Title	Assessment of Functionality Development Dynamics
Author(s)	SHIN, Jae-Ho; WATANABE, Chihiro
Citation	年次学術大会講演要旨集, 23: 365-368
Issue Date	2008-10-12
Type	Conference Paper
Text version	publisher
URL	http://hdl.handle.net/10119/7576
Rights	本著作物は研究・技術計画学会の許可のもとに掲載するものです。This material is posted here with permission of the Japan Society for Science Policy and Research Management.
Description	一般講演要旨

１Ｇ０９

Assessment of Functionality Development Dynamics

○Jae-Ho SHIN, Chihiro WATANABE (東京工業大学)

1. Introduction

Sustaining level of functionality development is decisive to firm's competitiveness and the efforts for maintaining the level of functionality development is critical for firm's strategy. Stimulated by these understandings, the analysis of the functionality development dynamism in a diffusion trajectory utilizing new approach supportive to optimization method is developed. The objective of this paper is to analyze the optimal level of functionality development in a diffusion trajectory. By means of the optimization theory, this research focuses on the issue of the optimal control problem for functionality development and production. On the basis of empirical analyses, this paper attempts to assess the optimal level of functionality development

Diffusion of innovative goods is induced by their ability to dramatically improve the performance of production processes, goods and services by means of innovation. This ability can be defined as functionality development (FD) (Watanabe et al., 2003).

An empirical analysis focusing on the diffusion trajectory of Japan's mobile phone is conducted. By means of the empirical analysis utilizing the bi-logistic growth model (Meyer, 1994), this paper attempts to demonstrate the optimal level of functionality development for 1st wave mobile phone and 2nd one, respectively.

2. Analytical Framework

This paper proposes a corresponding optimal control problem with the infinite horizon and solves it using the Pontryagin maximum principle (Pontryagin et al., 1962). In this problem, the control variables are investments intensity in the production factors, and the utility is the integral discounted consumption index. Applying the Pontryagin maximum principle, the optimal investment policy and the corresponding trajectories of the FD, which maximize the utility function, can be observed (Krasovskii, 2006).

2.1 Model's Main Variables

In order to analyze the optimal level of FD, the following main variables can be demonstrated:

$t \in [t_0, +\infty)$ – Time on the infinite horizon;

$y = y(t)$ – Production;

$n = n(t)$ – Technology driving force for capacity;

$f = f(t) = \dfrac{n(t)}{y(t)}$ – Functionality development;

$z = z(t) = \dfrac{y(t)}{n(t)} = \dfrac{1}{f(t)}$ – Production to capacity;

$s = s(t) = \dfrac{\dot{n}(t)}{y(t)}$ – Investment intensity.

2.2 Dynamics of Production

First, an attempt to demonstrate a numerical analysis depicting the dynamics of production in a diffusion trajectory is conducted.

$$\dot{Y} = a \cdot Y(t) \cdot (1 - \frac{Y(t)}{N(t)}) = a \cdot Y(t) \cdot (1 - z(t))$$

$$Y(t_0) = Y^0$$

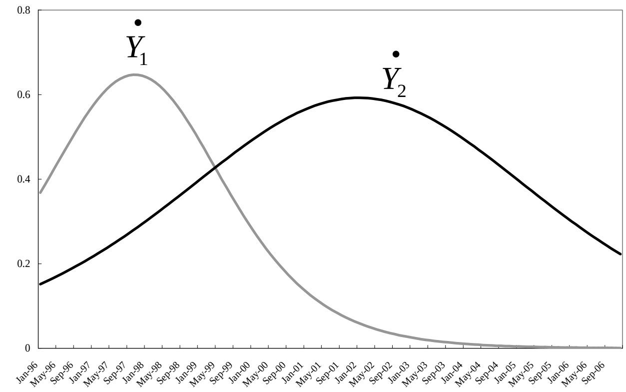

Figure 1. Trend in Dynamics of Production.

where $\dot{Y}_1$: dynamics of production in 1st generation mobile phone and $\dot{Y}_2$: dynamics of production in 2nd generation mobile phone.

