<table>
<thead>
<tr>
<th>項目</th>
<th>内容</th>
</tr>
</thead>
<tbody>
<tr>
<td>項目</td>
<td>動的な関数における複雑さの発展</td>
</tr>
<tr>
<td>作者</td>
<td>并川 之淳</td>
</tr>
<tr>
<td>指導教員</td>
<td>橋本 敬</td>
</tr>
</tbody>
</table>
Evolution of Complexity in Dynamical Functions

Jun Namikawa

School of Knowledge Science,
Japan Advanced Institute of Science and Technology

March 2001

Keywords: Dynamical Functions, Homomorphism, Complexity of Functions, Redundancy, Functional Entropy.

Abstract

A new approach to study the evolution of complexity is presented as a formal system describing a dynamic change of functions. We study, analytically and numerically, how complexity of functions changes by transformation.

A dynamical function $f_n : S_n \rightarrow S$ ($S_n \subseteq S$) is defined by

$$f_{n+1} = g \circ f_n \circ g_n^*$$

where $g : S \rightarrow S$ and an indexed family $\{g_i^*\}_{i=0}^\infty$ with $g_n^* : S_{n+1} \rightarrow S_n$ are $g|S_n \circ g_n^* = i_{n+1}$. Here, $i_{n+1} : S_{n+1} \rightarrow S$ is inclusion mapping.

For any function $g$, $f_n$ and $S_n \subseteq S$, if $x \in g_n^*(S_{n+1})$ then $g \circ f_n(x) = f_{n+1} \circ g(x)$.

Thus the Eq. (1) is homomorphic mapping in $g_n^*(S_{n+1}) \subseteq S_n$.

We propose measures of complexity of functions, “redundancy” and “functional entropy”. Redundancy evaluates the number of inputs giving the same output in a function. Functional entropy measures the degree of randomness of outputs from a function when inputs are given randomly. Given any function describing a dynamical system, these measures identify whether it has periodicity. Thus, these measures represent complexity of functions. If there is an isomorphism between $f$ and $f'$, then both functions have the same value for each measure.

We find that these measures have the following characters:

1. If $g$ is an injection, max redundancy decreases.
2. If $g_n^*$ is an surjection, average redundancy increases.
3. If $g$ is an injection and $g_n^*$ is an surjection, redundancy is invariable.

Copyright © 2001 by Jun Namikawa
We study how redundancy and functional entropy change in terms of numerical simulations of Eq. (1), by choosing logistic map \( ax(1 - x) \) and tent map \( a(0.5 - |x - 0.5|) \) as function \( g \), and \( x^2 \), \( x + 0.3 \) (mod1), and \( \frac{2}{\pi}\arctan(2x - 1) + 0.5 \) as the initial function \( f_0 \). Here, the domain and range of \( g \) and \( f_n \) are chosen to be \([0, 1]\). As results of simulations, we confirm that 1) redundancy changes largely for logistic map and not so large for tent map; 2) functional entropy is associated with lyapunov exponent.