) \ln r) + \psi(fy - gr) \quad (25)$$

and represents the utility flow.

Its maximum by parameter r is determined by

$$\frac{\partial H}{\partial r} = \frac{1-\alpha}{\alpha} (\beta_1 + \beta_2) \frac{1}{r} - g\psi = 0 \quad (26)$$

So its maximum value is attained at the optimal R&D investment r^0

$$r^0 = \frac{1-\alpha}{\alpha} \frac{\beta_1 + \beta_2}{g\psi} \quad (27)$$

ψ is marginal price of production y expressed as $\psi = \partial W / \partial y$ where W is optimal value.

Halmiton-Jacobi equation depicts that a trajectory of the optimal position can be expressed

$$\frac{\partial W}{\partial t} + H(y, r, \frac{\partial W}{\partial y}) = \frac{\partial W}{\partial t} + H(y, r, \psi) = 0 \quad (28)$$

Optimal trajectory with respect to y is

³ Consumer behaves to taste for diversity in consumption represented by number of available varieties to which technology contributes to increase [1].

$$\frac{\partial}{\partial y} \left(\frac{\partial W}{\partial t} \right) + \frac{\partial H}{\partial y} = \frac{\partial}{\partial t} \left(\frac{\partial W}{\partial y} \right) + \frac{\partial H}{\partial y} = \frac{\partial \psi}{\partial t} + \frac{\partial H}{\partial y} = 0 \quad (29)$$

Utility function (24) requires the following Hamiltonian in addition to the Hamiltonian (25):

$$H^*(y, r, \psi^*) = e^{-\eta(s-t)} \left(\ln y + \frac{1-\alpha}{\alpha} ((\ln b - \beta_2 \ln(\theta + \rho)) + (\beta_1 + \beta_2) \ln r) \right) + \psi^* (fy - gr) \quad (30)$$

Under the optimal trajectory condition $\frac{\partial H}{\partial r} = \frac{\partial H^*}{\partial r} = 0$,

$$\frac{\partial H}{\partial y} = \frac{1}{y} + f\psi = 0 \quad (31)$$

$$\frac{\partial H^*}{\partial y} = e^{-\eta(s-t)} \frac{1}{y} + f\psi^* = 0 \quad (32)$$

$$\therefore \psi^* = e^{-\eta(s-t)} \psi \quad (33)$$

$$\frac{\partial H^*}{\partial y} = e^{-\eta(s-t)} \left(\frac{1}{y} + f\psi \right) = e^{-\eta(s-t)} \frac{\partial H}{\partial y} \quad (34)$$

Assume y in an optimal trajectory in (29),

$$\frac{\partial H^*}{\partial y} = -\frac{\partial \psi^*}{\partial t} = -(\eta e^{-\eta(s-t)} \psi + e^{-\eta(s-t)} \dot{\psi}) \quad (35)$$

From equations (34) and (35),

$$\frac{\partial H}{\partial y} = \eta\psi - \dot{\psi} \quad (36)$$

Therefore, for dynamics of the conjugate variable ψ one can compose the adjoint equation:

$$\dot{\psi} = \eta\psi - \frac{\partial H}{\partial y} = \eta\psi - \frac{1}{y} - f\psi \quad (37)$$

Combining equations (12) and (26), and changing (37), the following closed system of differential equations are obtained:

$$\frac{\dot{y}}{y} = f - \frac{1-\alpha}{\alpha} (\beta_1 + \beta_2) \frac{1}{y\psi} \quad (38)$$

$$\frac{\dot{\psi}}{\psi} = \eta - \frac{1}{y\psi} - f \quad (39)$$

Introducing notation $z = y\psi$ for the production cost and summarizing equations (38) and (39) the following differential equation is obtained:

$$\dot{z} = \eta z - \left[\frac{1-\alpha}{\alpha} (\beta_1 + \beta_2) + 1 \right] \quad (40)$$

By solving this differential equation, the following equation can be obtained:

$$z = z(t) = \frac{1}{\eta} \left[\frac{1-\alpha}{\alpha} (\beta_1 + \beta_2) + 1 \right] \quad (41)$$

Substituting solution (41) into optimal control (27), the relation between the optimal investment r and the optimal production y is obtained:

$$r = \frac{1}{\varepsilon - 1 + (\beta_1 + \beta_2)} \cdot \frac{\eta}{g} y \quad (42)$$

In case the number of available varieties $n(s)$ in equation (16) is under constant returns to scale with respect to r and T , $\beta_1 + \beta_2 = 1^4$.

Under these conditions:

$$\frac{r}{y} = \frac{\eta}{\varepsilon g} \quad (43)$$

Equation (43) suggests that the optimal R&D intensity depends on the elasticity of substitution ε , the discount rate η and the discounted marginal productivity of technology g , and its level increases as ε and g decrease and η increases.

4. Concluding Remarks

- (i) Increasing significance of optimal R&D control is identified by demonstrating the stagnation of R&D intensity, marginal productivity of technology, technology substitution for scarce resources and a decrease in assimilation capacity leading to a vicious cycle between R&D and growth.
- (ii) On the basis of a concept of constructing a virtuous cycle trajectory between R&D investment, technology stock and economic growth, a R&D investment model based on the optimal control theory postulated by Pontryagin is constructed to satisfy customer's tastes for diversity in consumption.

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⁴ The empirical analysis on the invention of innovative goods in the Japanese manufacturing industry over the period 1975-1996 [9] demonstrates that: using number of patent application as a proxy of innovation goods, β_1 (0.34) + β_2 (0.62) = 0.96 \approx 1