2.3 Dynamics of Innovation Productivity

Dynamics of production to capacity (the second phase variable) controlled by control parameter $s(t)$ can be depicted as follows:

$$\dot{z}(t) = (\frac{Y(t)}{N(t)})' = \frac{\dot{Y}(t)\cdot N(t) - \dot{N}(t)\cdot Y(t)}{N^2(t)} = \frac{\dot{Y}(t)}{N(t)} - \frac{\dot{N}(t)}{Y(t)}\cdot(\frac{Y(t)}{N(t)})^2$$

$$= a\cdot(\frac{Y(t)}{N(t)})\cdot(1-z(t)) - s(t)\cdot(\frac{Y(t)}{N(t)})^2 = a\cdot z(t)\cdot(1-z(t)) - s(t)\cdot z^2(t)$$

$$,\quad z(t_0) = z^0 \cdot$$

where $s = s(t)$: control variable. z (t): the second phase variable.

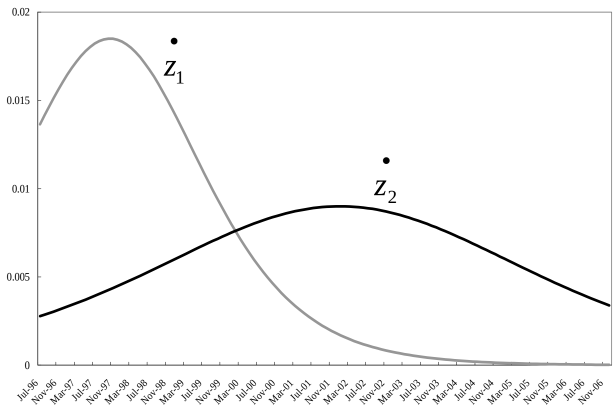

Figure 2. Trend in Dynamics of Innovation Productivity.

2.4 System's Dynamics

$$\dot{Y} = a\cdot Y(t)\cdot(1-z(t))$$

$$\dot{z} = a\cdot z(t)\cdot(1-z(t)) - z^2(t)\cdot s(t)$$

This dynamics of innovation productivity can be expressed as follows:

$$\dot{z} = a\cdot z(t)\cdot(1-z(t) - \frac{s(t)}{a}\cdot z(t))$$

Let $s(t) = s_0 > 0$,

$$\dot{z} = a\cdot z(t)\cdot(1-z(t) - \frac{s_0}{a}\cdot z(t)) \cdot$$

2.5 Stationary Point

$$1 - z - \frac{s_0}{a}\cdot z = 0$$

$$1 - z\cdot(1+\frac{s_0}{a}) = 0 \qquad\qquad 0 < z = (\frac{a}{a+s_0}) < 1$$

$$1 - z\cdot(\frac{a+s_0}{a}) = 0$$

3. Optimal Control Problem of FD

$$x(t) = FD(t) - 1 \Leftrightarrow FD(t) = x(t) + 1 \Leftrightarrow \dot{FD}(t) = \dot{x}(t)$$

where x: the first phase variable; Y: the second phase variable; and s: investment control.

3.1 Growth Dynamics for Production

$$\dot{Y}(t) = a\cdot Y(t)\cdot(1-z(t)) = a\cdot Y(t)\cdot(1-\frac{1}{x(t)+1})$$

3.2 Growth Dynamics for FD

$$\dot{x}(t) = \dot{FD}(t) = s(t) - a\cdot(FD(t)-1) = s(t) - ax(t)$$

3.3 System's Dynamics for Optimal Control Problem

$$\dot{x}(t) = s(t) - a\cdot x(t),\, x(t_0) = x^0,\quad 0 \le s(t) \le A < 1$$

$$\dot{Y}(t) = a\cdot Y(t)\cdot(\frac{x(t)}{x(t)+1}) \quad,\quad Y(t_0) = Y^0$$

3.4 Utility Function (*Integrated Logarithmic Consumption Index*)

Consumption $= Y(t)\cdot(1-s(t))$.

$$J(x(t),y(t),s(t)) = \int_0^{+\infty} e^{-\rho\cdot t}\cdot(\ln Y(t)+\ln(1-s(t)))dt$$ - Logarithm of consumption index.

where $e^{-\rho\cdot t}$: discount factor.

