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Evolution of Complexity in Dynamical Functions

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Abstract

A new approach to study the evolution of complexity is presented as a formal system describing a dynamic change of functions. We study, analytically and numerically, how complexity of functions changes by transformation.

A *dynamical function* $f_n : S_n \to S$ ($S_n \subseteq S$) is defined by

$$f_{n+1} = g \circ f_n \circ g_n^* \tag{1}$$

where $g : S \to S$ and an indexed family $\{g_i^*\}_{i=0}^{\infty}$ with $g_n^* : S_{n+1} \to S_n$ are $g|S_n \circ g_n^* = i_{n+1}$. Here ,$i_{n+1} : S_{n+1} \to S$ is inclusion mapping.

For any function g, f_n and $S_n \subseteq S$, if $x \in g_n^*(S_{n+1})$ then $g \circ f_n(x) = f_{n+1} \circ g(x)$. Thus the Eq. (1) is homomorphic mapping in $g_n^*(S_{n+1}) \subseteq S_n$.

We propose measures of complexity of functions, "redundancy" and "functional entropy". Redundancy evaluates the number of inputs giving the same output in a function. Functional entropy measures the degree of randomness of outputs from a function when inputs are given randomly. Given any function describing a dynamical system, these measures identify whether it has periodicity. Thus, these measures represent complexity of functions. If there is an isomorphism between f and f', then both functions have the same value for each measure.

We find that these measures have the following characters:

1. If g is an injection, max redundancy decreases.
2. If g_n^* is an surjection, average redundancy increases.
3. If g is an injection and g_n^* is an surjection, redundancy is invariable.

We study how redundancy and functional entropy change in terms of numerical simulations of Eq. (1), by choosing logistic map $ax(1-x)$ and tent map $a(0.5-|x-0.5|)$ as function g, and x^2, $x+0.3$ (mod1), and $\frac{2}{\pi}\arctan(2x-1)+0.5$ as the initial function f_0. Here, the domain and range of g and f_n are chosen to be $[0,1]$. As results of simulations, we confirm that 1) redundancy changes largely for logistic map and not so large for tent map; 2) functional entropy is associated with lyapunov exponent.