Adjoint Variables

$\psi_1=\psi_1(t)=e^{\rho t}\psi_1^*(t)$ and $\psi_2=\psi_2(t)=e^{\rho t}\psi_2^*(t)$ - Prices of production and functionality development.

3.5 Stationary Hamiltonian

$$H(x,Y,\psi_1,\psi_2,s)=\ln Y+\ln(1-s)+\psi_1\cdot(s-a\cdot x)+\psi_2\cdot a\cdot Y\cdot\frac{x}{x+1}$$

- Hamiltonian function.

where ψ_1 : shadow price for *x*; and ψ_2 : shadow price for *Y*. (ψ_1 and ψ_2 : adjoint variables).

3.5.1 Necessary Condition of Maximum for Hamiltonian Function

$$\frac{\partial H}{\partial s}=-\frac{1}{1-s}+\psi_1=0 \quad\Leftrightarrow\quad s=1-\frac{1}{\psi_1}=\frac{\psi_1-1}{\psi_1}$$ - Necessary Condition of maximum.

Maximized Hamiltonian

$$H(x,Y,\psi_1,\psi_2)=\ln Y-\ln\psi_1+\psi_1\cdot(1-\frac{1}{\psi_1}-a\cdot x)+\psi_2\cdot a\cdot Y\cdot\frac{x}{x+1}$$

3.5.2 Adjoint Equations (*Dynamics of Prices*)

$$\dot{\psi}_1=\rho\cdot\psi_1(t)-\frac{\partial H(x(t),Y(t),s(t),\psi_1(t),\psi_2(t))}{\partial x}=\rho\cdot\psi_1(t)+a\cdot\psi_1(t)-a\cdot\frac{\psi_2(t)\cdot Y(t)}{(x(t)+1)^2}$$

$$\dot{\psi}_2=\rho\cdot\psi_2(t)-\frac{\partial H(x(t),Y(t),s(t),\psi_1(t),\psi_2(t))}{\partial Y}=\rho\cdot\psi_2(t)-a\cdot\frac{\psi_2(t)\cdot x(t)}{(x(t)+1)}-\frac{1}{Y(t)}$$

where $\dot{\psi}_1$: changing price for *x*; and $\dot{\psi}_2$: changing price for *Y*.

Cost Variables

$C_1=C_1(t)=\psi_1(t)\cdot x(t)$,

$C_2=C_2(t)=\psi_2(t)\cdot Y(t)$ -cost variables.

Dynamics of Cost Variables

$$\dot{C}(t)=\dot{\psi}_1(t)\cdot x(t)+\psi_1(t)\cdot\dot{x}(t)+\dot{\psi}_2(t)\cdot Y(t)+\psi_2(t)\cdot\dot{Y}(t)$$

3.5.4 Stationary Equations of the Hamiltonian System

Stationary Points of the Hamiltonian System

$$1-\frac{x}{C_1}-a\cdot x=0 \quad , \quad \rho\cdot C_1-\frac{1}{\rho}\cdot\frac{a\cdot x}{(x+1)^2}+\frac{C_1}{x}-1=0$$

$$C_2=\frac{1}{\rho}$$

Solution of Stationary Equations of the Hamiltonian System: the First Equation

$C_1=\dfrac{x}{(1-a\cdot x)}$ - expression for cost of FD; and

$x=\dfrac{C_1}{a\cdot C_1+1}$ - expression for FD.

Solution of Stationary Equations of the Hamiltonian System: the Second Equation

$$\frac{(a+\rho)\cdot\rho}{a}\cdot C_1=\frac{x}{(x+1)^2}$$

Solution of Stationary Equations for Functionality Development

$$\frac{(a+\rho)\cdot\rho}{a}=\frac{1-a\cdot x}{(x+1)^2}$$

3.6 Stationary Level for FD

$$FD=\frac{-a+(a^2+4\cdot q\cdot(a+1))^{\frac{1}{2}}}{2\cdot q}$$

3.6.1 Suboptimal Level for FD

$$FD=\frac{a}{2\cdot(a+\rho)\cdot\rho}\cdot(-a+(a^2+4\cdot\frac{(a+\rho)\cdot\rho\cdot(a+1)}{a})^{\frac{1}{2}})$$

3.6.2 Optimal Level of Investments for Stationary Level of FD

$$s^* = a\cdot(FD^*-1) = a\cdot\left(\frac{a}{2\cdot(a+\rho)\cdot\rho}\cdot\left(-a+\left(a^2+\frac{4\cdot(a+1)\cdot(a+\rho)\cdot\rho}{a}\right)^{\frac{1}{2}}\right)\right)$$

Optimal Feedback Rule

$$s = s(x) = 1-\frac{x}{(C_1^* + \omega(x-x^*))}$$ – Suboptimal feedback rule.

4. Empirical Analysis

This empirical analysis, based on mathematical development and numerical analysis, calibrates the data using Japan's mobile phone (Watanabe et al., 2008) (1st and 2nd generation) as follows:

Table 1 Stationary Level of FD

	1st generation	2nd generation
ρ	0.0303	0.0145
FD^*	4.223	6.306

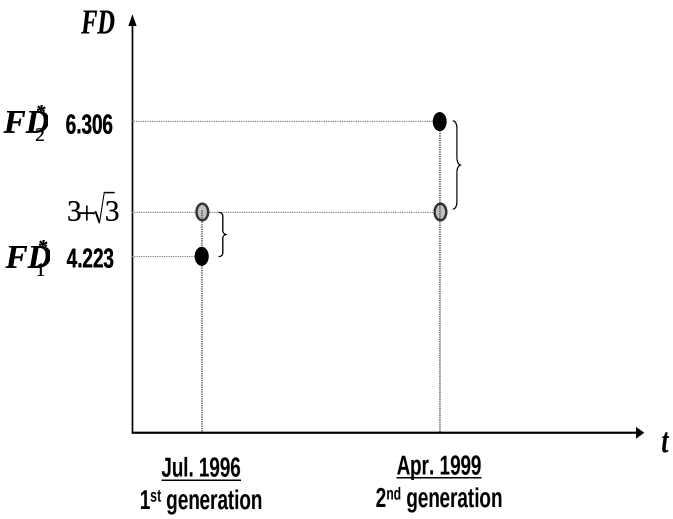

Figure 3. Assessment of Optimal Level of FD.

5. Conclusion

In light of the significance of the sustainable *FD* for firms' profitability in their new innovation in a competitive market, this analysis attempted to identify the optimal levels of *FD* in a diffusion trajectory based on the logistic growth model.

By means of the optimal theory postulated by Pontryagin, mathematical equations identifying optimal *FD* level and corresponding optimal level of investment were developed.

Taking Japan's mobile phone (1st and 2nd generation) diffusion trajectories, their level of *FD* and investment were examined.

Based on the results of analysis, it is shown that the *FD* level of 1st generation MP is little bit higher than optimal level. On the other hand, in case of 2nd generation MP, the *FD* level is lower than optimal level.

These results suggest the significance of firms' *FD* strategy for maintaining and enhancing their competitiveness and profitability.

Further analyses should be focused on the empirical analysis of more comprehensive optimal strategies including optimal trajectories option for FD.

References

[1] Watanabe, C., Asgari, B., and Nagamatsu, A.: Virtuous Cycle between R&D, Functionality Development and Assimilation Capacity for Competitive Strategy in Japan's High-Technology Industry, Technovation 23 (11) 879-900, 2003.

[2] Meyer, P.S., Bi-Logistic Growth, Technological Forecasting and Social Change 47 (1) 89- 102, 1994.

[3] Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V., Mishchenko, E.F., The Mathematical Theory of Optimal Processes, Interscience, New York, 1962.

[4] Krasovskii, A.N, Assessment of the Impact of Aggregated Economic Factors on Optimal Consumption in Models of Economic Growth, Interim report IR-06-050, IIASA, 2006.

[5] Watanabe, C., Moriyama, K., and Shin, J.H., Functionality Development Dynamism in a Diffusion Trajectory: *A Case of Japan's Mobile Phones Development*, Technological Forecasting and Social Change, 2008 (in print).